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INTERMEDIATE PHYSICS

by

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INTERMEDIATE PHYSICS

Introduction

1. Study of Nature.—Hold a piece of stone under the surface of water. When let free, it sinks to the bottom. If you were asked, "Would another piece of stone sink to the bottom or rise to the surface when let free in water ?" you would say at once, "It will sink." On being asked what makes you say so you would say that millions of pieces of stone have at different times been tried and not one of them has ever been known to rise to the surface. There is no reason why this untried piece should behave differently.

The above illustration is only a particular case of the general principle that *what the forces of Nature have been found to do once, they will, under similar circumstances, do again.* This principle is known as the **Uniformity of Nature.**

The principle just stated means in other words that the actions of the forces of Nature are governed by certain laws called the *Laws of Nature.* The discovery of these laws formed the basis of human knowledge. By careful observation and close reflection on the causes of natural phenomena, the mass of knowledge gained was classified into a system which was called *Natural Philosophy.*

The ancients included in the Natural Philosophy, *Astronomy, Botany, Zoology, Chemistry, Medicine, Mechanics,* and even *Astrology.* Nowadays the term is very much restricted in its scope. It stands only for that branch of science of matter and energy which deals with the laws expressing the relation between the physical phenomena and their causes. This branch is now usually called *Physics.* It is divided into *Mechanics Heat, Light, Sound, Frictional Electricity, Magnetism, Current* and *Electricity.*

Before we deal with these branches we shall briefly explain what we mean by matter.

2. Matter.—Whatever occupies space and can affect one or more of our senses is called *Matter.* Wood, stone, iron, oil, steam etc., are all different kinds of matter. Each different kind of matter is called a *substance,* while a particular portion of a substance is called a *body.* The quantity of matter in a body is called its *mass.*

Matter is found to exist in three states. In the air we breathe, the water we drink, and the bread we eat, we have examples of these states, which are called gaseous, liquid and solid. From our everyday experience we get a fairly good idea of the difference between the different states. We know, for instance, that a solid does not change its shape or size, nor can its particles be moved apart. A liquid, on the other hand, has no shape of its own, it merely takes up the shape of the vessel containing it. It can easily be stirred, which shows that its particles can easily be moved away from each other. But it has a volume of its own that cannot be changed unless it is subjected to a considerable pressure.

A gas has neither shape nor volume of its own. It not only takes up the shape of the vessel containing it but also fills it up completely. The hand can be moved far more freely in it than in a liquid, which shows that the particles of a gas can move about far more easily than the particles of a liquid.

3. Structure of Matter.—The question arises, how can we explain this difference in the behaviour of the various states of matter ?

All bodies can be split up into smaller and smaller parts, and if they be continuous in structure, there ought to be no limit to their divisibility. But facts like compression of a gas with increase of pressure or diffusion of liquids and gases or contraction and expansion of bodies with change of temperature lead us to the conclusion that all substances are composed of minute particles of matter which do not touch each other. These minute particles are called *molecules*. We define a *molecule* as the smallest particle of a substance which can exist in a free state as such. Each molecule is composed of *atoms*. In simple cases like the molecules of hydrogen, oxygen, table salt, each molecule is composed of two atoms, whereas in other cases it is composed of many atoms. (In the case of blood, a molecule is composed of about two thousand atoms.) By an *atom* we mean the smallest portion of an element which can take part in a chemical reaction. In other words, *a molecule is the physical unit* and *an atom, the chemical unit*.

Since the odour of camphor, musk, etc., can be detected at a considerable distance it is obvious that the molecules break away from their surface and fly about. If a piece of lead is placed on a thin sheet of gold, after some time, the molecules of gold are found to penetrate into the lead showing thereby that the molecules of gold must be in motion. The diffusion of liquids and the expansibility of gases lead us to the conclusion that the molecules of liquids and gases also are in motion.

The hypothesis that bodies are made of molecules which are in motion enables us to explain the differences in the behaviour of different states of matter.

In solids the motion of a molecule is restricted to a limited space, and although it is in constant vibration, its position with respect to the other molecules is relatively fixed. It seldom leaves altogether its mean position.

In liquids a molecule is free to move in any direction, but it is so close to the other molecules that it is never completely free from their influence.

In gases the average distance between molecules is very large as compared with the dimensions of a molecule, and this results in almost complete absence of cohesion. Consequently a molecule is quite free in its motion. It flies about in all directions with a great velocity. On account of its motion it collides very frequently with other molecules. It is on account of this free motion of the molecules that a gas always endeavours to expand. When the molecules strike against the walls of the vessel, they exert force (*i.e.*, pressure) on the walls.

Perhaps it will be interesting for the student to know that the number of molecules in one cubic centimetre of hydrogen at atmospheric

pressure and at 0°C. is 27×10^{18} and that they fly about with a velocity of 1840 metres per second (or about 4,000 miles per hour). Each one of these molecules suffers about 10,000,000,000 (*i.e.*, 10^{10}) collisions per second.

It may be mentioned here that molecules are so small that they cannot be seen even with the help of best microscopes. The size of the molecules of different substances is different, but on the average the diameter of a molecule is equal to $\frac{1}{2500000}$ millimetre. To realize how small this size is, imagine a drop of water to be magnified to the size of the earth, each molecule would now be of the size of a football.

4. Measurement of Quantities.—Accurate knowledge about physical phenomena has been acquired chiefly through careful measurement of different quantities. As a matter of fact measurement is the very basis of Science. Maxwell says :

"The most important step in the progress of every science is the measurement of quantities. Those whose curiosity is satisfied with observing what happens have occasionally done service by directing the attention of others to the phenomena they have seen ; but it is to those who endeavour to find out *how much* there is of anything that we owe all the great advance in our knowledge."

Every measurement is essentially a comparison. A quantity to be measured is compared with another quantity of the same kind, called the *unit* ; and to convey a complete notion about the given quantity to others we need only say that it is so many units. In other words, a complete statement of the measurement of a physical quantity consists of two parts ; a pure number, which states the number of times the unit is contained in the given quantity ; and the name of the unit.

Let us take an example. You are asked to write to a Japanese student in Tokyo the size of your science laboratory. To do this you must first choose the unit in which you wish to express the dimensions and then state the number of units which its length and breadth contain. Suppose you write to him that your laboratory is 30 ft. $\times$ 60 ft. He will understand you completely only if he understands what a foot is. Otherwise you will have to explain to him the connection between the foot and the unit of length with which he is familiar. If you want to avoid such explanations you must use international units. Fortunately, for all the physical quantities with the measurement of which we are concerned in this book the international units have been fixed upon. Before we consider them it will be better to note that since a quantity is measured in terms of a unit of a similar quantity, we must have *at least* as many different units as there are different kinds of physical quantities. As a matter of fact we have many more, because there are so many units for each kind of physical quantity. It would be impossible to remember all these units. Fortunately, they are not independent. It is found that if we select the units of length, mass and time we can fix the magnitude of the size of the units of other quantities. The units of length, mass, and time are, therefore, called the *fundamental units* and those of other quantities the *derived units.* The unit of area, for instance, is a derived unit, for it is the area of a square whose side is of unit length. The unit of volume is the volume of a cube, each side of which is of unit length and so on.

In selecting the unit it should be borne in mind that it must be unalterable and easily available for comparison with other standards which are supposed to be its copies. Bearing this in mind it will be clear why it is necessary that the standard unit must be kept in custody somewhere at a safe place.

We shall now discuss the fundamental units.

5. Units of Length.—In the British Empire the unit of length is the **yard**, which is defined by an Act of Parliament as follows :—

"The straight line or distance between the centres of transverse lines on the two gold plugs in the bar deposited in the office of the Exchequer shall be the genuine standard yard at 62° Fahrenheit and if lost, it shall be replaced by its copies."

The copies referred to are preserved at the Royal Mint, the Royal Society, the Royal Observatory, and the Houses of Parliament.

The yard is sub-divided into three equal parts, each of which is called a **foot** (ft). The foot is further sub-divided into 12 equal parts called **inches.**

For measuring large distances a multiple of the unit yard like a furlong or a mile is used.

The yard is used in the British Empire by engineers and commercial people only. In scientific work the unit employed is the **metre.** This is the international unit.

It was defined by a law of the French Republic in 1725 as the distance between the centres of two lines engraved upon the polished surface of platinum-iridium bar at the temperature of melting ice. The bar is deposited in the national archives at Sevres near Paris.

The metre has its multiples and sub-multiples as follows :

1 kilometre equal to 1000 metres.

1 decimetre ,, ,, $\frac{1}{10}$th of a metre.

1 centimetre ,, ,, $\frac{1}{100}$th of a metre.

1 millimetre ,, ,, $\frac{1}{1000}$th of a metre.

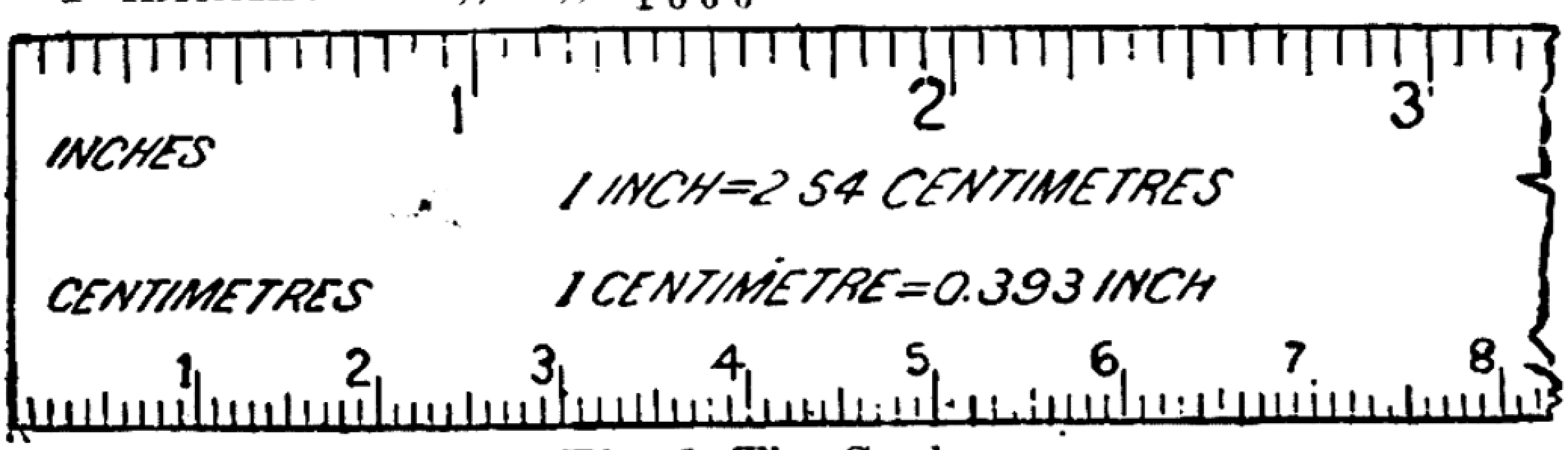


Fig. 1. The Scale.

Comparison of the metre with the yard shows that

$$1 \text{ metre} = 1{\cdot}094 \text{ yards} = 39{\cdot}37 \text{ inches}$$

and $$1 \text{ inch} = 2{\cdot}54 \text{ centimetres.}$$

To measure very small lengths, like the wavelengths of light, a special unit called an **angstrom** (written A°) is used. It is equal to 1×10^{-8} cm. This length is comparable with the size of an atom. Visible light has wavelengths ranging from 4000 to 7600 A°.

It may be mentioned here that for the purpose of securing a permanent, unalterable standard of length, the standard metre has been measured in terms of one particular wavelength of Cadmium vapour emitted in an electric discharge, and the value of

$$1 \text{ metre} = 1{,}553{,}164{,}13 \text{ wave lengths.}$$

This shows that the wave length of the cadmium vapour referred to is the ultimate standard of length.

6. Units of Mass.—There are two units of mass corresponding to the two units of length ; one, that is commonly used in the British Empire, and the other, the international unit, that is used in scientific work. The first unit is called the **Pound**. It is the mass of a lump of platinum kept at the Standards Office at Westminster. It is sub-divided into 16 parts called ounces (*oz.*); for heavy masses, multiples of this unit like the hundredweight, and the ton are used. A hundredweight=112 lb., and a ton*=2240 pounds.

The second unit is called the **gram**. It is one thousandth part of the mass of the piece of platinum-iridium called the "Kilogramme-International", which is kept at Sevres near Paris.

The gram has its multiples and sub-multiples as follows :

1 kilogram equal to 1000 grams.
1 decigram ,, ,, $\frac{1}{10}$th of a gram.
1 centigram ,, ,, $\frac{1}{100}$th of a gram.
1 milligram ,, ,, $\frac{1}{1000}$th of a gram.

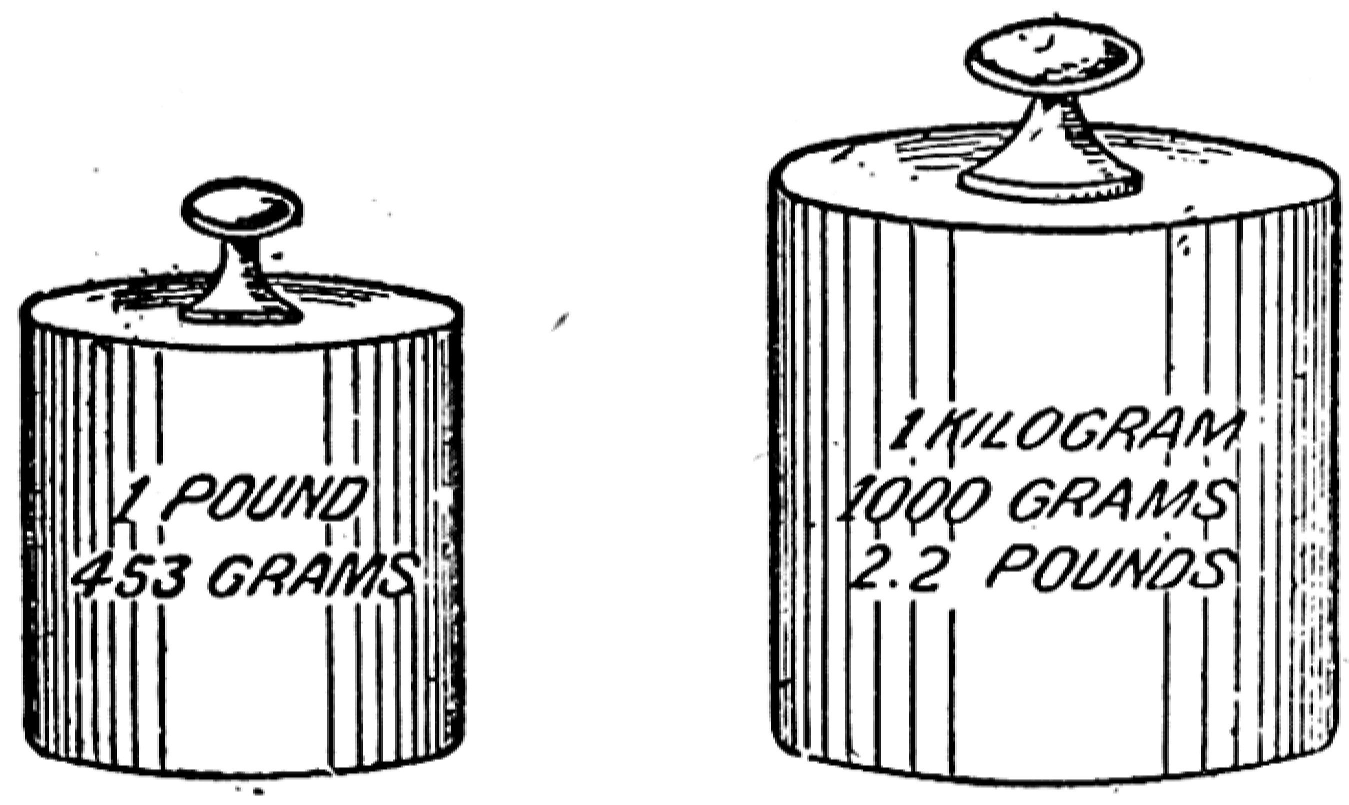


Fig. 2. The Pound and Kilogram.

A kilogram is approximately equal to 2·2 pounds.

The student will do well to remember that 1 cubic foot of water weighs 62·4 lb., 1 litre weighs 1 kilogram, 1 gallon 10 lb. and 1 pint 568 grams.

A cursory glance at the units mentioned above will show that the International system renders the calculations extremely simple, because to convert or reduce a unit to its multiples or sub-multiples we have to divide or multiply by some power of 10, and this can be done at once by moving the decimal point or adding or taking away the cyphers.

7. Units of Time.—The idea of time in its simplest form is the recognition of the succession of events. The setting or rising of the sun supplies an excellent example of the succession of events, even to a cursory observer. Hence, no wonder that this event has been made use of in measuring time. In actual practice, however, instead of taking the period between two sunrises or two sunsets, the interval between two successive noons is taken as the basis of the measurement of time. Since this interval varies slightly from day to day on account of the motion of the earth round the sun, the average value of this interval is

*In America a ton is equal to 2,000 lbs.

taken as the unit of time. This unit is called the mean **Solar Day.** 1/86,400 th part * of it is called the mean solar second, or for shortness a **second**. The mean solar second is the fundamental unit of time, and it is the same in both the systems.

To sum up, we have two systems of units, the British and the International. In the first system the units are the *foot, pound,* and *second.* This system is briefly called the *F.P.S.* system. In the second system the units are the *centimetre, gram,* and *second.* This system is called the *C. G. S.* system.

8. Measurement of Length, Mass and Time.—To measure length mass and time we have generally to note the position of an index on a scale. For instance, time is measured by noting the position of the hands on the dial of a watch or a clock, mass by noting the position of a pointer on the scale of a balance and length by noting the position of the ends of the object against the graduations of a measuring scale. In all these cases the head should be placed in the correct position *viz.,* directly above the scale or in front of it as the case may be, otherwise an error known as parallax error will be introduced.

Measurement of Length.—It is only seldom that a length to be measured is equal to an exact number of divisions on the measuring scale ; in general, it is either a little too large or a little too small. For everyday purpose it is enough to measure it to the nearest tenth of the smallest scale division. With a little practice this can easily be done by the eye alone. But when a greater precision is required in reading, the fractions, as is often the case in scientific work, we use a device called *Vernier.* It consists of an auxiliary scale which slides along the main scale. In Fig. 3 is shown an instrument in which vernier is used for measuring fractions accurately. It is called *Vernier Calipers* or *Sliding Calipers.* It consists of a steel bar with a scale *S* engraved on it. The bar is furnished with two jaws, *A* and *B*, projecting at right angles to it. The jaw *A* is fixed but the jaw *B* can slide backwards and forwards and has a scale *V* marked on it. *S* is the main scale and *V*, the vernier

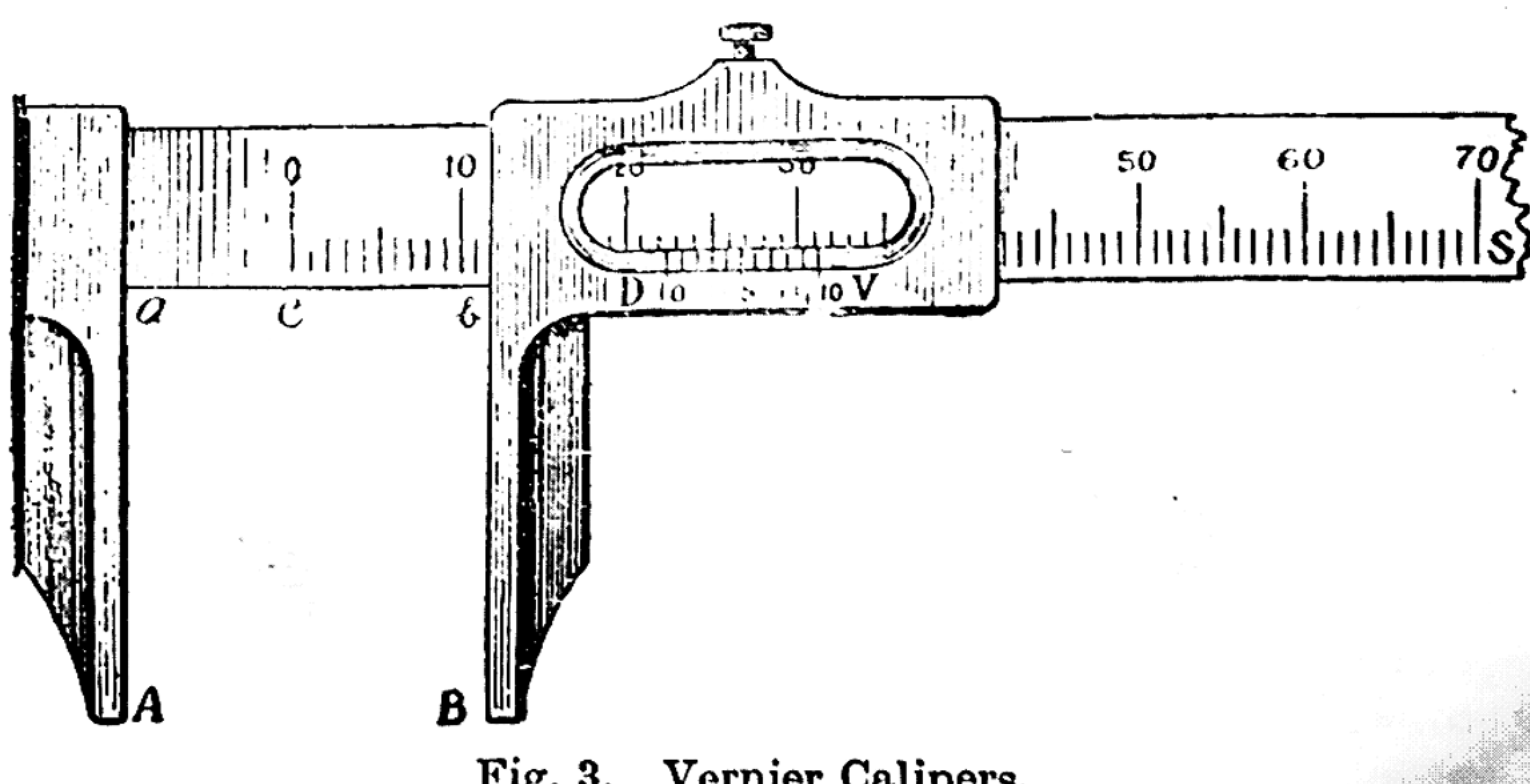


Fig. 3. Vernier Calipers.

scale. In general the vernier is divided into 10 parts although in some cases it may be divided into 20 or more parts. The vernier divisions

* The figure 86,400 comes from dividing the day into 24 hours and the hour into 60×60 *i.e.* 3600 seconds.

are either a little larger or a little shorter than the divisions of the main scale ; usually they are slightly shorter. Let us see how a length is measured with sliding Calipers.

Move the jaw B towards A until A and B touch each other and notice that the zero of the vernier scale is opposite the zero of the main scale.* This means that the distance between the jaw A and zero on the scale (*i e.*, aC) is equal to the distance between the jaw B and zero on the vernier (*i e.*, bD) and hence the position of zero of the vernier on the scale S gives the length to be measured.

Place the object whose length is to be measured between the jaws A and B and push the sliding jaw B towards A until the jaws touch the object lightly on both sides and read the division opposite the zero of the vernier. If it is opposite a scale division the length of the object is equal to an exact number of divisions and if not, the length is equal to a certain number of divisions *plus* a fraction of a division.

To understand how a fraction of a division is read with the help of the vernier, take two strips of cardboard, one 20 cm. in length and 5 cm. in width and second 10 cm. long and 2 cm. wide. Draw on the bigger piece a scale such that each division is equal to 1 cm., and on the smaller a scale such that its 10 divisions are equal to 9 cm. in all.

Put the smaller strip against the longer as shown in Fig. 4 so that its zero is opposite the 5th division of the main scale and its

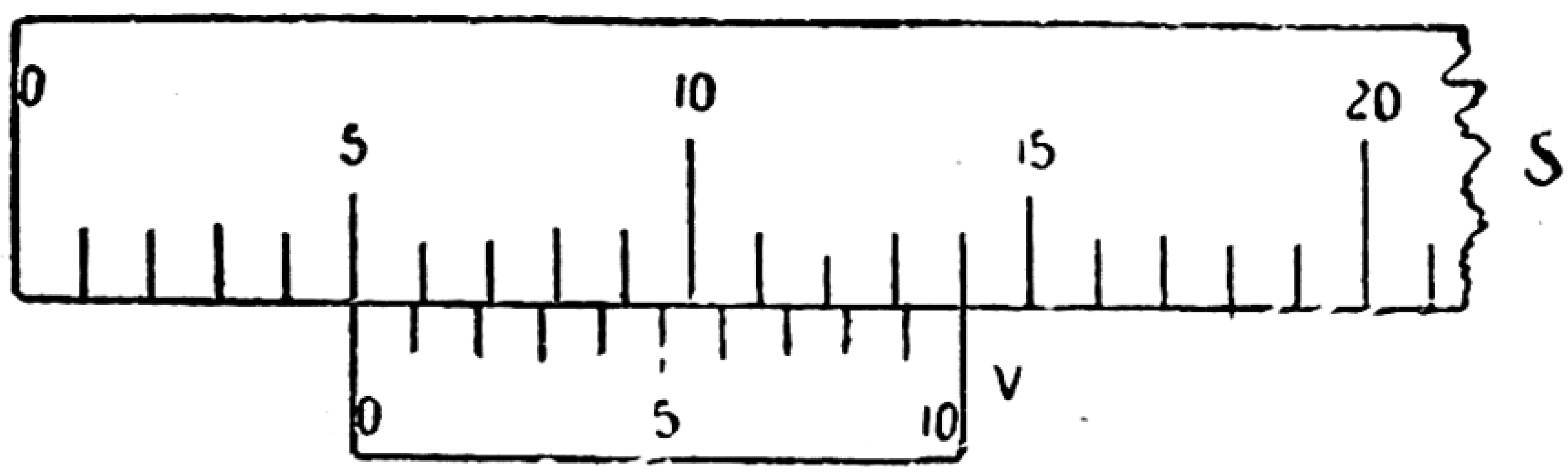


Fig. 4. Principle of Vernier.

10th division is opposite the 14th of the scale. Since 10 divisions of the smaller scale, called vernier divisions, (written for brevity $V.D.$) are equal in length to nine scale divisions (written as $S.D.$), 1 $V.D.$ is equal to 0·9 $S.D.$ or the difference in length of 1 $S.D.$ and 1 $V.D.$ is equal to 0·1 $S.D.$ We can write it as

$$1\ S.D. - 1\ V.D. = 0{\cdot}1\ S.D.$$

This difference is called the *Vernier Constant* or the *least count* of the Vernier.

From the above example, it follows that if n vernier divisions were equal in length to $n-1$ Scale divisions, each $V.D.$ would be equal to $\frac{n-1}{n}$ or $\left(1-\frac{1}{n}\right)$ $S.D.$, the least count of the vernier would be equal to $\frac{1}{n}$ of the $S.D.$

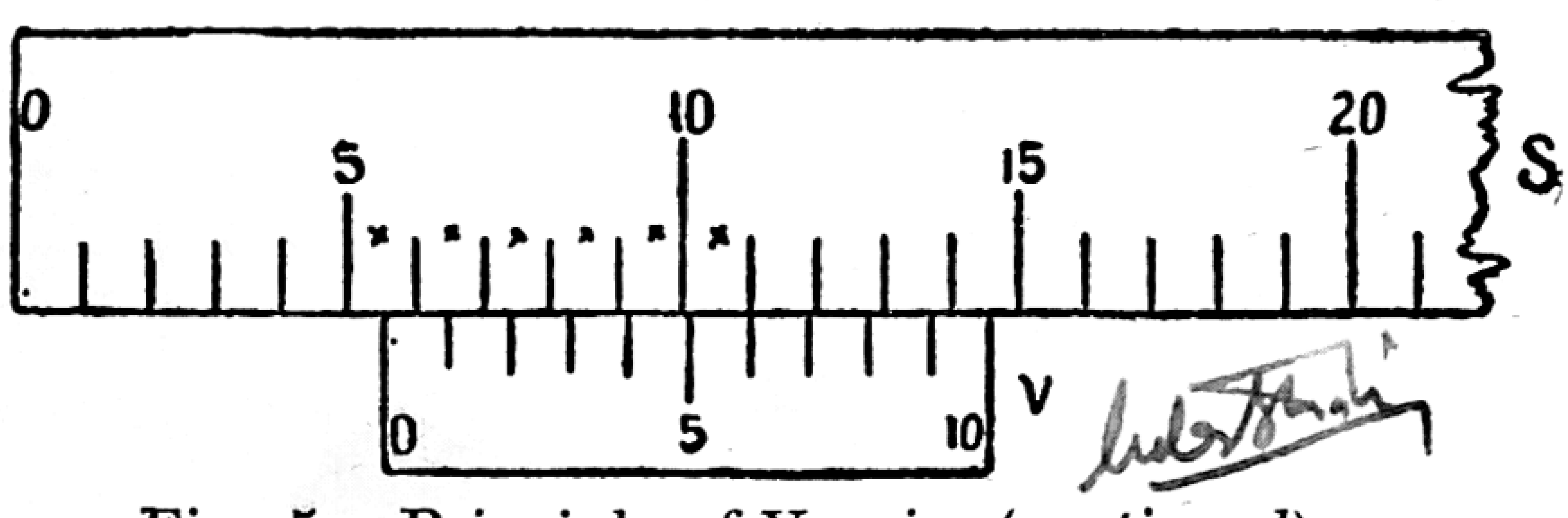


Fig. 5. Principle of Vernier (*continued*).

* We suppose that there is no zero error in the instrument.

Now move the smaller strip a little forward so that the zero of the vernier is a little farther off than the 5th division of the scale (Fig. 5). Let us read this fraction. Call it x. It is obvious from the figure that the 6th $V.D.$ is opposite the 11th scale division. The total length consisting of the fraction plus 6 vernier divisions is equal to the length of 6 scale divisions (from 5th to 11th, marked with crosses). Writing it in the form of an equation we get :—

$$x+6V.D.=6S.D.$$

or $$x=6(1S.D.-1V.D.)=6\times\text{vernier constant}$$

Since vernier constant $=0{\cdot}1$ $S.D.$

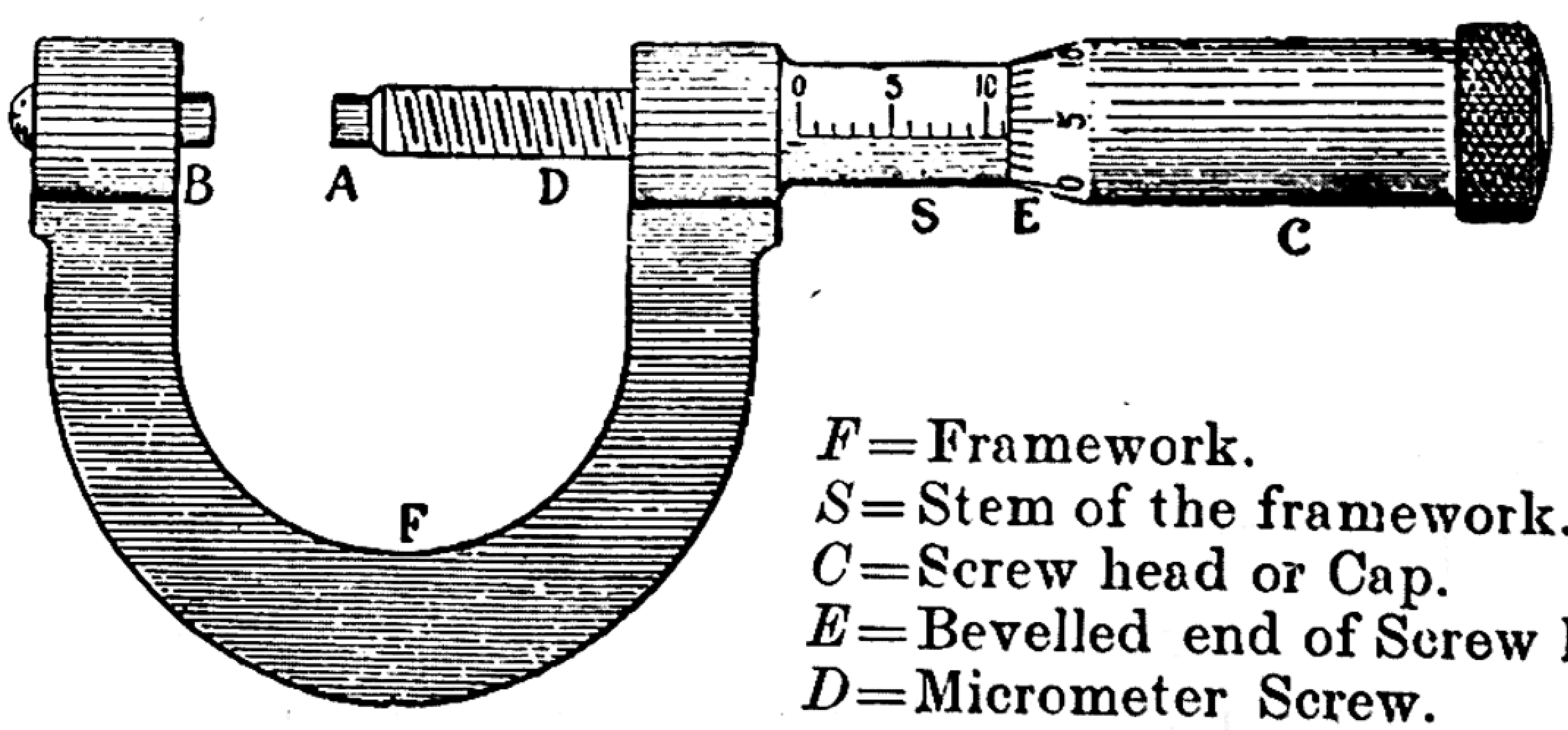
$$\therefore \quad x=0{\cdot}6 \; S.D.$$

If the vernier constant had been equal to $\frac{1}{n}$ of the $S.D.$ the fraction would have been $\frac{6}{n}$ of the scale division. If in place of the sixth division any other division say mth had come opposite a scale division the fraction would have been $\frac{m}{n}$ of $S.D.$

Hence the rule to read a fraction is to *multiply the number of the vernier division which coincides with a scale division by the vernier constant.*

We have taken for granted that when jaw B is pushed backwards to come in contact with A, the zero of the vernier comes opposite the zero of the scale ; if that is not the case the instrument has a **zero error**. In such a case first find the zero error and subtract it from all subsequent readings to get the true length of the object.

Micrometer Screw Gauge.—Ordinary sliding calipers read to the tenths of a millimetre ; if greater accuracy is required in measuring small lengths like the diametres of wires, the micrometer screw gauge or as it is briefly called the screw gauge is used. A common form of this instrument is shown in Fig. 6. It consists of screw D which works in



F=Framework.
S=Stem of the framework.
C=Screw head or Cap.
E=Bevelled end of Screw head.
D=Micrometer Screw.

Fig. 6. Screw Gauge.

a nut fixed inside the cylindrical portion S called stem of the frame F. To the other end of the frame is fixed a steel plug B, with a carefully planed face. The plug B and the end A of the screw D, which is also planed, serve as two jaws. On the stem is engraved a line

parallel to the axis of the screw, and is divided either into millimetres or half millimetres according as the pitch of the screw *i.e.*, the distance through which the screw moves in one complete revolution, is 1 millimetre or half millimetre. The screw is fixed on the right-hand side to the cap or screw head C. The end E of the screw head is bevelled and is divided into 50 or 100 parts. If the pitch of the screw is half a millimetre and the circumference of the screw head is divided into 100 parts, one division of the screw head corresponds to $\frac{1}{100}$th of half a millimetre, or to $\frac{1}{200}$ or 0·005 mm. This shows that we can easily read lengths up to $\frac{1}{200}$th of a millimetre with a screw gauge. The smallest length which a screw gauge can read is called the *least count* of the instrument. To use a screw gauge first find the pitch of the screw and the least count of the gauge and then see if there is any zero error in the instrument, *i.e.*, see if the zero of the scale on the screw head comes opposite the zero on the stem when the two jaws are in contact. If it does not, take the reading on the screw head which is opposite the zero on the stem. This is a zero error. It must be subtracted from all subsequent readings.

Now place the object whose length is to be measured between the jaws A and B and slowly turn the screw head till the object is *just* held. The reading on the stem S shows the number of complete revolutions and the reading on the head E gives the fraction of the revolution.

It is important to remember that the jaws should just hold the object ; they should not be screwed up tightly because otherwise not only will the readings differ widely but the instrument will also be damaged. To avoid this extra pressure or straining some gauges are provided with a *ratchet* arrangement, which on being turned ordinarily advances the screw but when the gap is closed the ratchet turns on itself without moving the screw.

Spherometer.—Another application of the screw to measure small lengths or thicknesses is met with in the spherometer. It consists of a metal frame standing on three pointed legs of equal length and arranged at the corners of an equilateral triangle. A fine and accurate screw pointed at its lower end passes through a nut fixed at the middle of the frame as shown in Fig. 7.

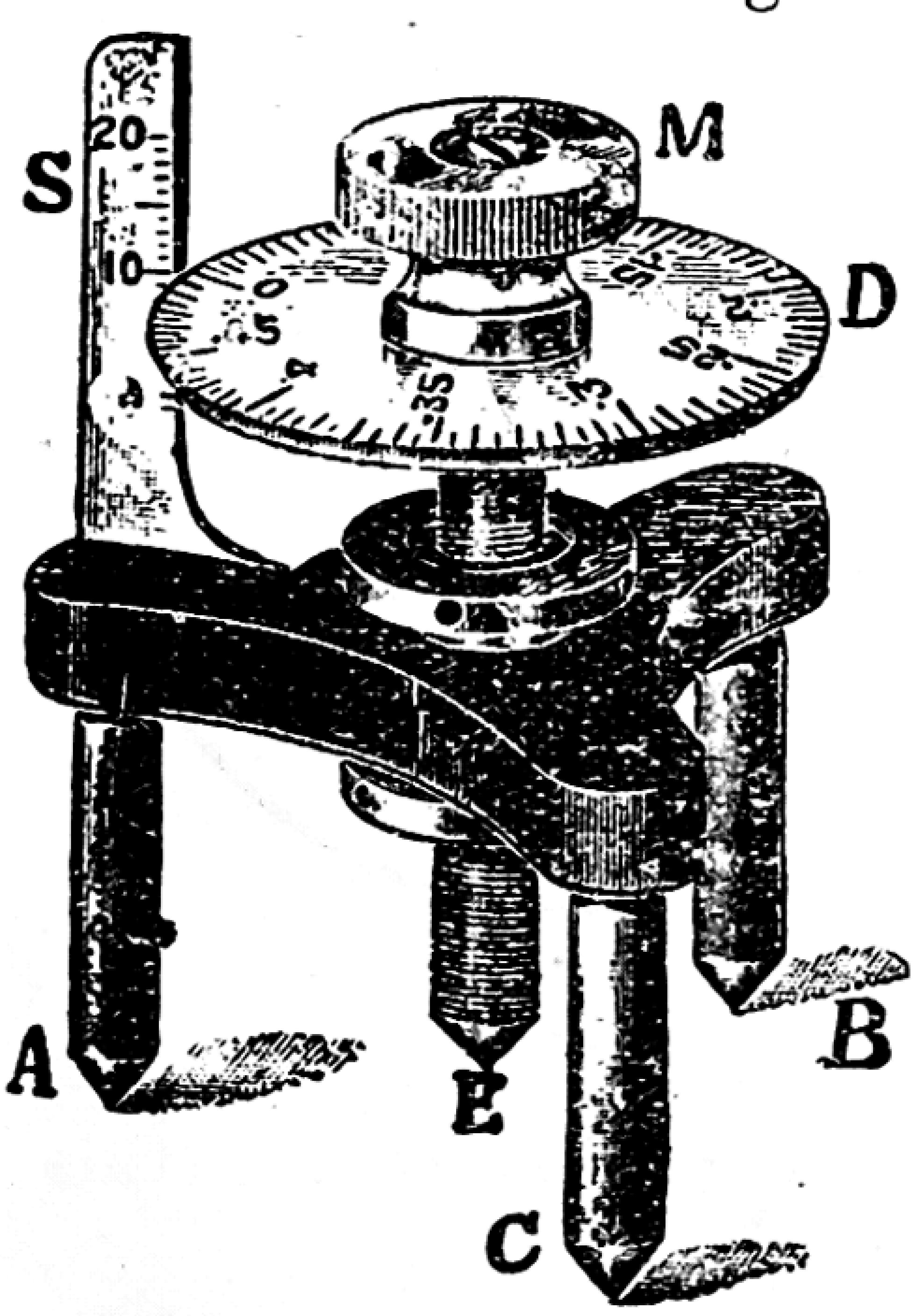


Fig. 7. Spherometer.

The screw forms, so to speak, the fourth leg and carries at its upper end a graduated disc D and ends in a milled head M. The position of the central leg can be read by means of a vertical scale S fixed to one of the outer legs. The vertical scale is divided into millimetres or half millimetres according as the pitch of the screw is 1 or 0·5 millimetre. If the pitch be 0·5 mm. and the disc be divided into 100 parts, one division of the disc scale corresponds to $\frac{1}{200}$ or 0·005 of a mm. The smallest length which a spherometer can read is called its *least count.*

To find the thickness of a thin object, say a cover glass, place the spherometer on a plane glass plate and move the screw upwards. Place the cover-glass below the lower end of the screw and move it down till its end just touches the upper surface of the cover-glass. Note the reading on the vertical scale as well as on the disc. Now remove the cover-glass and move down the screw till its end touches the glass plate. Again take readings on the vertical scale and the disc. From these two sets of readings the thickness of the cover-glass can be calculated.

The reading on the vertical scale gives the total number of revolutions and the reading on the disc the fraction of a revolution.

The spherometer can also be used to determine the radius of curvature of a spherical surface. To do this place the spherometer on the surface and screw up or down the central leg until it just touches the surface. It will do so when the spheromter *just* begins to revolve round the central leg. In this position take readings on the scale S and disc D. Now place the spherometer on a plane glass plate and take the readings when the central leg just touches the plate. From these two sets of readings find the height or depth as the case may be and calculate the radius of curvature R with the help of the formula,

$$R=\frac{a^2}{2h}+\frac{h}{2},$$

where h = height or depth of the point E, and
a = distance of the central leg from any one of the outer legs.

Proof.—Let $ALNM$ represent the sphere of which the given spherical surface forms a part. Imagine a vertical section passing through one of the legs say, A, the central leg E and the centre O of the sphere. It will be a circle as shown in Fig. 8. The third leg B of the spherometer is not visible in this position. E is the position of the lower end of the central leg when it touches the spherical surface and D, where it would touch a flat surface. DE represents h, AD represents a and OE or ON represents R.

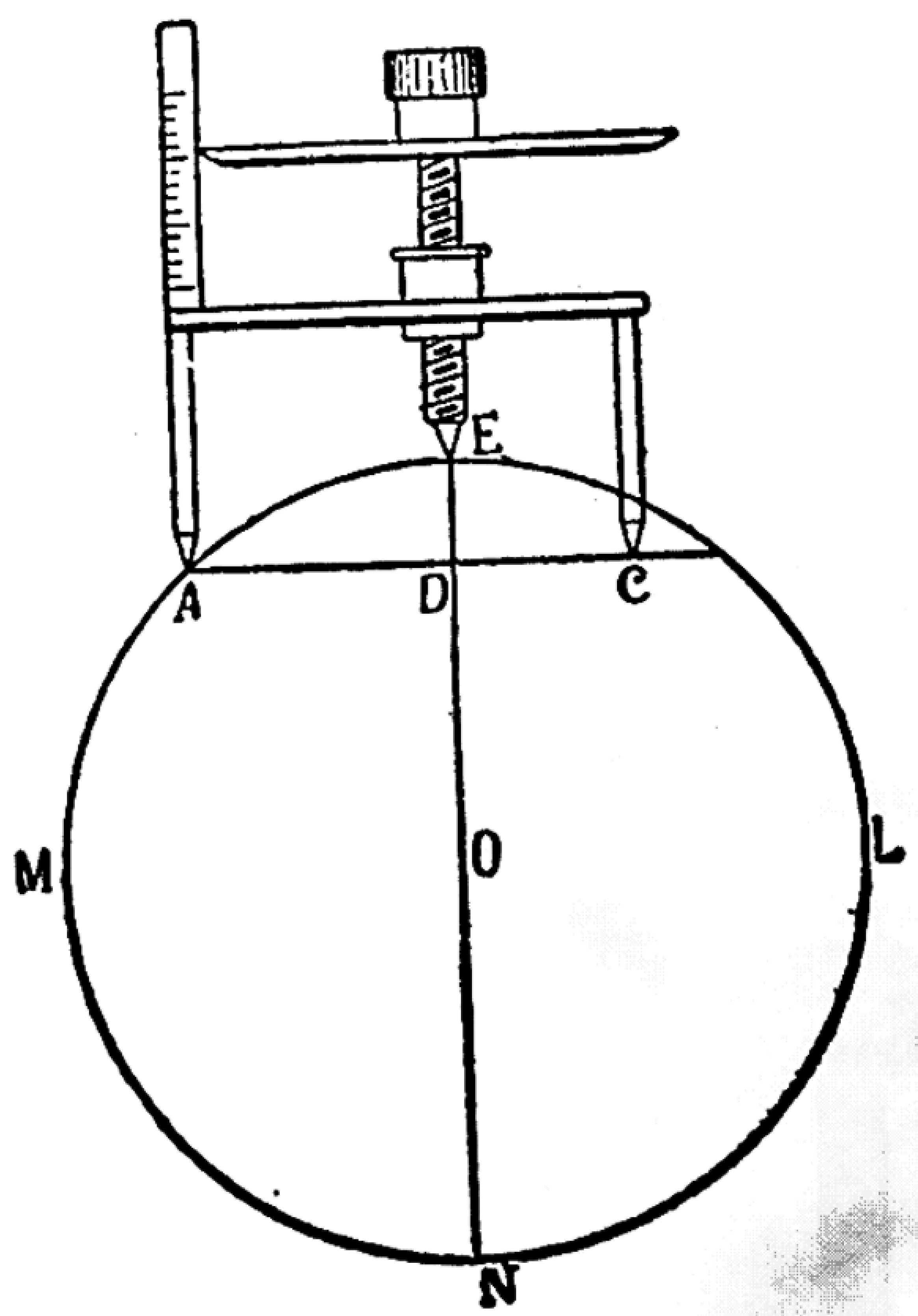


Fig. 8.

From the geometry of the figure we have

$$AD^2=ED.DN$$
$$=h(2R-h)$$

or $$a^2=2Rh-h^2$$

or $$2Rh=a^2+h^2$$

or $$R=\frac{a^2}{2h}+\frac{h}{2}.$$

It can be proved easily that

$$a=\frac{l}{\sqrt{3}},$$

where l is the distance between any two outer legs; substituting this value of a we get

$$R=\frac{l^2}{6h}+\frac{h}{2}.$$

If we neglect $\frac{h}{2}$ the relation simplifies to $R = \frac{l^2}{6h}$ or $\frac{a^2}{2h}$. For practical purposes this approximate value of R is sufficiently accurate.

Measurement of Mass.—The mass of a body is usually measured by means of a balance. As to what type of a balance should be used depends upon the mass of the body and also upon the accuracy required. Ordinary balances are not very accurate, hence in scientific work, where we generally require an accurate measurement of mass, we use special balances called *analytical balances*. They enable us to measure the mass accurately to a fraction of a milligram. These balances are fitted inside glass cases to protect them from dust and draught. To measure the mass of a body, we place the unknown body in one pan and standard weights in the second and allow the beam to swing. The weights are changed till the pointer moves over an equal number of divisions on either side of the zero position. When it does so, the mass of the body is equal to the mass of the standard weights. We have supposed here that the balance is a true one. If it be not so, to find the true weight we have to use special methods. We shall discuss them as well as the theory of the balance later on.

Measurement of Time.—Time is ordinarily measured by means of clocks or watches, which are machines constructed to run with great uniformity. For measuring short intervals of time we generally use stop-watches or stop-clocks which are so called because they can be started and stopped at will by pressing a lever. For accurate work the stop-watch or clock should first be compared with a standard clock called the chronometer. Ordinary stop-watches enable us to measure time up to $\frac{1}{10}$th of a second but it is possible nowadays to get pocket stop-watches which read up to $\frac{1}{50}$th or $\frac{1}{100}$th of a second. In such watches the second hand makes one revolution every second. To measure extremely short intervals we require elaborate arrangements and if the student wishes to know something about them he should consult some advanced book on Physics.

PART I

MECHANICS

CHAPTER I

Velocity and Acceleration

9. Formerly Mechanics, which literally means "contrivances", was the science of Machines, for it dealt with the devices which were helpful in lifting or moving heavy bodies. Nowadays this branch of science is called applied mechanics, and by **Mechanics** we understand that branch which deals with the state of rest or motion of bodies. It is also defined as *that branch of science which deals with the action of force or forces on matter.* Before beginning the study of mechanics, let us first understand what is meant by Force.

10. **Force.**—When we push or pull, we say we are exerting force. For instance, suppose we lift a weight of 20 seers from the ground. The earth pulls the weight downwards, we pull it upwards. If our pull is greater the weight rises upwards. While lifting the weight we experience a certain sensation. It is this sensation which gives us the idea that we are exerting force. In this example the effect of our pull is to produce motion in a body. Now let an inanimate agent which cannot exert force also produce motion in the given weight. Since the effect produced is the same, we infer that the cause also is similar. Hence the inanimate agent is said to exert force. For example, the pull of the earth brings the weight of 20 seers down when it is released, *i.e.*, it produces motion in it just what we did when we exerted force : hence the earth is said to exert force on the weight. Take next the case of an archer who bends his bow and shoots an arrow. Here motion is produced in the arrow by the bow, and hence the bow is said to exert force on the arrow. The force may not necessarily produce motion, it may simply tend to do so : for instance, the arrow, so long as it is held by the hand, does not fly off although the bow is exerting force all the time.

The force may not, however, always produce or tend to produce motion ; it may, on the other hand, stop or tend to stop motion. For instance, when we hold a weight in our hand the earth tends to produce motion by pulling it down, but the force exerted by our hand stops it. If a weight heavier than the one we can support be placed in our hand, our effort will tend to support it, but will be unsuccessful. Even in this case our hand is exerting force. These examples show that we can recognize force not only when it produces or tends to produce motion, but also when it stops or tends to stop motion. Hence *Force is described as a push or pull which produces or tends to produce, stops or tends to stop motion.*

11. Motion of a Body.—A body is said to be in motion if it changes its position, whereas it is said to be at rest if its position remains unchanged. Motion, hence, is change of position. Rest and motion are both *relative*. A motor car is said to be at rest if it remains stationary in the garage day after day. The man sitting in a moving car is at rest with respect to the car if he does not move in it ; although actually he is sharing the motion of the car with respect to the earth.

Similarly by saying that the car is at rest in the garage, we mean that it does not move with respect to the house in which it is kept. But the house itself is sharing the motion of the earth, which is moving round the sun.

Thus, all that we mean when we say that a body is at rest is that it is not changing its position with respect to its surrounding objects. In other words *rest is relative* only. Motion also is relative, because in the case of the moving car it is only its motion with respect to the earth that we know ; but surely that is not its real motion because the earth itself is moving with respect to the sun. In other words we do not know the *absolute* motion of a body : all that we know is that it is moving at such-and-such speed with respect to certain bodies regarded as fixed.

12. Velocity.—To describe fully the motion of a body we must state not only the rate at which it moves but also the direction. The rate at which a body moves or the *rate of change of its position is called speed*. If it moves slowly, its speed is small ; and if it moves quickly, its speed is great. To specify speed we need not know the direction in which the body moves. Suppose a man is walking on foot along the road running between Jullundur and Amritsar. To know his speed we must know only the distance he travels during a particular time If he goes 4 miles in an hour, we say his speed is 4 miles an hour. But this statement does not completely specify his motion ; for it does not state whether he is going from Jullundur to Amritsar or from Amritsar to Jullundur. To give a complete idea we must mention which way he is going. Both these facts are included in the word *Velocity, which means the rate at which a body moves along a particular line.* In the above example, if the direction, from Jullundur to Amritsar, is taken as positive, the velocity of the man when he goes from Jullundur to Amritsar is plus 4 miles an hour, whereas it is minus 4 miles an hour if he goes from Amritsar to Jullundur. His speed in both the cases is the same but velocity is opposite, for it is positive in one case and negative in the other.

13. Uniform and Variable Velocities.—*If a body moves over equal distances in equal intervals of time, however small, it is said to move with a uniform speed*, and if in addition to the speed the direction also remains the same, it is said to move with a *uniform velocity*. But if, on the other hand, the body moves over unequal distances, its speed, and therefore, velocity, is *variable*. If speed remains the same, but the direction changes, even then the velocity is variable. For instance, an aeroplane flying in a circle might move with the same speed, but its velocity is undergoing a change of direction every moment and hence is variable. It is important to note that the *velocity is variable no matter whether it is the rate of motion that changes or the direction*. While

defining uniform speed or uniform velocity we have used the words equal intervals of time, *however small*, and not equal intervals of time only. To explain fully the significance of the word "however small" we shall consider an example.

Suppose a railway train leaves Amritsar for Calcutta and covers this distance in 40 hours, stopping at important stations. Suppose further that during every 12 hours it passes over 360 miles. Now, if we fix upon 12 hours as our interval of time, we shall have to say that its speed is uniform, for it travels equal distances during these intervals. But actually such is not the case, for not only does it stop at the important stations, but also on the bridges and within the yards of the stations, its speed is less than at other places. If a passenger were to note the time the train takes in going from one milestone to the next for a distance of about 5 miles somewhere between two stations and were to find that it takes two minutes to cover each mile, he would be justified in saying that the train travels with uniform speed over that distance. A still surer test, however, would be to note the time the train takes in going from one telegraph pole to the next for about 8 to 10 poles and if the time be the same, the speed is uniform. Therefore, remember that when we say that a train runs with a uniform speed of 30 miles an hour we mean that it covers one mile in every two minutes, half a mile in each minute and 44 ft. in each second. If, in addition, the direction does not change, the velocity is uniform.

An excellent example of a uniformly moving body is met with in the case of the earth. We know that it rotates on its axis in a period of always the same duration, which is equal to the mean solar day. A point lying on it moves through equal distances in equal intervals, however small. In one minute a point on the equator moves through 17·3 miles.

Let us now consider the relation between the distance passed over, the velocity, and the time.

14. Relation between distance passed over, velocity and time.—If a body moves with a uniform velocity of u ft. per second, it will cover u ft. in each second. In t seconds it will pass over ut ft.

Writing S for the distance passed over from the starting point, we can express what is said above in the form of an equation

$$S=ut \qquad \ldots \qquad (1)$$

We can write relation (1) also as $u=\dfrac{S}{t}$ ... (2)

Relation (2) gives us an expression for the velocity of a moving body.

Caution. It is clear from this relation that *velocity is distance passed over per unit time. It is not correct to speak of it only as so many feet.* We must express it as so many ft. per second or ft. per minute, etc. For brevity, feet per second is sometimes written as ft./sec.

15. Variable Velocity.—When a body traverses unequal distances in equal intervals of time, or when its direction of motion changes, it is said to move with a *variable velocity*. To measure it at any point we

determine the distance passed over in an infinitely small time. Suppose, for instance, the body covers an indefinitely small distance ds, including the point, in an infinitely small time dt, the ratio $\frac{ds}{dt}$ gives the velocity at that point.

If a body moves with a variable velocity and covers a total distance of S ft. in t seconds, $\frac{S}{t}$ is spoken of as the *average velocity* of the body during t seconds. It is such a velocity that a body moving uniformly with it would cover the same distance in t seconds as it does when moving with the variable velocity.

16. Acceleration.—There are very few bodies in this world which move with a uniform velocity; most of them move with a variable velocity. To describe completely the motion of all such bodies we have to deal with the rate at which their velocity changes. For instance, it is not enough to know that the velocity of a train when it starts from a railway station, goes on increasing gradually: we must know the rate at which it increases or, as it is said in Physics, its acceleration. By *acceleration* is meant *the rate of change of velocity*. If the velocity increases, the acceleration is positive, if it decreases, the acceleration is negative. Negative acceleration is sometimes spoken of as *retardation*. Even if the magnitude of a velocity does not change, but the direction changes, the body will have an acceleration.

17. Uniform Acceleration.—Acceleration may be uniform or variable. It is said to be *uniform if the velocity increases by equal amounts in equal intervals of time, however small*. But if, on the other hand, the increase in velocity is not regular, the acceleration is said to be *variable*.

Mark the words *however small*. To explain their importance we shall consider the following examples :—

(1) Suppose we are sitting in a 4-oared boat, rowing 15 strokes to the minute. Let the boat attain a velocity of three miles an hour in 6 strokes, so that during each stroke the velocity increases by half a mile an hour. One stroke takes 4 seconds, hence the velocity of the boat increases in each interval of 4 seconds, by 0·5 mile an hour, and if 4 seconds be our unit of equal interval, the acceleration is uniform; but in reality that is not the case, for the increase in velocity is brought about mostly when the oars are in water. In other words if we fix upon 1 second as our interval the acceleration is not uniform.

(2) Let us next take an example of a falling stone. Its velocity increases regularly, however small the interval may be. This supplies us with an example of uniform acceleration. We shall discuss this case fully later on.

If v be the final velocity, u the initial velocity, t the time during which the change of velocity, $v-u$, takes place, and a the acceleration, we can express uniform acceleration as

$$a=\frac{v-u}{t} \qquad \text{... (3)}$$

Relation (3) can be written as, $v=u+at$ ... (4)

We could obtain this relation directly also. For the velocity of the body is u ft. per second to begin with and after one second it becomes $u+a$: in the next second the velocity increases by a ft. per second again, and hence at the end of 2 seconds it becomes $u+2a$, and so on. After t seconds the velocity v will be equal to $u+at$.

It is clear from relation (3) that *acceleration is the change in velocity brought about in a unit time.* The student at this stage must understand clearly the full significance of acceleration as the change in velocity introduced in a unit time. Suppose a mail train moving at the rate of 30 miles an hour has its velocity increased to 45 miles an hour. Evidently the velocity has increased by 15 miles an hour. But in what time, we do not know. It may have occurred in two minutes or three minutes or in any other interval of time. Since we do not know the time in which the increase in velocity of 15 miles an hour has been brought about, we cannot find the acceleration. But if it is given that the above increase of velocity is brought about in one minute, let us find the acceleration.

Since in one min. the velocity increases by 15 miles/hour ;

in one sec. it „ „ 15/60 miles/hour ;

or „ „ „ „ „ „ $\frac{1}{4}\times 1760\times 3$ ft./hour ;

„ „ „ „ „ „ „ $\frac{440\times 3}{60}$ ft./min ;

„ „ „ „ „ „ „ $\frac{440\times 3}{60\times 60}$ ft./sec.

This is the value of acceleration by our definition. In place of writing that in one second the increase in velocity is $\frac{440\times 3}{60\times 60}$ or $\frac{11}{30}$ ft. per sec., it is usual to write that the acceleration is $\frac{11}{30}$ ft. per sec. per sec. For brevity per sec. per sec. is often written as $\frac{1}{\text{sec.}^2}$.

Caution. The student should thoroughly grasp the meaning of the words *per second per second.*

It is absurd to talk of acceleration as so many feet per second, for it would then be velocity only. The unit of time must come twice.

18. Velocity-time Graph.—The nature of motion of a body can be more clearly visualised by means of graphs than by means of the formulæ proved above. Let us, for instance, consider the motion of a train whose velocity at successive times is given in the following table :

Time in seconds	0	5	10	15	20	25	30
Velocity in ft./sec.	0	11	22	33	44	44	44

Take any point O as origin and represent the velocity along the

axis of Y and the corresponding times along the axis of X, and plot the points which indicate the velocity of the train at different times. The curve passing through these points is called the *velocity-time* Graph. Such a graph shows at a glance how the velocity changes with time The graph in the above case is seen to consist of two parts, OB and BC. The first part shows that the velocity increases with time. The ratio $\frac{AD}{OD}$ or $\frac{BE}{OE}$ is called the slope of the curve. Since AD gives the increase in velocity in time OD, $\frac{AD}{OD}$ gives the rate of change of velocity, *i.e.*, acceleration. Hence we learn that the *slope of the velocity-time curve gives the acceleration of the moving body.*

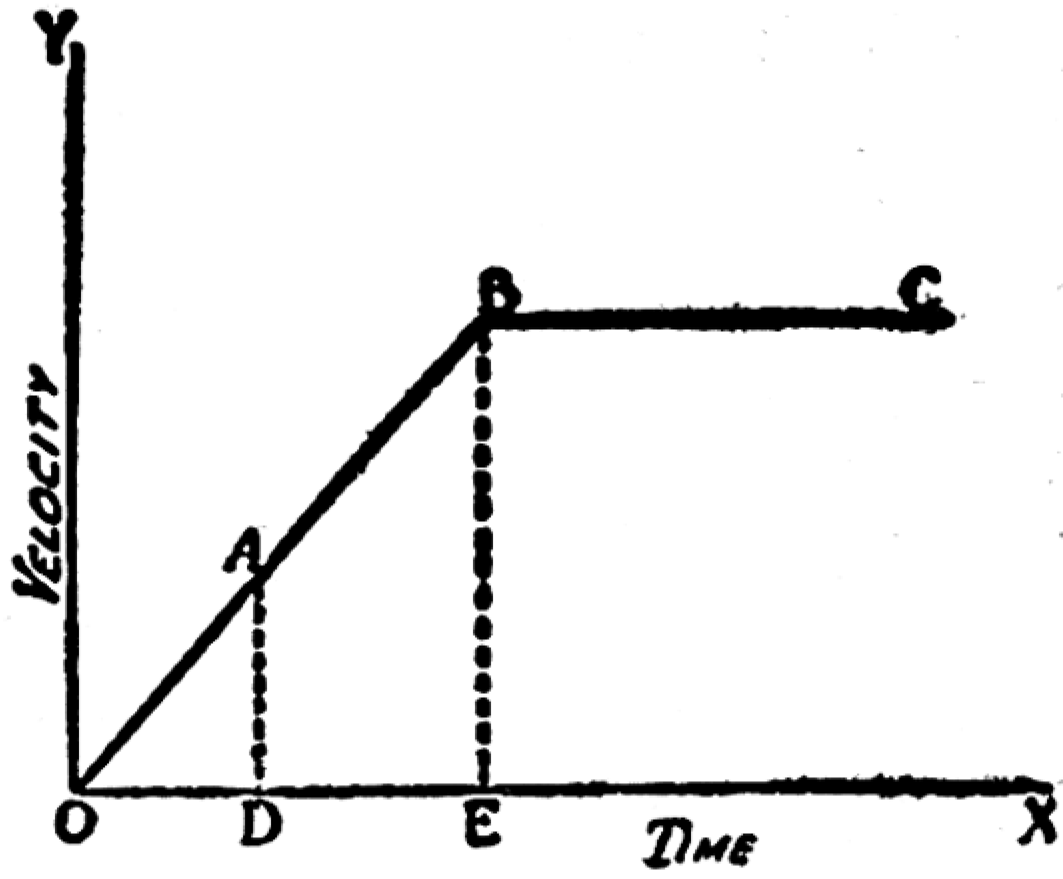


Fig. 9.

Since $\frac{AD}{OD}=\frac{BE}{OE}$, it is clear that the acceleration is constant in the first part of the curve. During this time the body moves with a uniform acceleration.

In the second part the curve is parallel to the time axis which indicatesi that there is no change in velocity with time, *i.e.*, the train moves with a uniform velocity.

If the acceleration had been variable the velocity-time graph would have been a curve of the type shown in Fig. 10. It is obvious from the figure that the slope changes from point to point. We can find the slope at any point of the curve by drawing a tangent at that point. For instance, at the point B the slope of the curve $OABC$ is $\frac{EF}{GF}$. Hence this represents the acceleration at the point B.

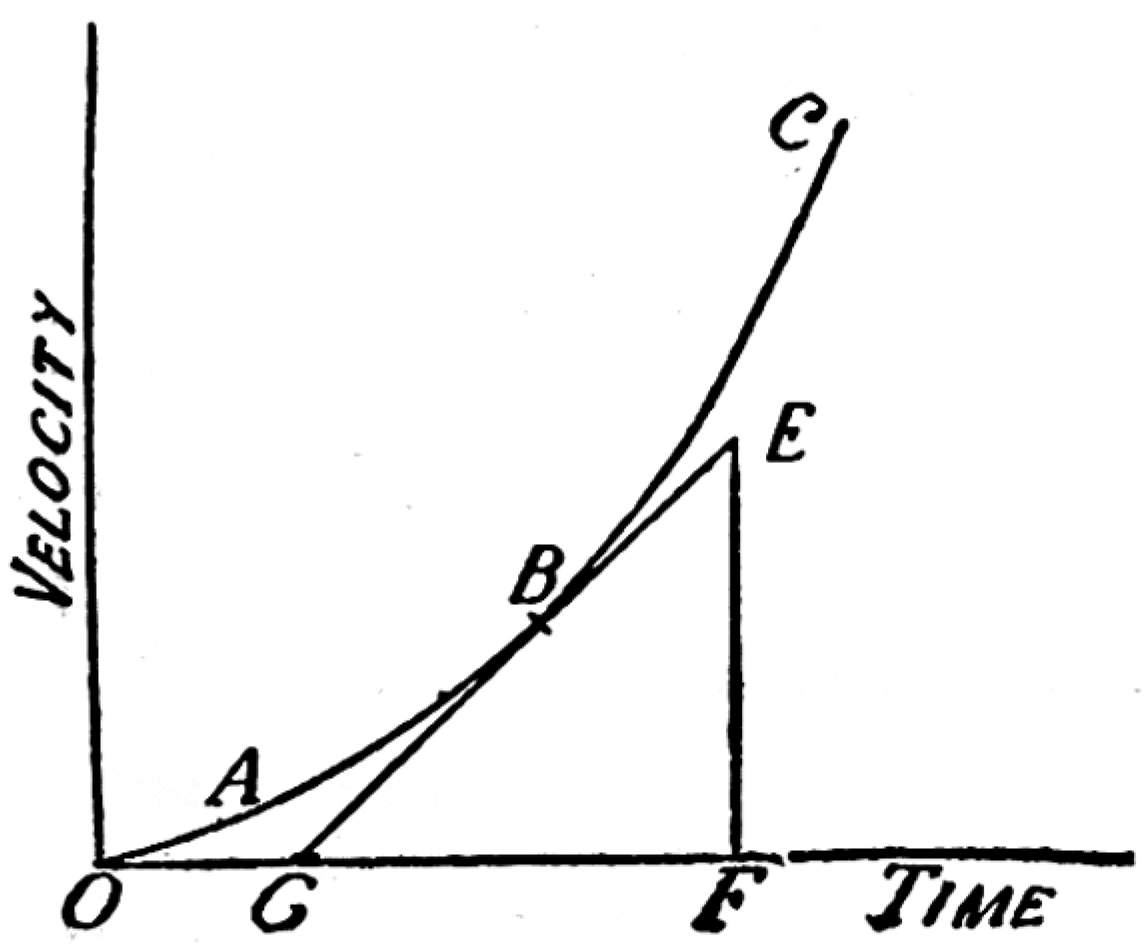


Fig. 10.

NOTE.—If the velocity-time graph is curved upwards as in Fig. 10 the acceleration increases with time ; if it is curved downwards as in Fig. 11 the acceleration decreases with time, and if it is a straight line, sloping upwards or downwards, the acceleration is uniform.

Let us consider two points, A and B, lying close to each other on the curve OC (Fig. 11). Draw the ordinates AD and BE. They represent the velocities at times say t_1 and t_2 so that $DE=t_2-t_1$. If the points are very close to each other, BE will be almost equal to AD, that is the change

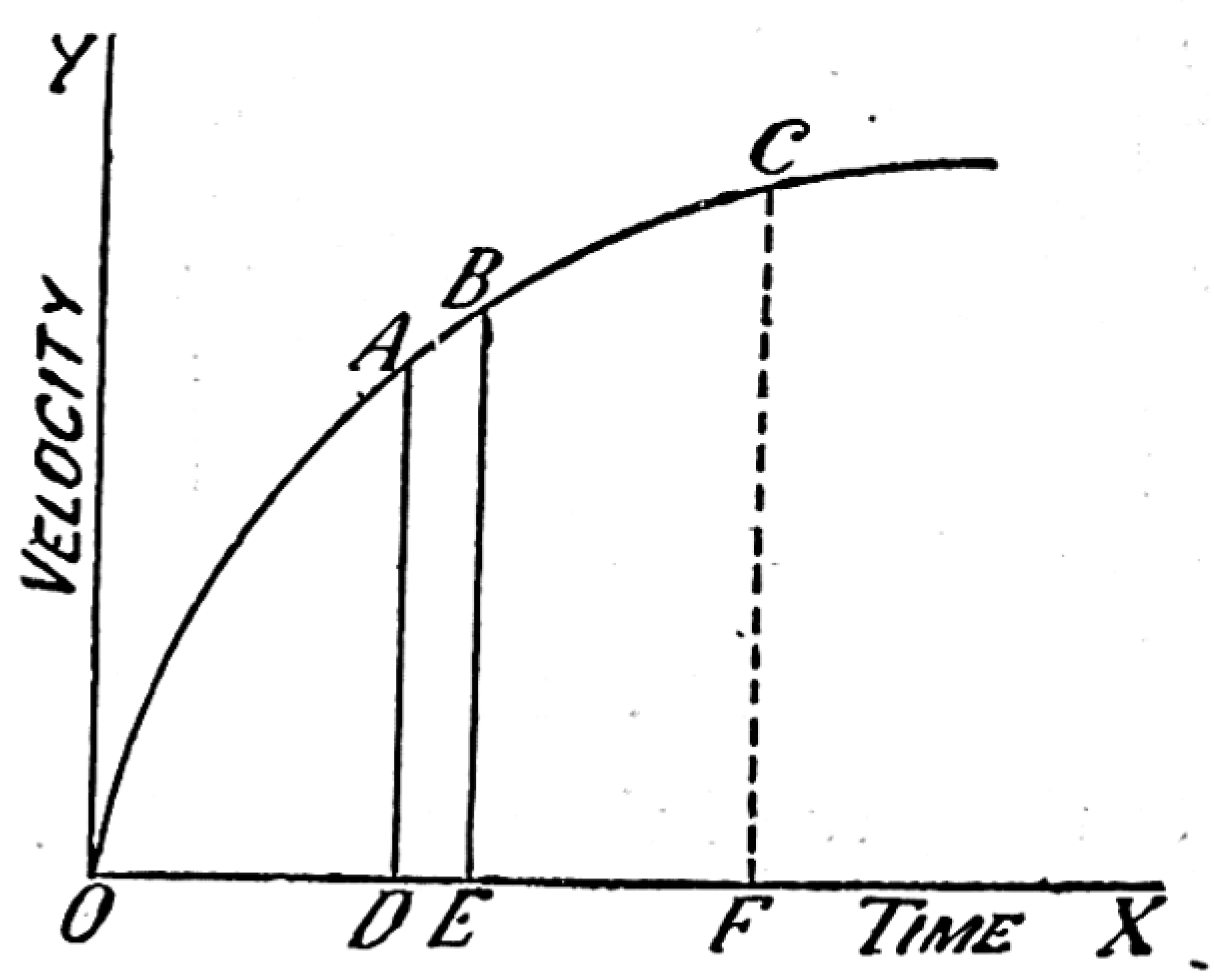


Fig. 11.

in velocity which occurs in interval t_2-t_1 will be negligible and the strip $ABED$ will be very nearly a rectangle. Hence the area of the strip will be equal to $AD\ (t_2-t_1)=v(t_2-t_1)$ where v is the velocity at the time t_1. But $v(t_2-t_1)$ represents the space passed over in the interval t_2-t_1, hence we learn that the area of the strip represents the space passed over by the body in the interval. If we add up the areas of such strips we can find the total space passed over by a body moving with acceleration during a given interval of time. For instance, space passed over in time OF is equal to the area of the figure OCF.

Let us try to find with the help of the velocity-time graph the space passed over in t seconds by a body starting from rest and moving with a uniform acceleration of a ft./sec². Since the body starts from rest its velocity-time graph will pass through the origin and since it moves with a uniform acceleration, the graph OC (Fig. 12) will be a straight line whose slope will give the acceleration. If OD represents t seconds, the velocity v at the end of this period will be equal to CD, and the space passed over will be equal to the area of the $\triangle\ OCD$ which is equal to $\frac{1}{2}vt$.

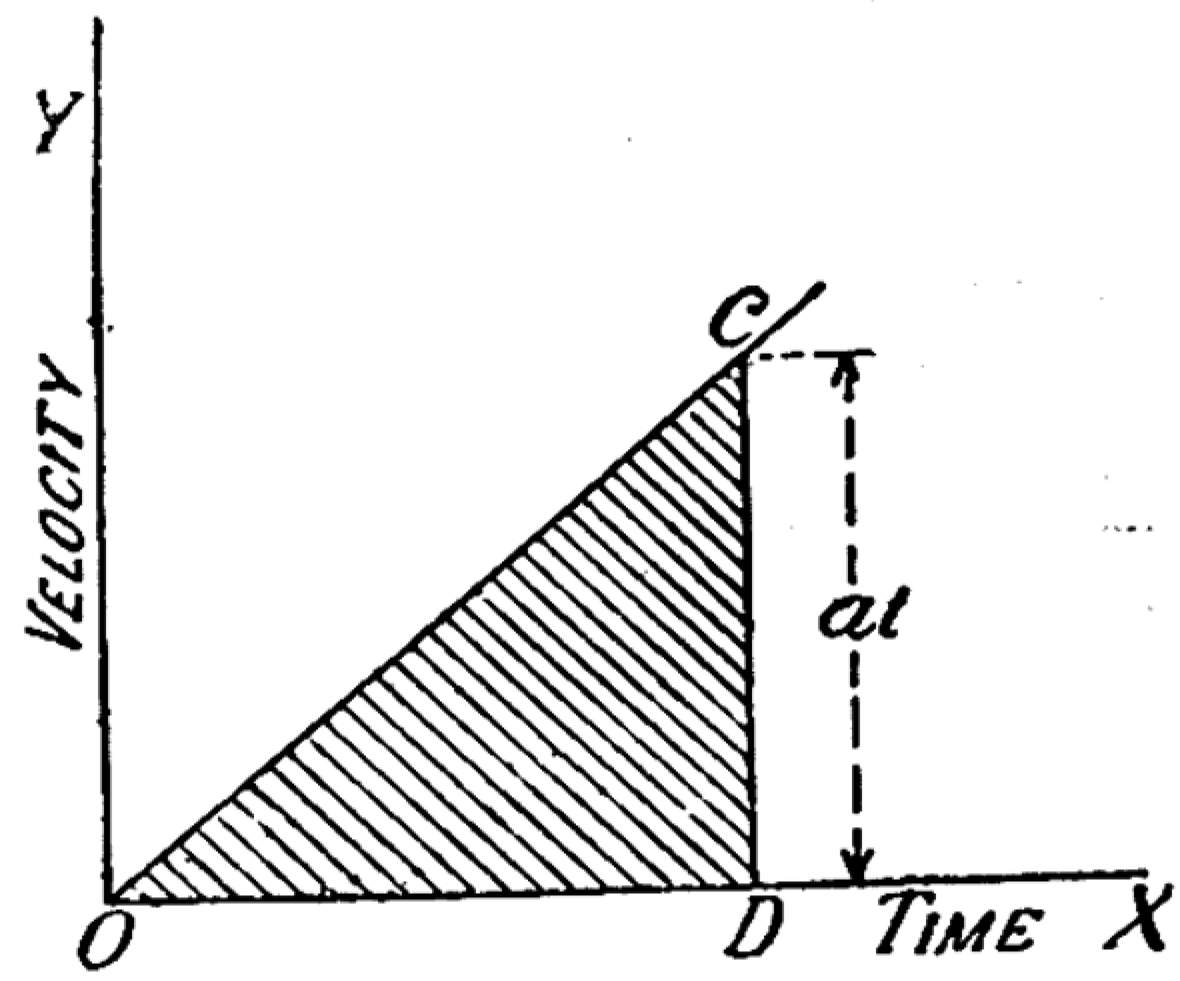


Fig. 12.

Hence space passed over in t seconds can be expressed by the relation

$$S=\tfrac{1}{2}vt \text{ feet.} \quad \ldots \quad \ldots \quad (5)$$

Since the slope $\dfrac{CD}{OD}$ = the acceleration $=a$,

$$CD=a.\ OD$$

$$\therefore \quad v=at.$$

Substituting this value of v in equation (5), we find that the space passed over by the body is given by

$$S=\tfrac{1}{2}at.t=\tfrac{1}{2}at^2 \ldots\ldots\ldots\ldots (6)$$

Now let us try to find with the help of velocity-time graph the space passed over in t seconds by a body which has an initial velocity of u ft./sec., and moves with an acceleration of a ft./sec². The velocity-time graph will in this case be a straight line as before, with this difference, however, that the line will meet the axis of Y at A (Fig. 13) where OA represents a velocity of u ft./sec. As before let OD represent t seconds and BD represent the final velocity v. The space passed over by the body in t seconds is equal to the area of the figure $OABD$. Since the figure is a trapezium, the

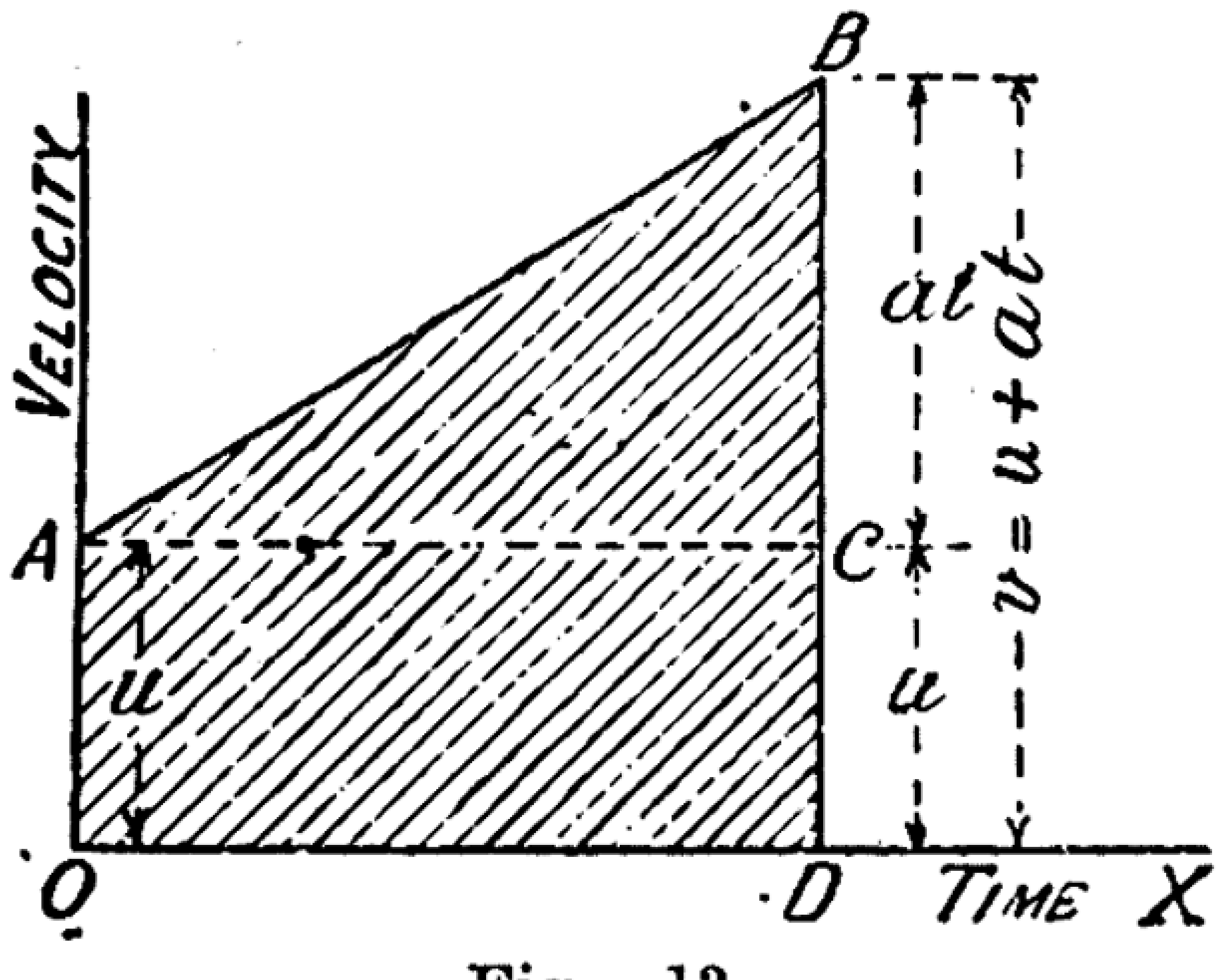


Fig. 13.

area

$$=\frac{AO+BD}{2}\times OD=\frac{u+v}{2}t.$$

Hence space passed over by the body can be written as

$$S=\frac{u+v}{2}t.$$

If the body had moved with a uniform velocity $\frac{u+v}{2}$, it would have travelled in t seconds the same distance. Hence $\frac{u+v}{2}$ is the average velocity during the interval. It is obvious that this velocity is also equal to the velocity at the middle of the interval.

We, therefore, learn that *the average velocity of a body moving with a uniform acceleration during a given interval is equal to half the sum of the initial and final velocities.* We also learn that the space passed over by a body moving with uniform acceleration is given by the relation,

$$S=\frac{u+v}{2}t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The area of the figure $OABD$ is also equal to the sum of areas of the rectangle $OACD$ (*i.e.*, ut) and $\triangle ABC$ (*i.e.*, $\frac{1}{2}at.\ t=\frac{1}{2}at^2$). In other words the space passed over in t seconds by a body starting with an initial velocity u ft./sec. and moving with a uniform acceleration a ft./sec.^{2} is given by the relation,

$$S=ut+\tfrac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

We could get this result directly by substituting the value $u+at$ for v in equation (7) as shown below :—

$$S=\frac{u+(u+at)t}{2}$$

$$=ut+\tfrac{1}{2}at^2.$$

When $a=$zero, *i.e.*, when there is no acceleration, equation (8) is reduced to $S=ut$, equation (1). Writing equation (4) as

$$v-u=at$$

and equation (7) as $\quad v+u=\frac{2S}{t}$

and multiplying the corresponding sides we get

$$v^2-u^2=2aS, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

a very useful relation for solving numerical problems.

18a. *Space passed over in the* n*th second.*—Sometimes it becomes necessary for us to find the space passed over by a body in any particular second, say for instance, nth. One method is to find the space passed over first in n seconds and then in $(n-1)$ seconds ; subtract the latter from the former, and get the result. Another method is to determine the average velocity during the nth second and to get from it direc-

tly the space passed over. As a matter of fact the average velocity itself is equal to the distance passed over, for the time is 1 second.

The velocity at the end of the $(n-1)$th second is

$$u+(n-1)\ a \text{ ft./sec.}$$

The velocity at the end of the nth second is

$$u+na \text{ ft./sec.}$$

Average velocity is $u+(2n-1)\ \frac{a}{2}$ ft./sec.

The space passed over during the nth second is therefore,

$$\left\{u+(2n-1)\ \frac{a}{2}\right\} \times 1 \text{ or } u+(2n-1)\ \frac{a}{2} \text{ ft.}$$

19. Bodies falling under Gravity.—We have already said on page 16 that in the case of a falling stone, the velocity increases at a uniform rate and that a falling stone is an example of a body moving with uniform acceleration. We shall see later on that the acceleration of such bodies, *i.e.*, of bodies falling freely under the action of gravity is approximately 32 ft./sec.^{2} Writing g for acceleration due to gravity the relations proved above can be written as ;

$$v=u+gt.$$
$$S=\tfrac{1}{2}gt^2$$
$$S=ut+\tfrac{1}{2}gt^2,$$

and

$$v^2-u^2=2gS.$$

While using these relations in solving problems we should be careful about the sign of g. For instance, if the initial velocity of a body thrown upwards is taken as positive, g must be taken as negative, but if there is no initial velocity, direction of g might be considered as positive. *It is immaterial whether g is taken as positive or negative, but whatever sign is given to it, the same sign must be used throughout a given problem.* We shall consider a few cases of special importance.

(1) *Find the velocity* of a body which starts from rest and falls through a height 'h' feet.*

To find the velocity with which the body falls on the ground, use the relation

$$v^2-u^2=2gh.$$

Taking the downward direction as positive and substituting 32 ft./sec.^{2} for g in the above relation we get

$$v^2-u^2=64h.$$

As the initial velocity is zero the relation is reduced to

$$v^2=64h$$

or

$$v=8\sqrt{h} \text{ ft./sec.}$$

Since the sign of v is positive it means the velocity is in the downward direction.

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*Neglect the air resistance in all problems in this and the following chapters unless stated otherwise.

Similarly it can be shown that the velocity with which a body must be projected to just reach a height h is also $8\sqrt{h}$ ft./sec.

(2) *A body is projected upwards with a velocity of u ft./sec., find the greatest height to which it will rise.*

When a body is thrown upwards, its velocity constantly decreases due to the downward pull of the earth. At the point of greatest height its upward velocity must be zero.† To find h use the equation

$$v^2-u^2=2gh,$$

bearing in mind that u is upward and positive and g is downward and therefore negative, and that at the highest point $v=0$. The above equation reduces to

$$-u^2=-2gh,$$

$$\therefore\ h=\frac{u^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$=\frac{u^2}{64}\text{feet.}$$

since g is 32 ft. per second per second.

(3) *Find the time to reach the highest point.*

To find the time we shall make use of the same fact, that velocity is zero at the highest point. Let us use the equation

$$v=u+gt.$$

If u is positive, g will be negative. Bearing this in mind, we get

$$0=u-gt$$

or
$$t=\frac{u}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

$$=\frac{u}{32}\text{ seconds.}$$

(4) *Find the time of fall from the highest point.*

Since the velocity at the highest point is zero, $v=0$.

The greatest height is $\frac{u^2}{2g}$ from (i); this means that the body will have to fall through this much distance to reach the ground; hence the space passed over $=\frac{u^2}{2g}$.

From this we can at once find the time of fall using the relations

$$S=\tfrac{1}{2}gt^2=\frac{u^2}{2g}.$$

Therefore
$$t^2=\frac{u^2}{g^2}.$$

or
$$t=\frac{u}{g}.$$

†For after that the velocity changes in direction, and the body begins to move downward instead of upward.

Note that the time of ascent was also u/g. Thus we see that the time of ascent to the highest point = the time of descent to the ground.

$$\text{The total time} = \frac{2u}{g}.$$

We can also directly find the total time by making use of the fact that the total space passed over during this time is zero.

(5) *Find the velocity with which a body returns to the ground when projected upwards with a velocity of u ft. per second.*

Total time of journey $= 2u/g$. Making use of this fact, let us find the velocity with which it falls on the ground. We know that

$$v = u + gt,$$

where u is positive and g is negative.

Substituting the value $\frac{2u}{g}$ for t, we get $v = u - g\frac{2u}{g} = -u$.

This shows that the velocity with which a body returns to the ground is numerically equal to the velocity with which it is projected upwards.

EXERCISES

1. A bullet is fired vertically upwards with a velocity of 1,600 ft. per second. After how many seconds will it return to the earth? What is the greatest height reached ?

$u = +1{,}600$ ft. per sec.

$g = -32$ ft. per sec. per sec.

$S = 0$

$t = ?$

$H = ?$

(i) To find t use the equation $S = ut + \frac{1}{2}gt^2$

$0 = 1600 \times t - 16t^2$

Therefore $t = 100$ seconds.

(ii) At the greatest height $v = 0$

Therefore $v^2 - u^2 = 2gS$ reduces to $-u^2 = 2gS$,

or $-1600 \times 1600 = -2 \times 32 \times H$

or $H = \dfrac{1600 \times 1600}{2 \times 32} = 40{,}000$ ft.

2. If a particle were moving at any instant at the rate of $7\frac{1}{2}$ ft. per second, after what time would its velocity become four times if the acceleration be 10 ft. per sec. per sec. and what distance would it travel ?

$u = 7\frac{1}{2}$ ft. per second.

$v = 30$ ft. per second.

$a = 10$ ft. per second per second.

$t = ?$

$S = ?$

The relation $v^2-u^2=2aS$ would give S at once, for all other quantities are known.

$$900-\frac{225}{4}=2\times 10S.$$

Therefore $$S=\frac{3600-225}{4\times 20}=\frac{675}{16}=42\frac{3}{16}\text{ ft.}$$

To know time use the formula

$$v=u+at,$$

or $$30=\frac{15}{2}+10\times t.$$

Therefore $$t=\frac{45}{2\times 10}=\frac{9}{4}=2\tfrac{1}{4}\text{ seconds.}$$

3. A train going 64 ft. per second is brought up in 64 ft. to avoid a collision. What is the acceleration in yards/sec.^{2}; in yards/hour^2, and in ft./sec.^{2}?

Use $v^2-u^2=2aS$. Since $v=0$, the relation is reduced to $-u^2=2aS$,

or $$a=-\frac{u^2}{2S}$$

$$=-\frac{64\times 64}{2\times 64}=-32\text{ ft. per sec.}^2$$

$$=-\frac{32}{3}\text{ yards per sec. per sec.}$$

To express acceleration of $-\frac{32}{3}$ yds./sec.^{2} in yds./hour^2 proceed as below :

In one second a velocity of $-\frac{32}{3}$ yds. per sec. is added ;

or in one hour the velocity of $-\frac{32\times 60\times 60}{3}$ yards per second is added ;

In ,, ,, ,, ,, $-\frac{32\times 60\times 60\times 60}{3}$ yards per minute is added.

or in one hour the velocity of $-\frac{32\times 60\times 60\times 60\times 60}{3}$ yards/hour is added.

So we see that the acceleration of -32 ft./sec^2. or $\frac{-32}{3}$ yds./sec.^{2} is equal to the acceleration of $-\frac{32\times 60^4}{3}$ yds./hour^2.

4. A train starts from Shahdara and for the first mile moves with a uniform acceleration, for the next four miles with a uniform velocity, and finally for one mile with a uniform retardation before coming to rest at the next station Kala Shah Kaku. It takes 16 minutes to cover this distance of 6 miles. Draw a velocity-time graph and find from it the maximum velocity of the train.

Since the train first moves with a uniform acceleration, then with a uniform velocity and finally with a uniform retardation, it is obvious that the graph will consist of three parts, straight line OA sloping upwards, line AB parallel to the time axis, and line BC sloping downwards. Let the maximum velocity represented by AD or BE be v miles per hour, and let the time taken by the train to travel the first mile be t_1 minutes, to cover the next 4 miles be t_2 minutes and finally to complete the last mile be t_3 minutes.

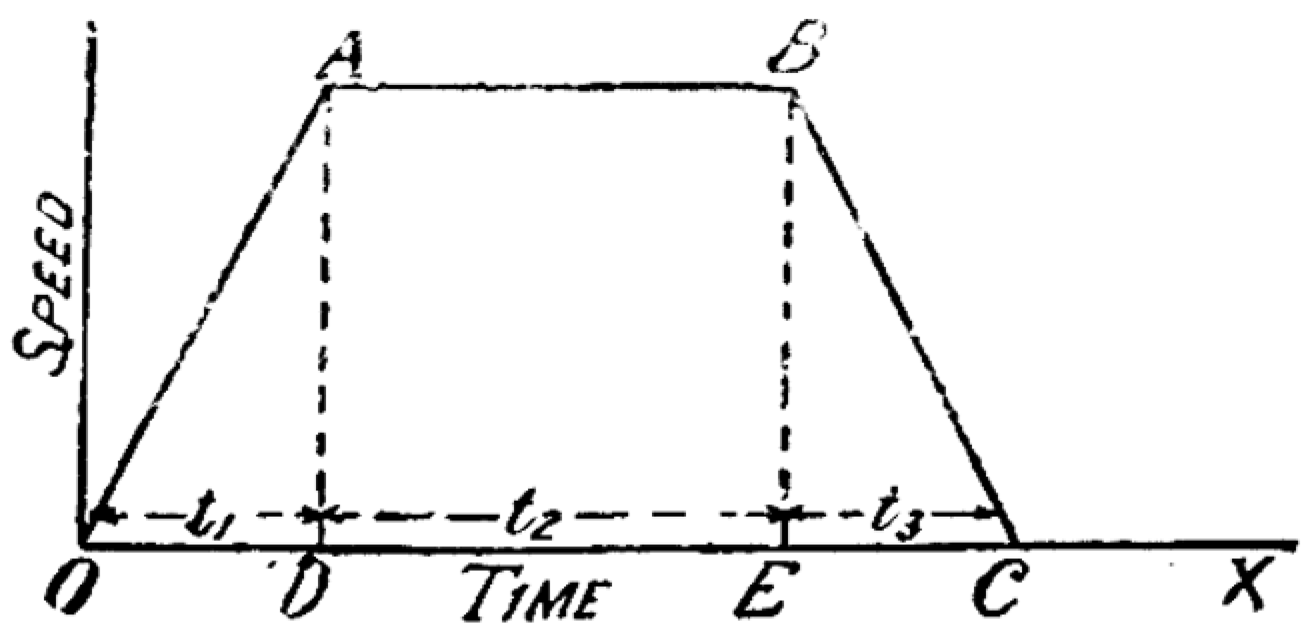


Fig. 14.

The area of the $\triangle OAD$ represents 1 mile, hence

$$\tfrac{1}{2}vt_1=1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (i)$$

The area of the $\triangle EBC$ also represents 1 mile, hence

$$\tfrac{1}{2}vt_3=1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (ii)$$

From equations (i) and (ii) it is clear that $t_1=t_3$.

The area of the rectangle $ABED$ represents 4 miles, therefore

$$vt_2=4 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (iii)$$

From equations (i) and (iii) we find that $t_2=2t_1$

Hence the total time taken by the train to go from Shahdara to Kala Shah Kaku $=4t_1=16$ minutes

$$\therefore \; t_1=4 \text{ minutes}$$

$$=\frac{1}{15} \text{ of an hour}$$

Substituting this value of t_1 in equation (i) we get

$$\frac{1}{2}v.\ \frac{1}{15}=1$$

whence $v=30$ miles/hour.

5. A stone is dropped from a balloon moving vertically upwards with a uniform velocity of 48 ft. per second; and is found to reach the ground in 4 seconds. Find the height of the balloon (i) when the stone was dropped, (ii) when the stone reached the ground.

The stone was moving upwards along with the balloon with a velocity of 48 ft. per second when it was dropped. Hence

$u=48$ ft. per sec. $\quad t=4$ sec.

$g=-32$ ft./sec.^{2} $\quad S=?$

To find S, use $S=ut+\frac{1}{2}gt^2$

or $$S=48t-16t^2=48\times4-16\times16$$
$$=192-256=-64 \text{ ft.}$$

Note that 64 ft. is the space passed over from the starting point in the downward direction, which has been taken negative in this problem. The actual distance passed over will be 36 ft. above the starting point, 36 ft. back from the highest to the starting point, and 64 ft. downwards.

(*ii*) To find out the height of the balloon at the time when the stone reached the ground, find the height through which the balloon rises in 4 seconds. To do so we have simply to make use of the relation $S=ut$.

Since $u=48$ ft.sec. and $t=4$ seconds

Therefore $S=48\times4=192$ feet.

Now since the balloon was already 64 ft. high at the time when the stone was dropped, the total height of the balloon when the stone reached the ground $=192+64=256$ ft.

6. Define uniform velocity.

A railway train 50 yards long passes over a bridge 290 ft. long at the rate of 10 miles an hour. How long will it take to completely pass over the bridge ? *Ans.* 30 seconds.

7. A particle is moving with a uniform acceleration and has an initial velocity of 100 ft./sec, ; at the end of 1 minute its velocity is 220 ft./sec. How far will it move in ten minutes and what will be its velocity then ? *Ans.* 420,000 ft. and 1,300 ft. per second.

8. If a body moves with an acceleration of 96 ft./min.^{2}, what is its acceleration in terms of centimetres/sec.^{2} ?

Ans. 0·813 cm./sec.^{2}.

9. A train going 40 miles an hour is brought up in 200 yards by the breaks to avoid a collision. What is the acceleration in miles/sec.^{2} ; in miles per hour per second, and in ft./sec.^{2} ?

Ans. (1) $\frac{11}{20250}$ miles/sec.^{2} (2) $\frac{88}{45}$ miles per hour per sec.

(3) $\frac{1936}{675}$ ft./sec.^{2}

10. A body describes 12 ft. in the 2nd second and 50 ft. in the 4th second of its motion. If the motion be uniformly accelerated, how far will it go in the next 3 seconds ? *Ans.* 264 ft.

11. A tram-car starts from rest and accelerates uniformly for 8 seconds to a speed of 10 miles per hour. It then runs at a constant speed, and finally is brought to rest in 40 ft. with a constant retardation. The total distance passed over is 250 yards. Find the value of the acceleration, the retardation, and the total time taken.

Ans. Acc.$=1·83$ ft./sec.^{2} ; Ret.$=2·69$ ft /sec.^{2} ; Time$=57·8$ seconds.

12. A motor car starts from rest and accelerates uniformly for 50 seconds to a speed of 30 miles/hour. It then runs at a constant

speed and is finally brought to rest in 132 feet with a constant retardation. The total distance passed over is 1 mile. Draw a speed-time graph and find therefrom the value of acceleration and of total time.

Ans. Acc. $= 1\frac{1}{15}$ ft./sec.^{2} : Ret. $= 7\frac{1}{3}$ ft./sec.^{2} Time. $=$ 2 min. and 18 sec.

13. A mail train is made to stop at a flag station for two minutes to enable another train to pass from the opposite direction. If the speed of the mail train were 40 miles/hour and were pulled up near the station with a retardation of 2 ft./sec.^{2} while stopping and accelerated at the same rate while getting up to full speed again, find the total time lost. *Ans.* 2 minutes 29·3 seconds.

14. A stone is dropped from the top of a cliff 160 ft. high. At the same time another stone is thrown vertically upwards with a velocity of 80 ft. per sec. Find when and where will they meet ?

Ans. After 2 seconds, 64 ft. from the top of the cliff.

15. A body falling freely under the action of gravity passes two points 96 ft. apart vertically in one second. Find from what height above the upper point it began to fall. (*P. U. 1937*) *Ans.* 100 ft.

16. The Empire State Building is 1248 ft. high. A stone is dropped from its top. Find the time taken by the stone to reach the ground. What will its velocity be on reaching the ground ?

Ans. Time 8·8 seconds ; velocity 282·6 ft./sec.

17. A stone after freely falling under the influence of gravity for one second strikes a pane of glass held horizontally. In breaking through the pane the stone loses half of its velocity. How far will it fall in the next two seconds ? *Ans.* 96 ft.

18. A body is projected upwards with a velocity of 120 ft. per second ; what is the greatest height to which it will rise, and when will it be moving with a velocity of 40 ft. per second ?

Ans. 225 ft. ; $2\frac{1}{2}$ or 5 seconds.

19. The intensity of gravity at Jupiter is 2·6 times as much as on the earth. How long will a body take to fall on Jupiter from a height of 167 ft. ? *Ans.* 2 seconds.

20. A balloon is going up at the rate of 80 ft. per second and when it is at a height of 800 ft. a stone is dropped. When, and with what velocity, does it hit the ground ?

Ans. 10 seconds ; 240 ft. per second.

21. A bomb is dropped from a plane flying horizontally at a height of 10,000 ft. How long will it take the bomb to fall to the earth if the air resistance were neglected ? *Ans.* 25 sec.

CHAPTER II

Composition of Motions and Velocity

20. Scalar and Vector Quantities.—Most of the quantities with which we deal in every day life are completely known if their magnitude is known. Mass, volume, energy, and time are just a few examples of such quantities. Such quantities are called *scalar**. To add them we use the arithmetical method ; for instance, if we have one litre of water in one jug and two litres in the second jug, on combining we get three litres of water.

There are some quantities, however, like force, velocity, and acceleration which are not completely known even when their magnitude is known. To know them fully we must know their direction as well. Such quantities whose direction as well as magnitude must be known are called *vector* quantities. To add two vector quantities the arithmetical addition does not serve the purpose except in the special case when their direction is the same. A question arises, how to add such quantities ? Before this question is answered it will be well to point out that a vector quantity can be represented by a straight line, the length of the line representing the magnitucde of the quantity to some chosen scale and the direction of the line the direction in which the quantity acts. Bearing this in mind let us see how we can add two motions or two velocities.

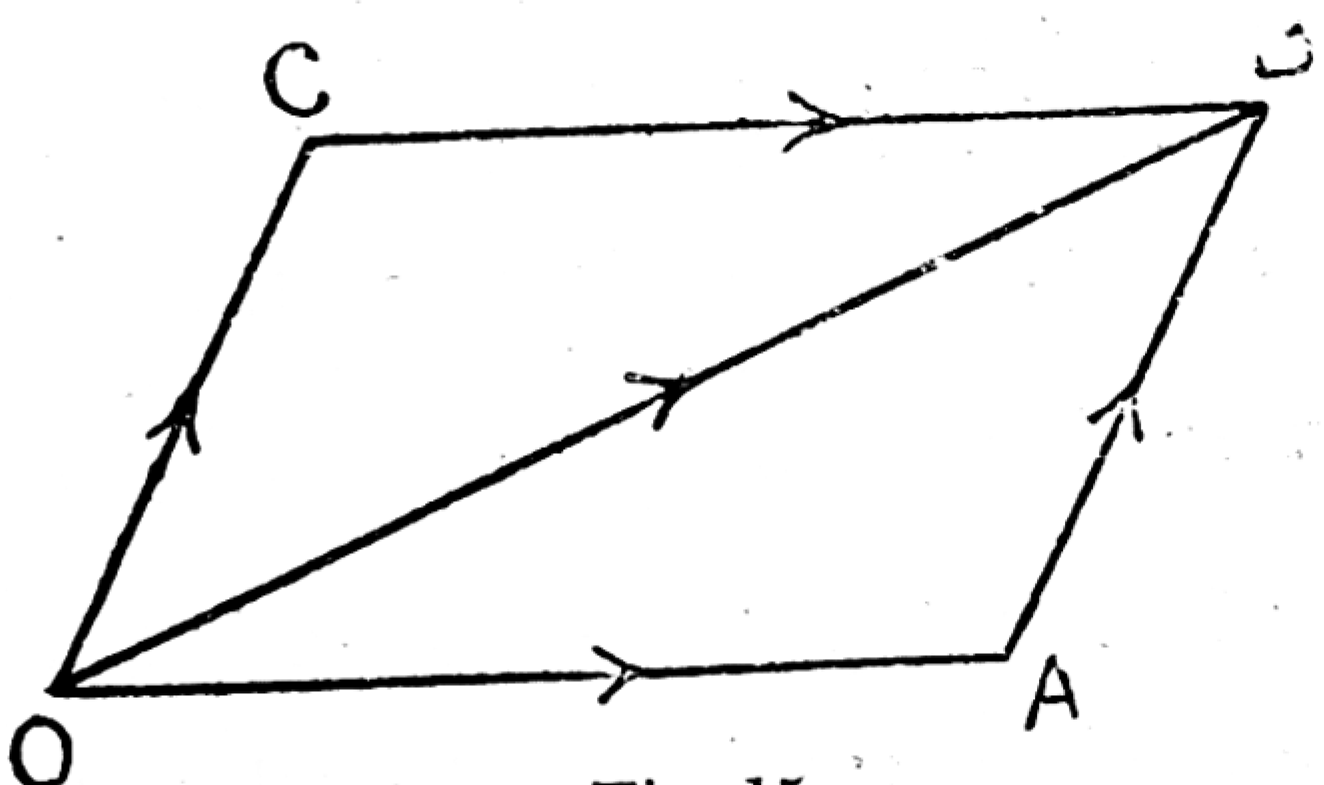


Fig. 15.

21. Composition of Two Motions.—Suppose a point O (Fig. 15) moves along OA, and when it reaches A it moves along AB. When it has reached B, its distance from O is not $OA+AB$, but OB. It would reach the same point B if it were first to go along $OC(=AB)$ and then along $CB(=OA)$. Thus we see that *the order in which the displacements are given to the point O is immaterial.* The point O would also reach the point B if a single displacement OB were given to it. The single displacement which produces the same effect as two separate displacements, is called the resultant displacement, and the two separate displacements, the component displacements. The resultant displacement is represented by the diagonal of the parallelogram (Fig.15) constructed on the two lines representing displacements as adjacent sides.

22. Composition of Two Velocities.—If the above displacements take place simultaneously in the course of 1 second, then OA

* The word *Scalar* comes from the word *Scala*, meaning ladder, a symbol for increasing or decreasing magnitudes.

and OC will represent velocities, and OB will represent the resultant velocity, which is equivalent to both OA and OC. It is not very difficult to imagine a body possessing two velocities simultaneously. Suppose, for instance, a man rows a boat from the point O across a stream with a uniform velocity of u ft./sec. while the stream flows with a uniform velocity of v ft. per sec. The boat will possess both these velocities simultaneously. To a person standing on the bank the boat will seem to move along OB, the diagonal of the parallelogram constructed with OA and OC as adjacent sides, OA being proportional to the velocity of rowing and OC to the velocity of the flow of the stream.

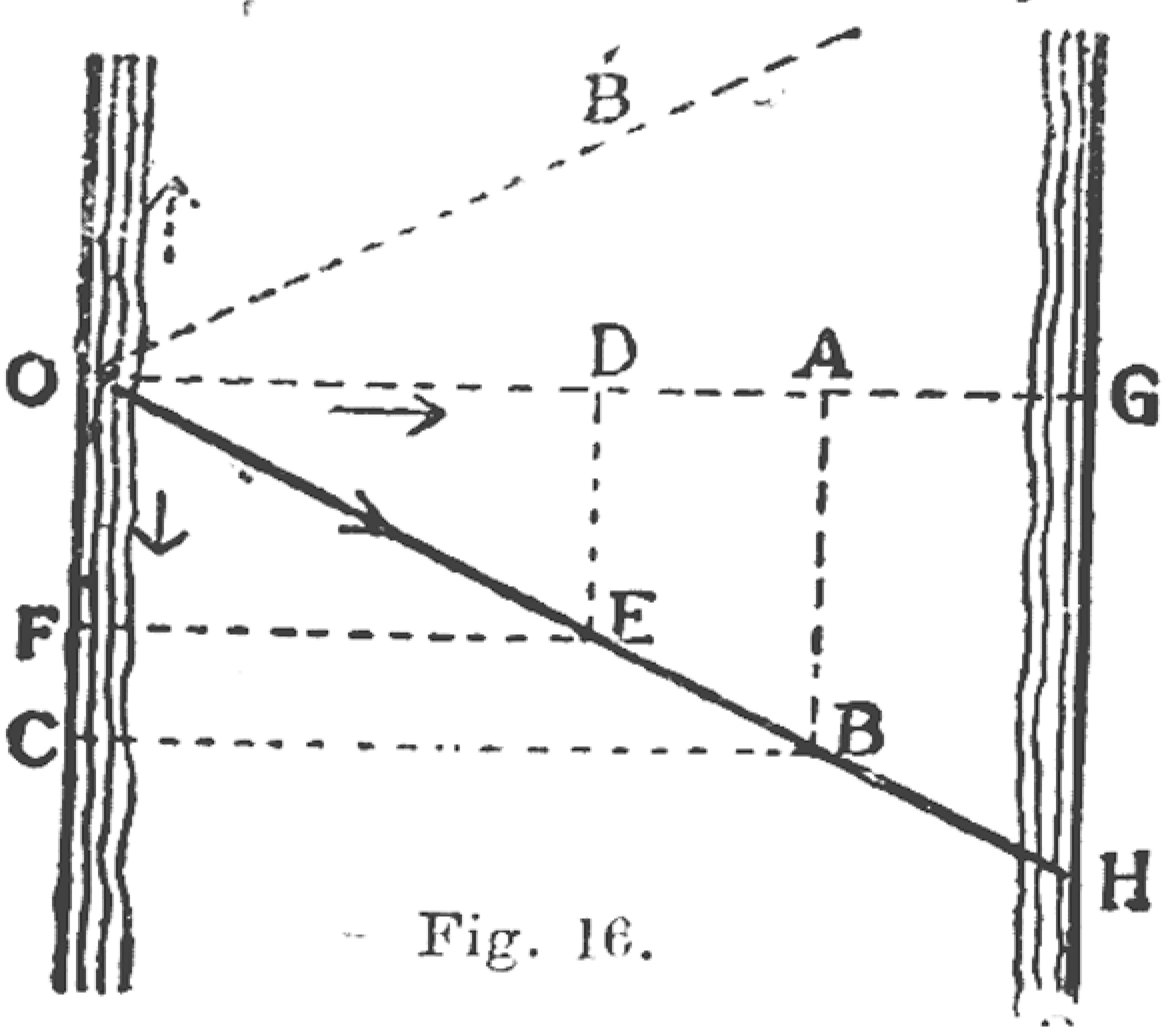


Fig. 16.

If OG represents the width of the stream, the boat will reach the opposite bank at the point H. To see that the boat will move along the diagonal OB, let us consider where it will be after $\frac{1}{2}$ second. In this time the boat will move across the stream $\frac{1}{2}u$ feet, and downstream $\frac{1}{2}v$ feet. This means that the boat will be at a distance of $\frac{1}{2}u$ feet along OA from O and $\frac{1}{2}v$ feet along OC. The ratio of the two distances will be $\frac{u}{v}$. Now whatever the duration of time might be, the ratio of the distances will always be $\frac{u}{v}$. It is clear from Fig. 16 that all the points for which the ratio is $\frac{u}{v}$, lie on the diagonal. Hence we find that if both the velocities be simultaneously impressed on the boat, it would move along the diagonal OB of the parallelogram $OABC$.

This law of addition is called the **parallelogram law**. It may be stated thus :

If a particle simultaneously possesses two uniform velocities, represented in magnitude and direction by the two sides of a parallelogram drawn from a point, the resultant velocity is completely represented by the diagonal passing through the same point.

Caution.—Before applying the parallelogram law it must be seen that both the *velocities are acting either towards or away from the point.*

23. Graphical Method of Determining the Resultant of Two Velocities.—To graphically find the resultant of two velocities, u and v, inclined to each other at an angle θ, draw OA to represent the velocity of u ft./sec. and on the same scale draw OC, making an angle θ, with OA to represent the velocity of v ft./sec. Complete the parallelogram and measure the diagonal passing through O. It represents the resultant in

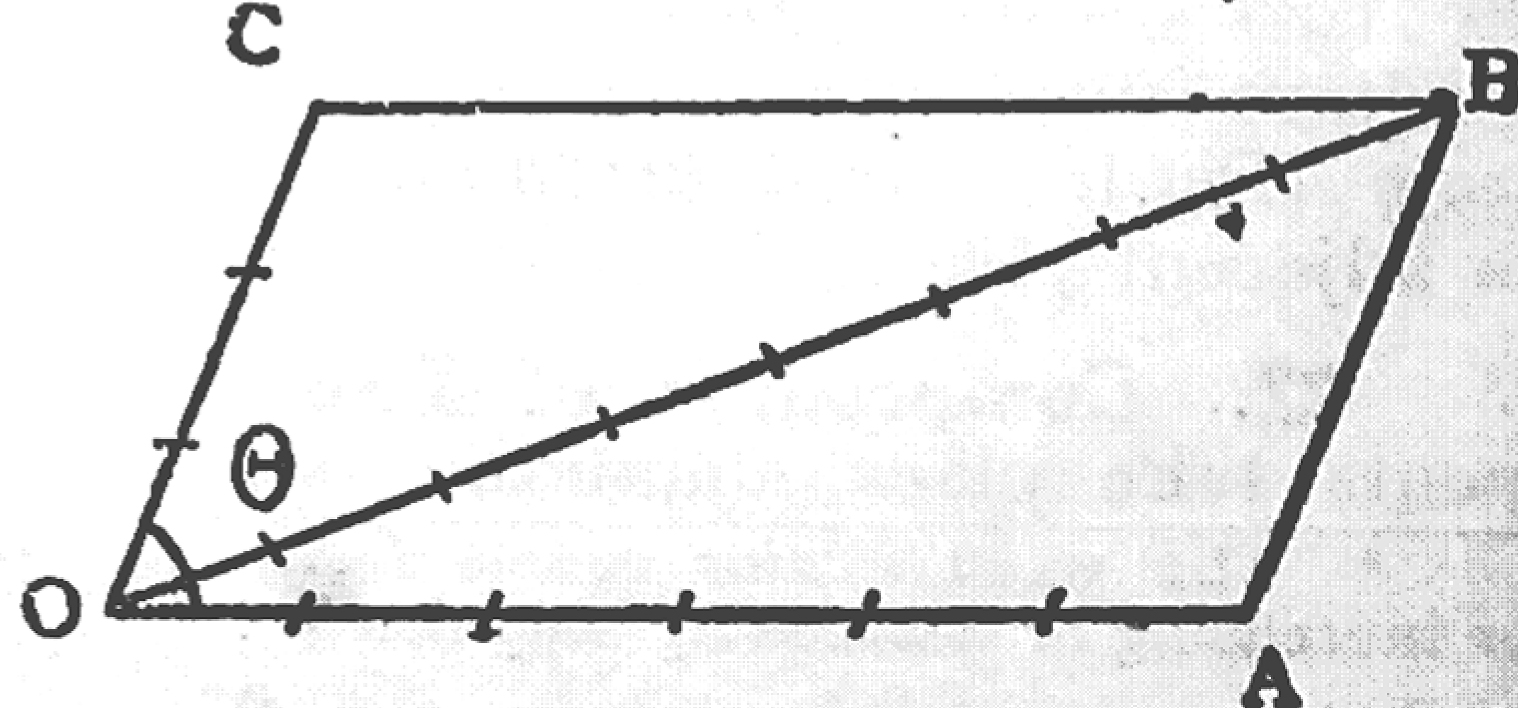


Fig. 17.

magnitude (on the same scale) as well as direction. To get accurate results use as large a scale as the paper permits because the larger the scale, the greater the accuracy with which the lengths can be measured.

This is only an approximate method. The imperfections are due to (*i*) error in measuring angles, and (*ii*) error in measuring lengths. This method is useful in cases where calculations are lengthy and hence tedious.

24. To find the Resultant by Calculation.—Let u and v be the two velocities which we desire to add, and θ the angle between them. Complete the parallelogram $OABC$ (Fig. 18). OB evidently represents the resultant R. From B draw BD perpendicular on OA produced. The angle $BAD = \angle COA = \theta$ and $\frac{BD}{AB} = \sin\ \theta$.

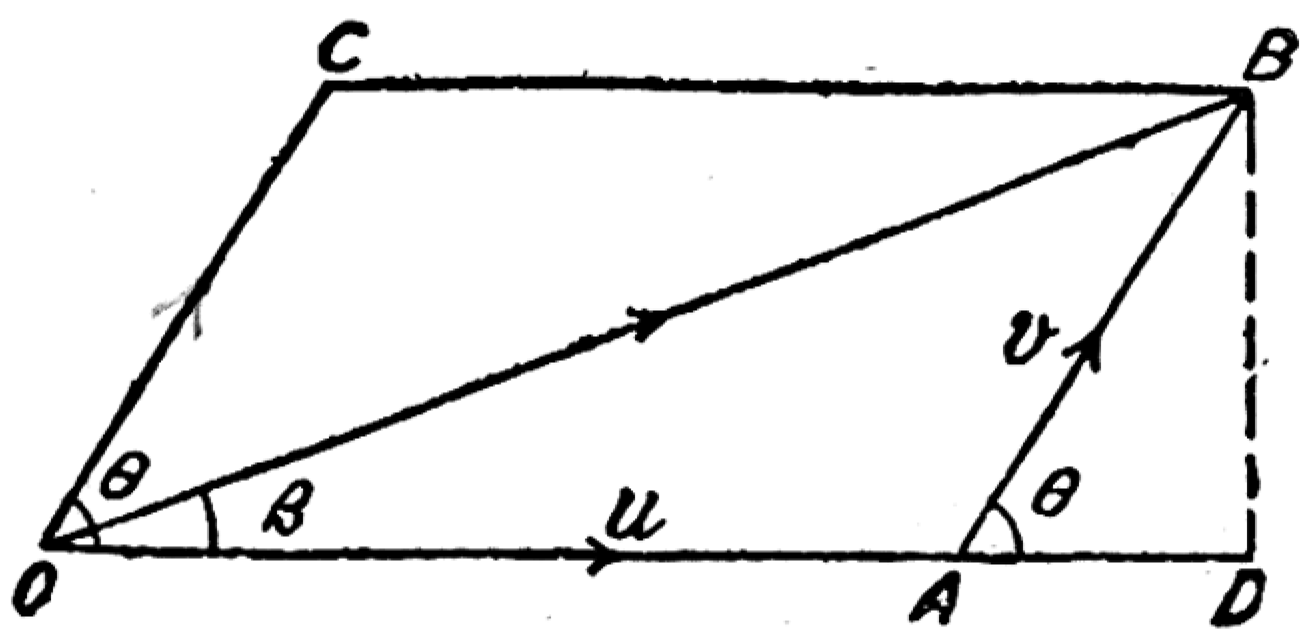


Fig. 18.

$\therefore\ BD = AB\ \sin\ \theta$, or $v \sin\ \theta$.
Similarly it can be shown that

$$AD = v\cos\ \theta.$$

We find from Fig. 18 that $OD = OA + AD = u + v\cos\ \theta$.

Consider the triangle OBD. Since it is a right-angled $\triangle$,

$$OB^2 = OD^2 + BD^2.,$$

Writing R for OB, $u + v\cos\ \theta$ for OD and $v\sin\ \theta$ for BD, we get

$$R^2 = (u + v\cos\ \theta)^2 + (v\sin\ \theta)^2$$
$$= u^2 + v^2 \cos\theta^2 + 2uv\cos\ \theta + v^2\sin^2\theta$$
$$= u^2 + v^2(\cos^2\theta + \sin^2\theta) + 2uv\cos\ \theta$$
$$= u^2 + v^2 + 2uv\cos\ \theta,$$

or
$$R = \sqrt{u^2 + v^2 + 2uv\cos\ \theta}\,.$$

This is a very useful relation : it enables us to find the resultant of any two velocities, u and v, inclined to each other at an angle θ.

To find the direction of the resultant proceed as follows :

Let the resultant make an angle β with OA :

$$\tan\ \beta = \frac{BD}{OD} = \frac{v\sin\ \theta}{u + v\cos\ \theta},$$

Let us calculate the value of R in some special cases.

(1) When θ is zero (*i.e.*, when the direction is the same) ;

$$\cos\ \theta = 1,$$

hence
$$R^2 = u^2 + v^2 + 2vv$$

or
$$R = u + v$$

It is just what is expected, because when direction is the same the component velocities simply add up

Notice that the resultant has the same direction as the two velocities.

(2) When $\theta = 90°$ (*i.e.*, the components are rectangular), $\cos\theta = 0$ and hence
$$R^2 = u^2 + v^2 + 2uv \times 0$$

or
$$R^2 = u^2 + v^2$$
as expected, for in a rectangle the diagonal is given by this result. Moreover, when $u=v$, $R^2=2u^2$, and the direction is given by

$$\tan \beta = \frac{u}{v} = 1, \ \beta = 45°$$

(3) When $\theta = 180°$, *i.e.*, the components are in opposite directions,

$$R^2 = u^2 + v^2 + uv \cos 180°$$
$$= u^2 + v^2 - 2uv$$
$$R = u - v.$$

When we have to add more than two velocities we first find the resultant of any two velocities, and then the resultant of this and the third velocity, and so on. The last resultant so found gives the resultant of all the velocities.

25. Triangle of Velocities.—The parallelogram law of composition of velocities explained in Art. 22 can be expressed in a slightly different form. We have seen that the two velocities OA and OC (Fig.17) are equivalent to a single velocity OB. Now $OABC$ is a parallelogram, $OC=AB$. Therefore AB represents the same velocity as OC. In the triangle OAB, the side OA represents one velocity, AB the second velocity, and OB (the third side) the resultant. Thus we see that if two sides of a triangle taken in order represent the component velocities, the third side in the opposite order represents the resultant. This method of composition of velocities is known as the Triangle of Velocities. It is stated as follows :

If a particle simultaneously possesses two uniform velocities represented by the two sides, OA, AB, of a triangle, taken in order, they are equivalent to a velocity represented by the third side, OB, in the opposite order.

A very important corollary which follows directly from the triangle of velocities is that, if a particle possesses three velocities simultaneously which can be represented by the three sides of a triangle taken in order the particle will remain at rest. Why ? It is because the velocity represented by the third side is equal in magnitude and opposite in direction to the resultant of the other two velocities.

26. Polygon of Velocities.—If a particle has a number of velocities we can find their resultant as explained below. Let the magnitude and direction of each velocity acting on the particle be represented by u, v, w, and r, as shown in Fig. 19.

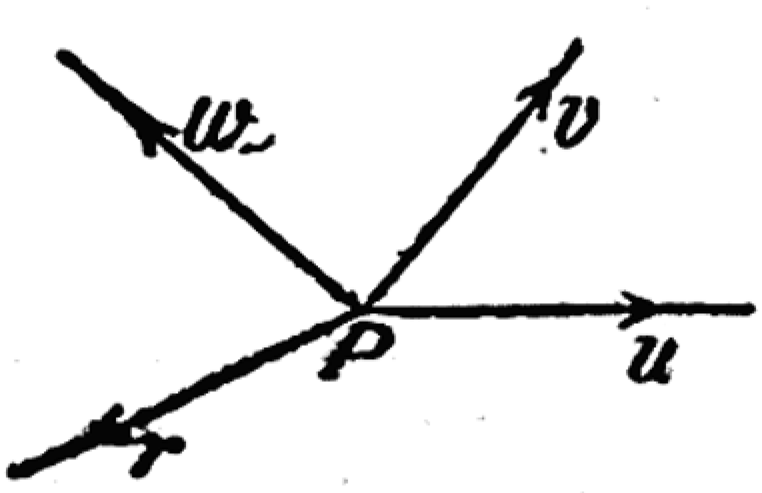


Fig. 19.

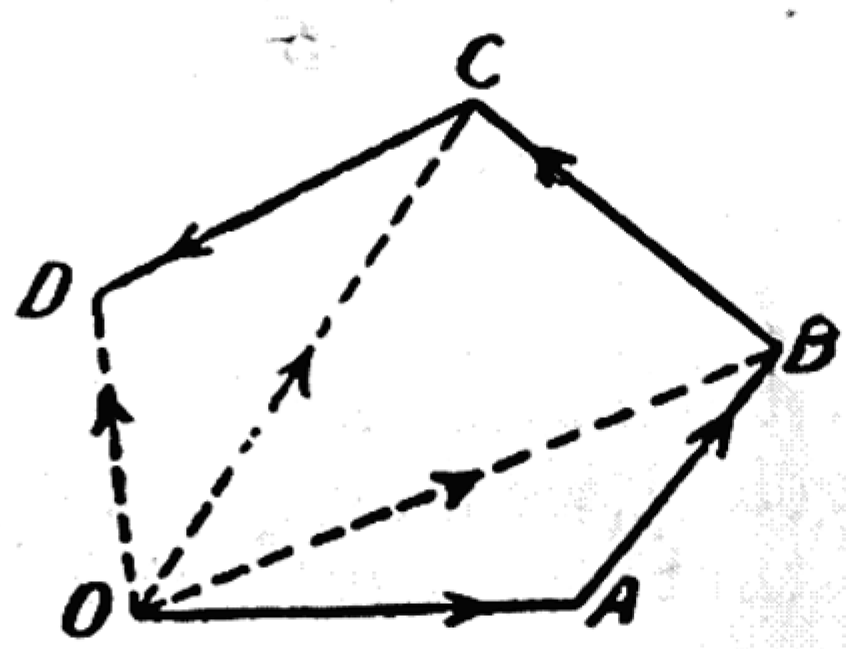


Fig. 20.

In order to find the resultant draw OA parallel and equal in length to u (Fig. 20). This will represent the velocity u. Now draw

AB parallel and equal to v : this will represent the velocity v acting on particle P. OB would represent the resultant of u and v. Draw BC parallel to w and equal to it in length. OC would represent the resultant of OB and BC. Next draw CD parallel to r and equal to it in length. OD would represent the resultant of OC and CD. Since OC is equivalent to BC and OB, or BC, OA and AB, we see that OD is equivalent to all the four velocities impressed on the particle.

This method of compounding velocities is called the Polygon of Velocities. It can be stated as follows :

If a particle possesses velocities represented by the sides, OA, AB, BC, CD,...KL of a polygon, the resultant velocity is represented by OL, the closing side of the polygon.

If L coincides with O, the polygon becomes a close one, and hence the resultant velocity vanishes. Consequently the particle would be at rest.

27. Resolution of Velocities.—So far we have been compounding two or more velocities into one : now let us see how to resolve a velocity into two components along given directions. Suppose OR is the velocity, and we want to resolve it into two components along AB and BC (Fig. 21). Draw through O a line parallel to AB, and through R parallel to BC, meeting each other at P. From the triangle of velocities we see that OP and PR are equivalent to OR. Therefore OP and PR are components in the required directions.

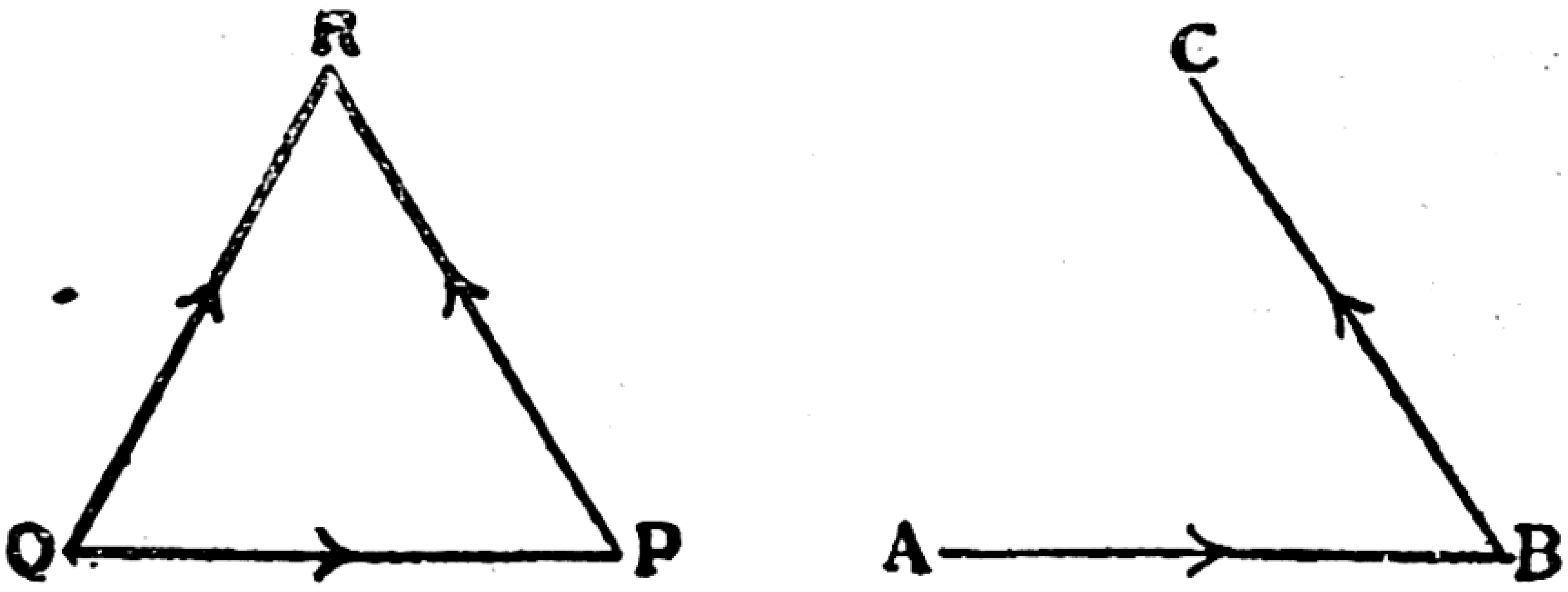


Fig. 21.

Generally we require two rectangular components. Suppose OR is the velocity (equal to u ft. per second) and we want to resolve it into two components at right angles to each other one of them along the line OB making an angle θ with OR (Fig. 22). Draw RB perpendicular to OB. Then the velocity OR is equivalent to OB and BR (from triangle law). We know that

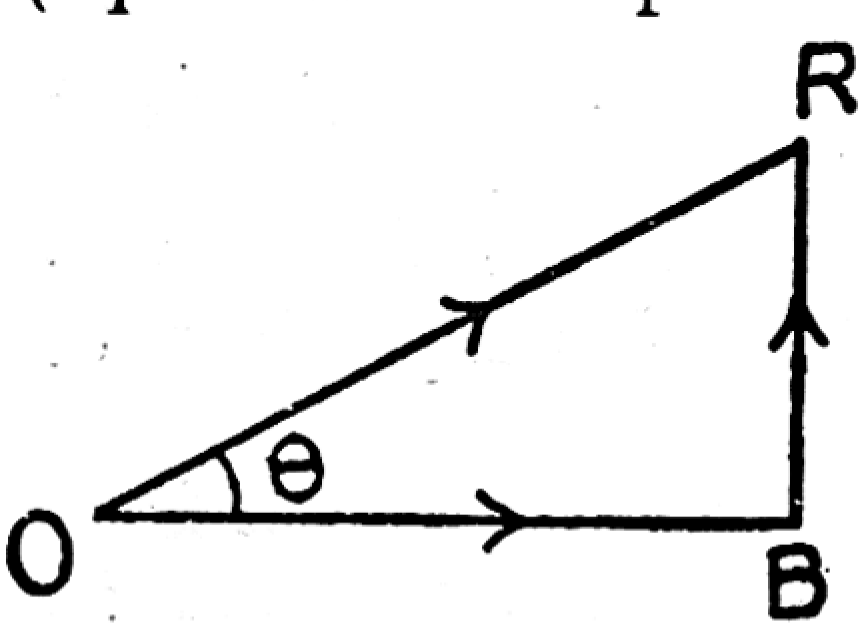


Fig. 22.

$$OB = OR \cos\theta = u \sin\theta.$$

and

$$BR = OR \sin\theta = u \sin\theta.$$

Therefore the rectangular components are $u \sin\theta$ and $u \cos\theta$.

The parallelogram, the triangle, and the polygon laws of velocities will hold good in the case of uniform accelerations as well. The method of resolution explained above will also hold good. Hence we shall not repeat these methods. We shall now consider the composition of a uniform motion with an accelerated one.

28. To Compound a Uniform Velocity with a Uniformly Accelerated Velocity.—Let us suppose that a man standing on the top of a tower throws horizontally a ball with a velocity of u ft./sec. ; on account of

gravity the ball will have acceleration in the downward direction. In one second it will pass over u ft. in the horizontal direction due to the horizontal velocity, and will fall vertically through $\frac{1}{2}g$ or 16 ft. due to gravity. Notice that we are supposing that the body starts with zero velocity in the vertical direction, as it had no velocity in that direction to begin with. If OP represents u ft. and PA 16 ft., then after one second the ball will be at A (Fig. 23). After the 2nd second the ball will be $2u$ ft. from the starting-point in the horizontal direction and $2g$ or 64 ft. below the horizontal line, and if $OQ=2u$ ft. and $QB=64$ ft. the ball will be at B; after the 3rd second it will be $3u$ ft. from the starting-point in the horizontal direction and $4{\cdot}5g$ or 144 ft. below the horizontal line, and if $OR=3u$ ft. and $RC=$ 144 ft. the ball will be at C, and, so on. Join the points O, A, B and C. The curve $OABC$ gives us the path of the ball. To find the form of this curve eliminate t from

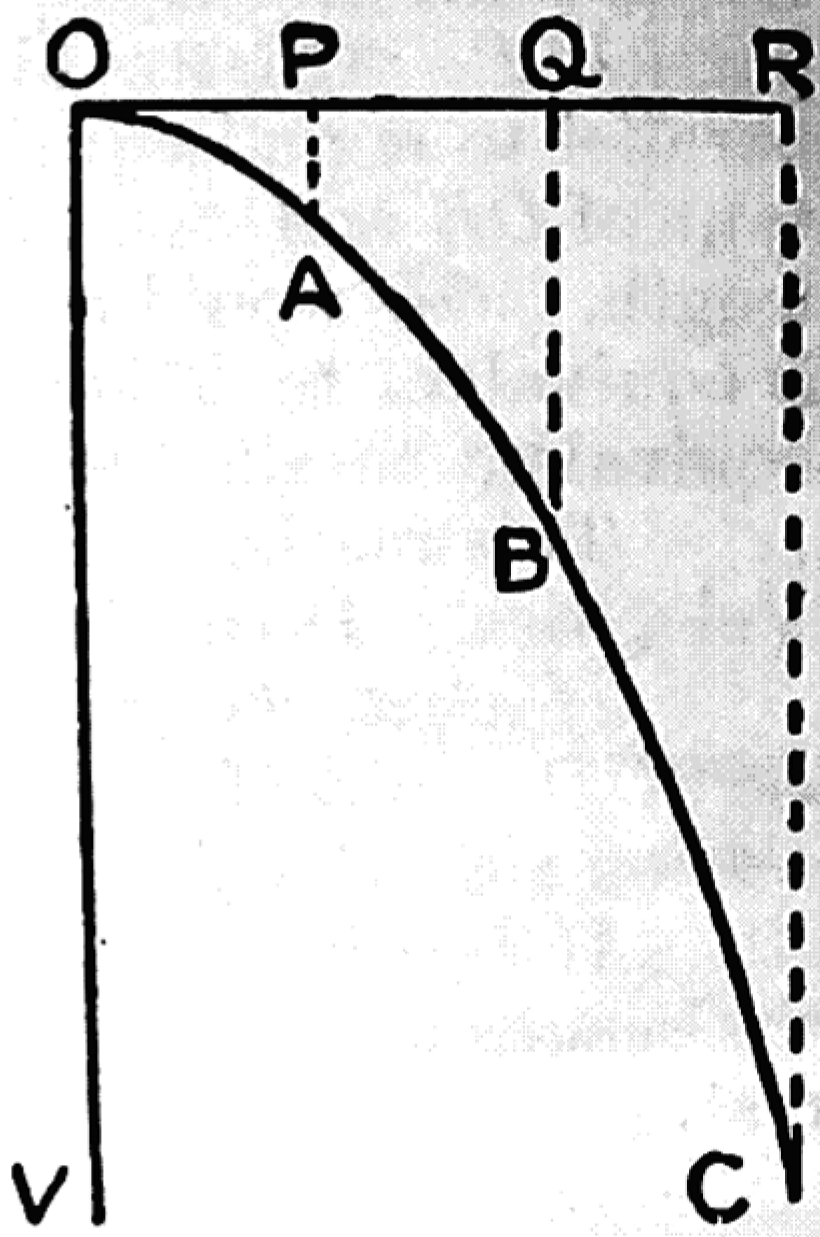


Fig. 23.

$$x=ut \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

and
$$y=\tfrac{1}{2}gt^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (b)$$

where x is the horizontal distance travelled by the ball in t seconds and y is the distance through which it falls down vertically. From relation (a) we know that $t^2=x^2/u^2$. Substituting this value of t^2 in equation (b) we get

$$y=\tfrac{1}{2}g.\frac{x^2}{u^2} \text{ or } x^2=\frac{2u^2}{g}.y$$
$$=ky \quad \ldots \quad (c)$$

where $k=\dfrac{2u^2}{g}$. The relation (c) is an equation of a parabola. Thus we learn that the path traversed by a body thrown horizontally with a certain velocity under the influence of gravity is a parabola.

Note that the horizontal velocity does not interfere with the vertical velocity produced by gravity. The two are quite independent of each other.

Suppose that a cannon ball is fired in a horizontal direction from a cannon with a velocity of 40 ft./sec., the cannon being fixed 144 ft. above the level of the ground. The ball would take 3 seconds to fall down, and in the mean time would travel 120 ft. in the horizontal direction. As said above the path described by it will be a parabola.

29. Relative Velocity.—So far we have been considering motion of bodies with respect to their surroundings which we regarded as fixed, but actually were not so. For instance, in the preceding article we have said that the path of a cannon ball fired horizontally from a cannon is a parabola. It is so with respect to the earth. But what it is with respect to space is difficult to tell. However, we are not the worse for it; for it would not help us very much even if we were to

know the actual path in space. All that we need know in practice is the path described by the ball and the velocity possessed by it with respect to the earth, considering the latter to be at rest.

As a matter of fact whenever we wish to find the relative motion of one body with respect to another, be this second body at rest or in motion, it is convenient to regard it at rest. We shall take some examples to clear this point.

(1) Consider two trains moving parallel to each other in the same direction with equal velocities, say 25 miles an hour, and suppose passengers, A and B (Fig. 24) look at each other. A, if he be unconscious of his own motion, will regard B at rest, which means in other words, that velocity of B, with respect to A is zero.

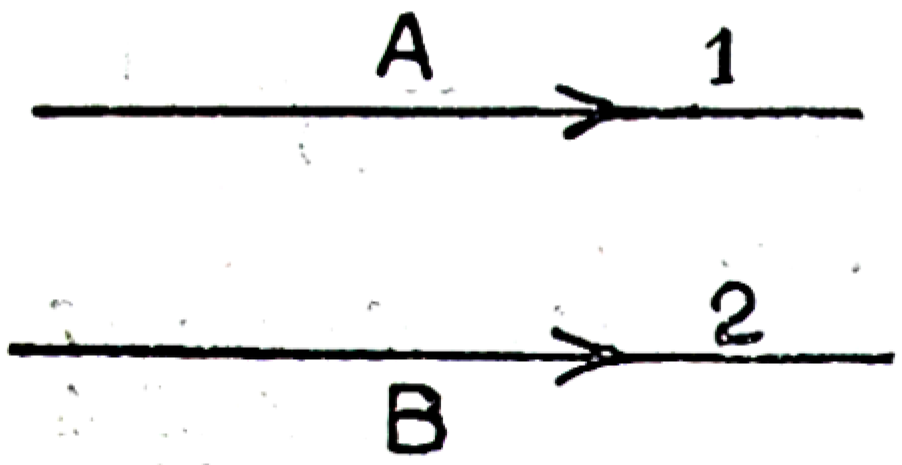


Fig. 24.

(2) Let the train No. 2 be moving faster than the train No. 1. Suppose the train No. 2 is going at the rate of 30 miles per hour, and the train No. 1 at the rate of 25 miles per hour. The passenger B will seem to the passenger A to be moving forward at the rate of 5 miles an hour.

(3) Let the train No. 2 move in the opposite direction to train No. 1. The passenger B will seem to the passenger A to be moving backward at the rate of 55 miles per hour.

If we analyse the above observations we shall find that in determining the velocity of B with respect to A we suppose A to be at rest. For instance, in the first example, if we impress a velocity of -25 miles an hour (*i.e.*, in the direction opposite to that in which the trains move) upon both the trains, the train No. 1, and hence passenger A, is brought to rest, and so is the case with the train No. 2 and passenger B. Therefore we say that B is at rest with respect to A. In the second example, when we impress upon the two trains and consequently on the two passengers a velocity of -25 miles an hour, passenger A is brought to rest whereas passenger B has a resultant velocity of 5 miles an hour. Similarly, in example (3), the resultant velocity of B becomes $-30+(-25)$, or -55 miles per hour. From this we arrive at the conclusion that :

The relative velocity of body B with respect to body A, when both of them are in motion, is obtained by compounding with the velocity of B a velocity equal and opposite to that of A.

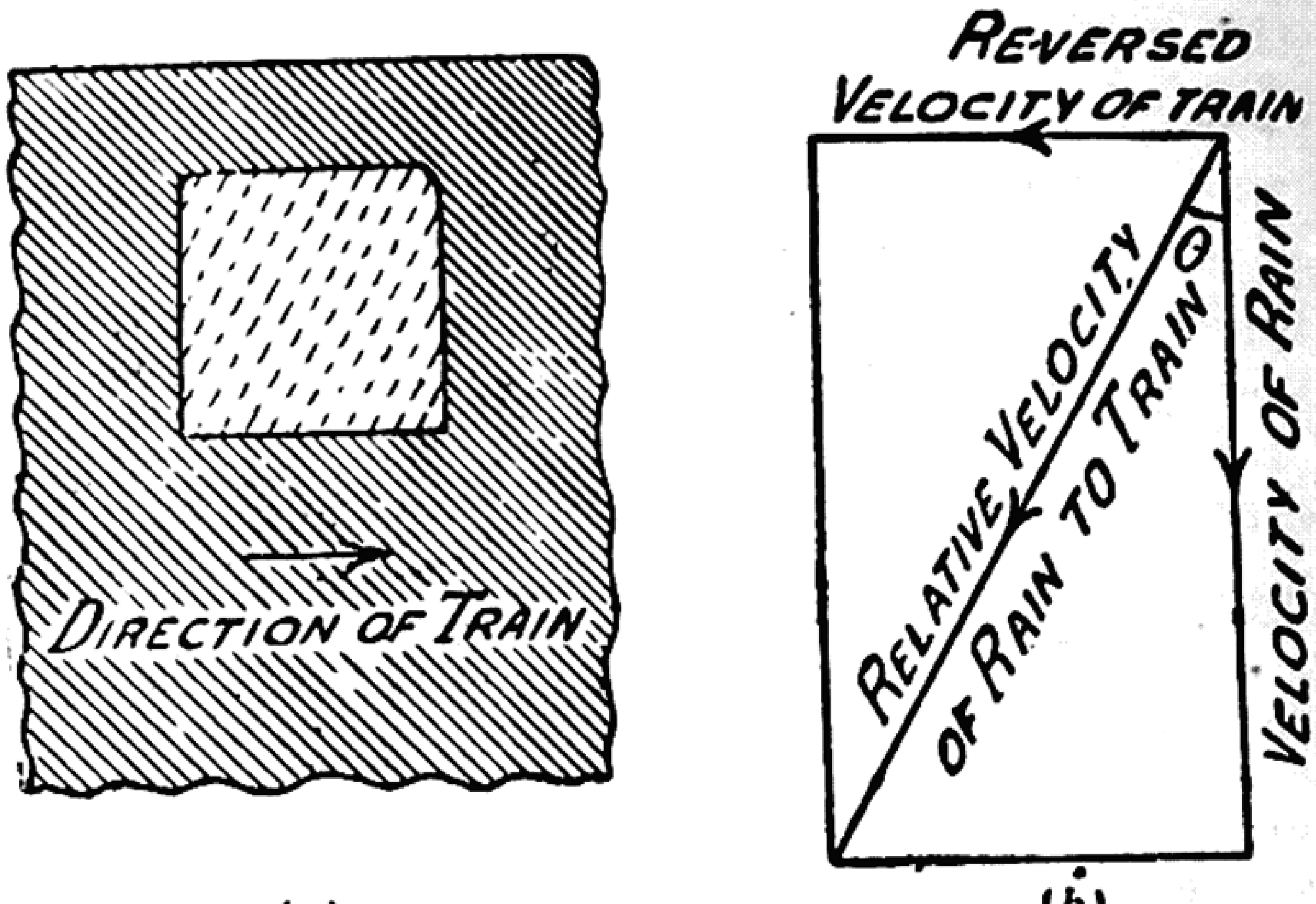
In order to understand thoroughly the principle explained above let us consider how rain falling vertically downward will appear to a passenger travelling in a train. When the train is standing still the rain appears to him to fall down along the vertical direction but when it begins to move the raindrops appear to be inclined away from the direction of motion of the train [Fig. 25 (*a*)]. As a matter of fact the faster the train moves, the more inclined would the raindrops appear to be. Let us see why ?

Suppose the train is going due east and the rain is coming down vertically. To find the relative velocity of rain with respect to the train, impress on the train as well as on the rain a velocity equal and opposite to that of the train [Fig. 25 (*b*)]. The train will be

brought to rest and the raindrops will appear to fall in the direction of the resultant of these two velocities. The angle of inclination θ with the vertical at which the rain appears to fall is given by the relation

$$\tan\theta = \frac{\text{velocity of train}}{\text{velocity of rain}}.$$

It is obvious from this relation that the greater the velocity of the train, the greater the value of tangent θ and hence of θ.



(a) (b)

Fig. 25.

We shall now take a numerical example. A man is walking due east at the rate of 2 miles/hour in a shower of rain which falls at an angle of 10° east of north with a velocity of 4 miles/hour. At what angle should he hold his umbrella to keep himself dry ?

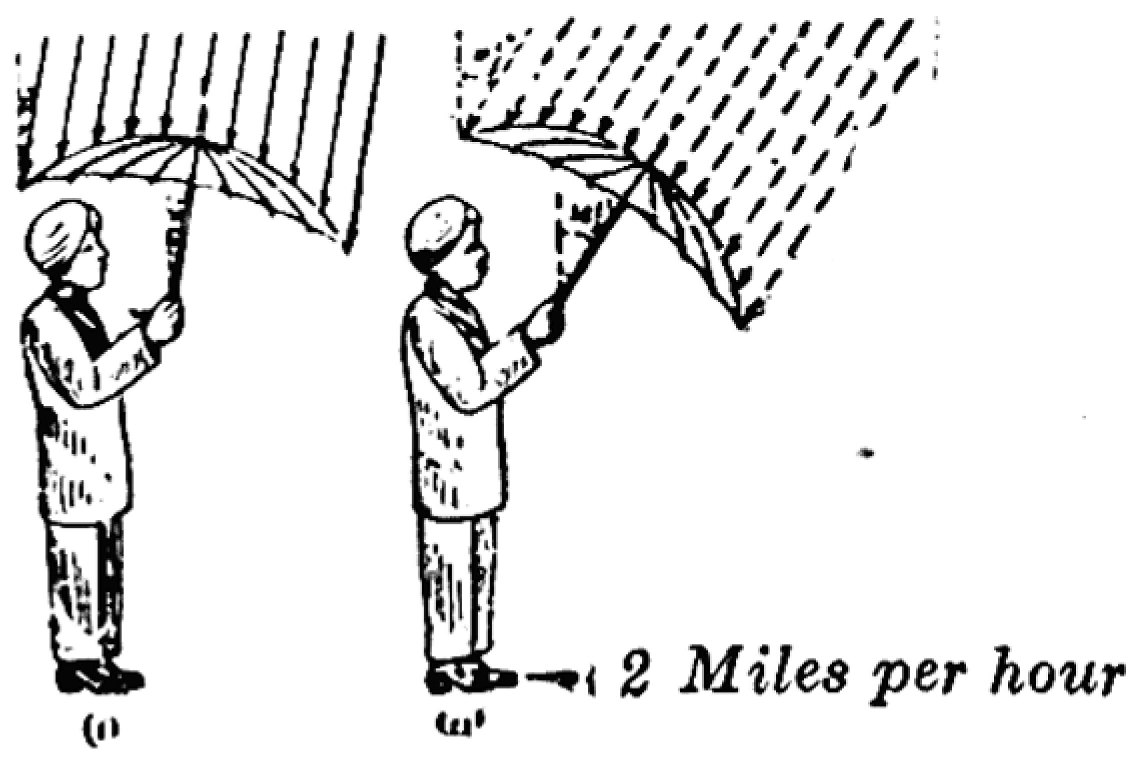


Fig. 26.

Let the rain come down with a velocity of 4 miles/hour along CO or BA and the man move with a velocity of 2 miles/hour along OA. To find the direction in which the rain will appear to come to the man, impress upon both the man as well as the raindrops a velocity of 2 miles/hour due west, *i.e.*, in the direction of AO. The direction of the resultant BO will give the apparent direction of the shower of rain.

$$\tan \angle BOA = \frac{BD}{OD} = \frac{4\sin 80^\circ}{2+4\cos 80^\circ}$$

$$= \frac{4\times 0{\cdot}9848}{2+4\times 0{\cdot}1736}$$

$$\therefore \quad \angle BOA = 55^\circ\text{-}36'$$

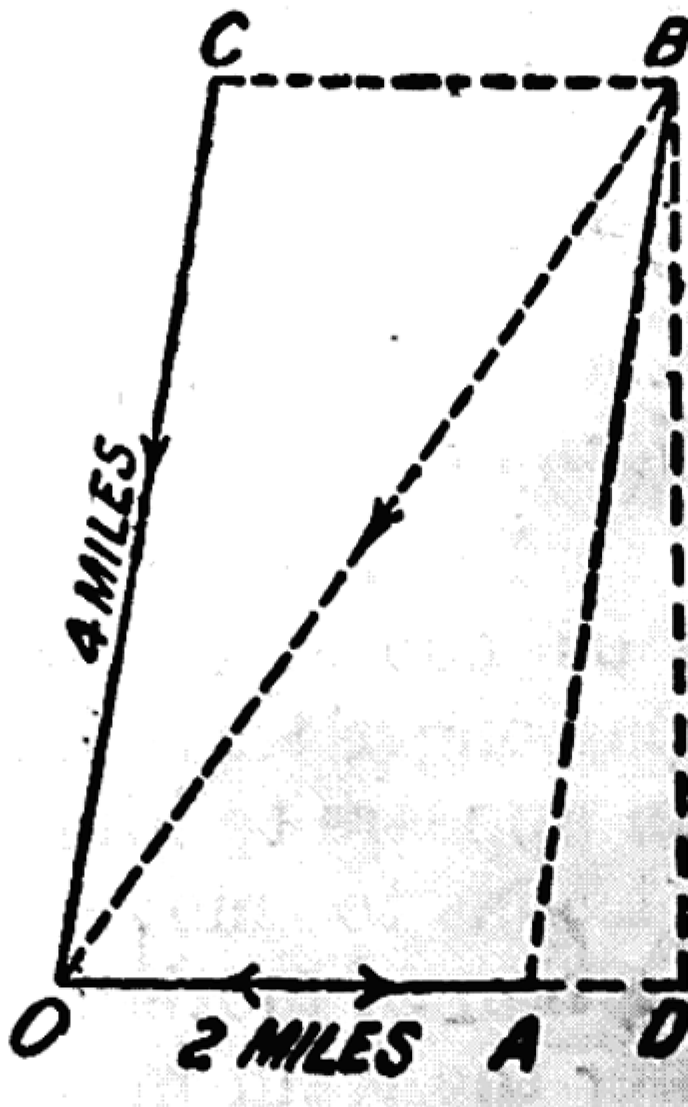


Fig. 27.

This shows that he should hold his umbrella making an angle of 34°-24′ east of north with vertical.

EXERCISES

1. An aeroplane pilot flies northward at a speed of 150 miles/per hour in a wind blowing at a speed of 20 miles/hour from the east. Calculate his speed with reference to the earth.

$$R=\sqrt{150^2+20^2}$$
$$=151 \cdot 3 \text{ miles/hour.}$$

The direction of his flight will be given by the relation $\tan \theta=\frac{20}{150}=0 \cdot 1333$ or $\theta=7°—40'$. This angle will be made by the aeroplane west of north.

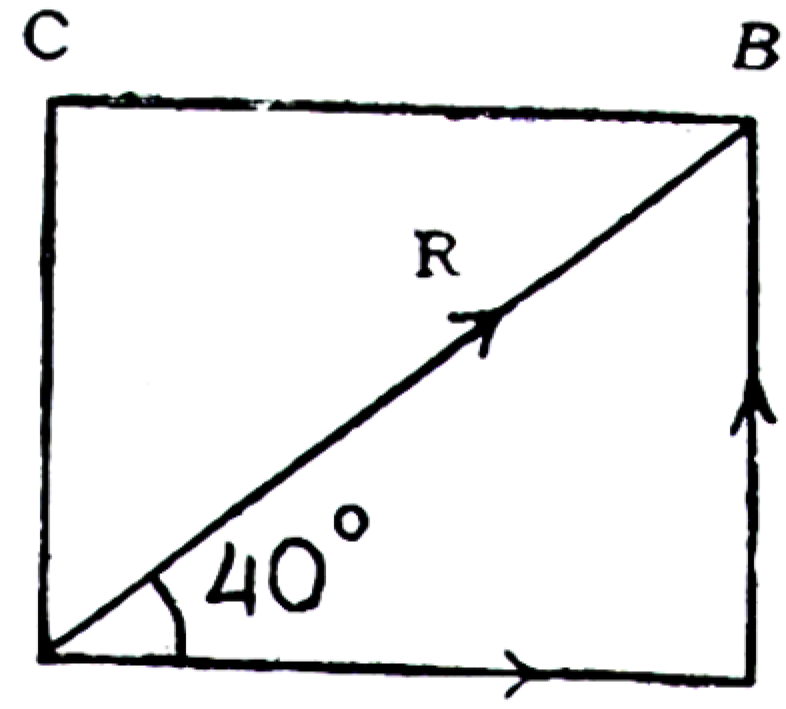


Fig. 28.

2. Find the rectangular components of a velocity of 60 ft./second ; one of them should make an angle of 40° with the original velocity.

The component along OA is $R \cos 40°$, and along AB or OC is $R \sin 40°$.

The component $OA=60 \cos 40°=60\times 0 \cdot 766=45 \cdot 96$ ft/.sec., and component $AB=60 \sin 40°$, or $60\times 0 \cdot 6428$
$=38 \cdot 568$ ft. sec.

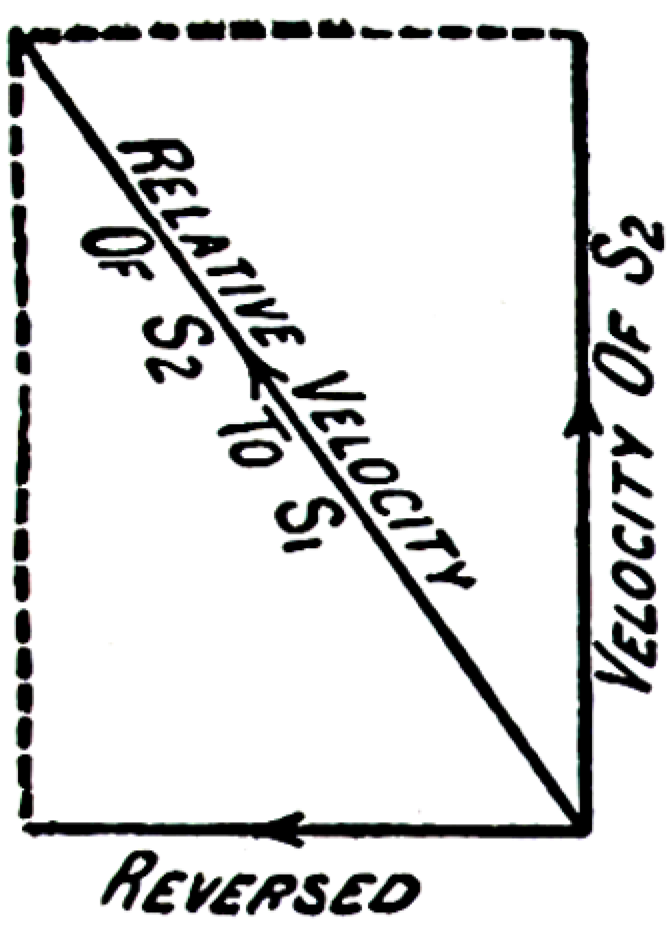


Fig. 29.

3. A ship S_1 is sailing due east at the rate of 12 miles/hour, and ship S_2 is sailing due north at the rate of 16 miles per hour, find the relative velocity of the second ship with respect to the first.

Impress upon both S_2 and S_1 a velocity of 12 miles per hour in the direction due west. The ship S_1 will be brought to rest and the ship S_2 will have a resultant velocity R which is given by the relation

$$R=\sqrt{16^2+12^2}=\sqrt{400}=20 \text{ miles/hour}$$

The direction in which S_2 will appear to sail to a passenger on S_1 will be given by

$$\tan \theta=\frac{12}{16}=0 \cdot 75$$
$$\therefore \quad \theta=37°.$$

It is seen from the figure that the angle θ is west of north.

4. A boat is rowed at right angles to the bank of a straight river at a speed half as fast again as the stream flows ; it reaches the opposite bank 2 miles below the starting-point. Find the breadth of the river and the distance rowed. *Ans.* 3 miles ; $\sqrt{13}$ miles.

5. A river 100 yards broad is running downwards at the rate $\frac{1}{2}$ mile per hour, and a swimmer, who can swim at the rate of 1 mile per hour wishes to reach a point just opposite. Along what line must he strike out, and how long will he take in crossing ?

Ans. 60° with the bank, 3·94 min.

6. On a particle two velocities are acting, one 4 ft./sec. due south and the other $2\sqrt{2}$ ft./sec. due north-east. Find the magnitude and the direction of the resultant. *Ans.* $2\sqrt{2}$ ft./sec. due south-east.

7. Two equal velocities have a resultant equal to either. At what angle are they inclined to each other ? *Ans.* 120°.

8. A velocity of 500 ft. per second is resolved into two components at right angles to each other. One of these is 250 ft. per second ; find the other. *Ans.* 250 $\sqrt{3}$ ft. per second.

9. One of the rectangular components of a velocity of 60 miles per hour is a velocity of 30 miles per hour ; find the other component. *Ans.* 30 $\sqrt{3}$ miles per hour.

10. Two trains start from a station with velocities of 25 and 40 miles per hour along two lines inclined at an angle of 60°. Find their relative velocity and the distance they will be apart in 10 minutes. *Ans.* 35 miles per hour, $5\frac{5}{6}$ miles.

11. A man is walking due east at the rate of 2 miles per hour in a shower of rainfall. The rain appears to him to come down vertically at the rate of 2 miles an hour. Find the actual velocity and the direction of the rainfall. *Ans.* $2\sqrt{2}$ miles per hour, 45° west of north.

12. A mail bag is to be dropped into a post office from an æroplane flying horizontally with a velocity of 60 miles/hour at a height of 640 feet above the ground. How far must the æroplane be from the post office at the time of dropping the bag so that the bag may fall directly into the post office ? *Ans.* 556 feet.

13. A bomb is dropped from a plane flying horizontally at a height of 10,000 ft. with a velocity of 120 miles/hour. With what velocity will the bomb fall on the ground and how far beyond the point vertically below the plane will the bomb strike the ground ? *Ans.* 819 ft./sec., 4,400 ft.

14. A stone is dropped from the window of a railway carriage moving at 40 miles/hour. If the window is 6 ft. above the ground, find the distance along the track where the stone strikes the ground. *Ans.* 35·92 ft.

CHAPTER III

The Laws of Motion

30. So far we have been considering motion in the abstract without any reference to the matter possessing it or to the forces producing it. Now let us consider the motion of real bodies under the action of forces. The laws of motion were first clearly enunciated by Newton. They run as follows :

First Law.—*Every body continues in its state of rest or of uniform motion in a straight line except in so far as it is compelled by the impressed forces to change that state.*

Second Law.—*The rate of change of momentum is proportional to the impressed force, and takes place in the direction of the force.*

Third Law.—*To every action there is an equal and opposite reaction.*

We shall consider these laws one by one.

31. First Law of Motion.—The first law contains two statements. The first of these, that a body continues in its state of rest except in so far as it is compelled by the impressed forces to change that state, seems self-evident. A book kept in an almirah in a library continues to be at the same place except when it is moved from there. The second part, that it will continue to move uniformly in the same straight line unless some force acts upon it, is rather difficult to understand in the beginning, because our everyday experience, that all moving bodies on the earth's surface stop after some time, *seems* to contradict it. A little reflection will, however, convince us that bodies stop, not because they have any tendency to do so but because of the frictional force of the earth's surface and of the resistance of the air. We have all observed that the more we eliminate the external forces, the longer the motion continues. Who, for instance, does not know that a ball, which stops after a short distance when rolled over a rough ground, goes a much longer distance when set rolling over a tarred road. A top, which rotates hardly for about 20 minutes in air, continues to rotate in the "sleeping" condition for more than 2 hours in an exhausted space. Experiments of this type lead us to the conclusion that if *all* the external forces were eliminated, the body would continue in its state of motion *for ever.*

Our everyday experience also serves to deepen our faith in the truth of this law. Let us see for instance, why a man, on stepping out of a rapidly moving train, falls to the ground with his face downward, if he does not run forward ? Before stepping out his whole body shares the motion of the train, but on jumping out, his feet are suddenly brought to rest whereas the upper part continues to move forward, with the result that he falls down.

The strongest proof, however, of the first law of motion as well as of the other two laws is indirect. On the basis of these laws the astronomical observations are predicted four years beforehand in the *Nautical Almanac.* The places of the sun, the moon and other satellites, the duration and the commencement of the eclipses, are all foretold to a wonderful degree of accuracy. Not even in a single case has our confidence in the truth of these laws been proved wrong, which shows that the laws underlying our calculations are without a flaw.

The property enunciated by the first law is called *inertia.* We define it as the inability of a material body to change by itself its condition of rest or of uniform motion in a straight line. It is on account of this property that matter requires force to change its state. Sometimes the first law of motion is referred to as the *Law of Inertia.* Inertia is a general property of matter, and is made use of to measure the mass of a body.

The first law states that unless some force from without acts upon a body, its motion continues unchanged.

The second law tells us how the motion changes when a force acts upon it.

32. Second Law of Motion.—It says that the rate of change of momentum is proportional to the impressed force and takes place in the direction of the force. Before we discuss this law of motion, it is necessary to explain what we mean by the term **Momentum**. A motor car moving slowly can be stopped more easily than when it is going fast. A cricket ball when hit lightly can be caught easily but when hit hard the catch is difficult. These examples show that the greater the velocity of a body the greater the force required to stop it.

This is not all, for a ping-pong ball and a cricket ball may be moving with the same velocity, but who does not know that a much greater force is required to stop the cricket ball than the ping-pong ball. This shows that the force required to stop a moving body, or what is the same thing, to move a body with a given velocity, depends upon two things, (a) its mass, and (b) its velocity. The product of the two quantities, mass and velocity is called *momentum.*

Suppose the momentum of a body to begin with is mu and after t seconds it is mv. The second law states that

$$F \propto \frac{mv-mu}{t}$$

or

$$\propto m\frac{v-u}{t}$$

where F is the force, m the mass of the body, and v and u denote the final and initial velocities and t the time during which the change in velocity takes place.

Since $$\frac{v-u}{t}=a, \text{ the acceleration.}$$

$$\therefore \quad F \propto ma,$$

or $$F=k\times ma$$

where k is some constant. This relation enables us to fix upon a unit of force.

If we define unit force as that much force which is just able to produce unit acceleration in unit mass, we can write the above equation as

$$1 = k \times 1 \times 1$$

or $$k = 1,$$

In other words in this supposition the equation $F = kma$ is reduced to

$$F = ma.$$

Since mass and acceleration are measured differently in the F. P. S. and C. G. S. systems there are two units of force.

When unit of the mass taken is 1 pound, and unit of acceleration 1 ft./sec.^{2}, the unit of force is called a **Poundal.** *It is that much force which produces in a mass of one pound an acceleration of 1 ft. per second per second.*

When unit of mass taken is 1 gram, and unit of acceleration 1 cm./sec.^{2} the unit of force is called a **Dyne.** *It is that much force which produces in a mass of one gram an acceleration of 1 cm. per second per second.*

The poundal and the dyne are termed the absolute units of force.

33. Impulse.—Let us consider once more the formula

$$F = ma = m\,\frac{v-u}{t}.$$

We can write it as $Ft = mv - mu$.

This relation tells us that the change in momentum is equal to the product of force and time. In all those cases where the force acts for a short time only, as, for instance, in the case of a blow or a collision, this relation is extremely useful, for it enables us to calculate the average force acting during the time. The product of force and time is called **Impulse.** It is equal to the total change of momentum produced in a body.

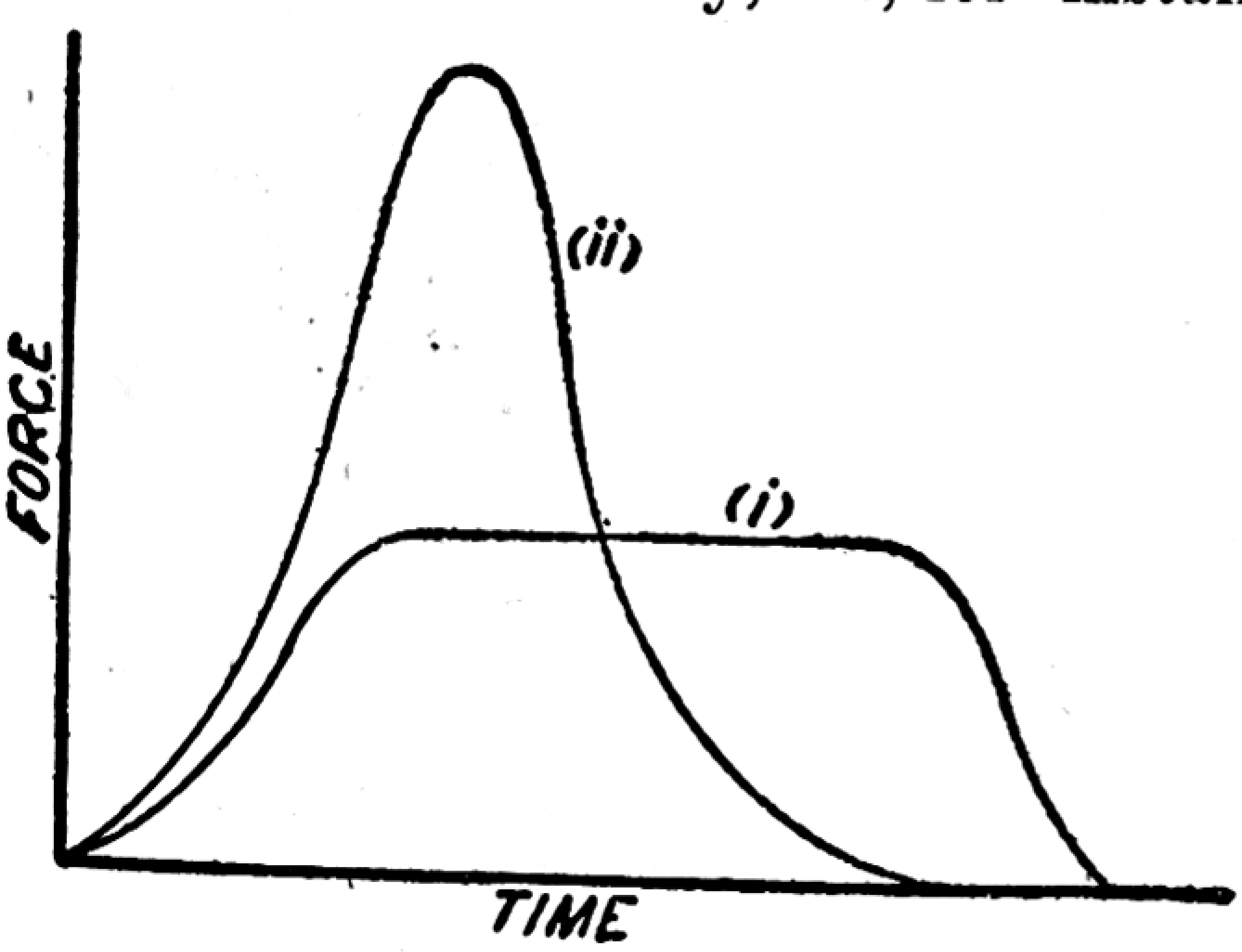


Fig. 30.

If we plot a force-time curve for a blow the area of the curve gives a measure of the blow or impulse. Curves (*i*) and (*ii*) have equal areas and hence represent equal impulses or blows. But it is obvious from the figure that the force in case of blow no. (*i*) is much less than in case of blow no. (*ii*).

Let us consider an example to illustrate the usefulness of the idea of impulse .

Example. A cricket ball of mass $5\frac{1}{2}$ oz. moving with a velocity of 40 feet per second is brought to rest in $\frac{1}{20}$ th of a second by a player. Calculate the average force applied by him.

$$\text{Momentum of the ball} = \frac{11}{2} \times \frac{1}{16} \times 40$$
$$= \frac{55}{4} \text{ F. P. S. units.}$$

Hence $\quad$ Impulse $= Ft = \frac{55}{4}$.

Substituting $\frac{1}{20}$ of a second for t we find that

$$\text{average force} = \frac{55}{4} \times 20 = 275 \text{ poundals.}$$

If the ball were stopped in $\frac{1}{10}$ th of a second, the average force exerted by the player would have been 137·5 poundals. This shows that by doubling the time, the force needed to stop the ball becomes one-half. It is to increase the time that in catching a ball a cricket player lowers his hands. If he were to catch the ball by keeping his hands still, he would require a much greater force to destroy the momentum of the ball. The force required may hurt his hands.

For the same reason a man who falls from a height on a pucca floor receives far more serious injuries than a man who falls on a heap of sand.

It is to reduce the force of the blows received by carriages when moving over uneven roads that they are fitted with springs.

34. Weight.—We have said in §19 that bodies falling freely under the influence of gravity fall towards the earth with an acceleration of g (*i.e*, 32 ft./sec.^{2}). Since by Newton's second law, mass × acceleration = force, it is clear that a body must be pulled downwards by the earth with a force mg, where m is its mass and g the acceleration. This downward force due to the attraction of the earth acting on a body is called its **Weight.** If we take two bodies, one of mass m and the other of mass m', their weights will be mg and $m'g$.

Dividing the weight of one by the weight of the other, we get

$$\frac{W}{W'} = \frac{mg}{m'g} = \frac{m}{m'}.$$

Thus we see that *the weights of two bodies at the same place are in proportion to their masses.* Of course the weight of a body is not a constant quantity since g is different at different places. The weight of a body, for instance, at the poles is greater than at the equator.

35. Distinction between Mass and Weight.—Unfortunately, hopeless confusion is often made by a beginner between the terms *mass* and *weight.* The confusion arises on account of the fact that we compare masses of bodies by *weighing* them. Let us see how. A man goes to a shop to buy 10 lb. of sugar. The shopkeeper, while weighing out sugar, compares not the mass but the weight of sugar with the weight of his standards, whereas all the time the buyer is thinking of the mass. He would be glad to have one handful more ; he does not bother himself about the force with which the earth pulls it down. He

has been thinking all along of the quantity. But when he carries it home, the inconvenience which he feels is due to the weight. Thus in this example in the first part we are concerned with mass and in the second part with weight. The quick succession of the ideas of mass and weight, coupled with the shopkeeper's weighing leads to confusion between the two notions.

To bring home to the student the difference between mass and weight we shall discuss a few examples.

(1) Suppose we weigh out 201 ounces of silver at the north pole with a *spring balance* and entrust it to a gentleman to hand it over to some banker in Colombo. The piece of silver when weighed there with the same spring balance will be found to weigh about 200 ounces only. During this transaction the quantity (mass) of silver has remained the same whereas the force with which the earth pulls it down has decreased. If it were taken to a place 4,000 miles above the surface of the earth, its weight would become 50 oz Let us suppose next it is taken to the centre of the earth ; it will have no weight at all there but its mass would not change.

(2) When a bullet strikes a target, the force it exerts depends on its mass and not on its weight. The force exerted on the target would be the same, on the surface of moon as on earth.

(3) If we spin a flywheel of an engine on the top of a mountain and on the plains with the same speed, we shall find that the force required at both the places is the same, although the weight will be different at the two places. It is due to the fact that the force required is proportional to the mass of the wheel, which remains constant.

These examples will help the student to think of the mass as something different from the weight.

It should be noted that an ordinary pair of scales helps us to compare masses and not weights, for both the substances in the pans are equally affected by gravity. [See § 34.] But in the case of a spring balance it is the weight that we measure and not the mass for the increase in the length of the spring depends upon the force pulling it downwards.

In Example (1) if we had sent an ordinary pair of scales along with the weights from the pole to Colombo, no change in the weight of silver would have been detected.

36. Gravitational Units of Force.—The absolute units of force, the poundal and the dyne, are too small to measure the actual forces with which we have to deal in our daily life. Hence the engineers use the weight of one pound and the weight of one gram as their units ; these are called the **Gravitational Units**. Since the value of g is different at different places the gravitational units are not constant. The actual variation in their value is, however, so small that for purposes of measuring forces we generally ignore it. So the units adopted are :

$$1 \ lb. \ wt. = g \text{ poundals} = 32 \text{ poundals.}$$
$$1 \ gm. \ wt. = g \text{ dynes} = 981 \text{ dynes.}$$

37. Comparison of Masses.—One method of comparing masses has already been explained. It is based upon the fact that the weights are proportional to the masses. This is the method adopted by a shopkeeper, who balances the weight of the bodies with the weights of his standards and thereby compares the masses. A second method is the direct application of the second law. Equal forces are impressed upon the two bodies whose masses we want to compare.

If their masses be m', and m'', and accelerations a' and a'', from Newton's second law ($F=ma$) we have

$$m'a'=m''a'',$$

or

$$\frac{m'}{m''}=\frac{a''}{a'}.$$

This shows that the masses are inversely proportional to the accelerations that are produced in them.

If

$$a'=a''$$
$$m'=m''.$$

38. Composition of Forces.—When a number of forces act upon a particle simultaneously, each one of them produces the same effect which it would do, if all others were absent. This is called the **Physical Independence of Forces.** We have already discussed on page 32 the case of a cannon ball projected horizontally. We saw there that the horizontal force produced its own effect (*i.e.*, motion in the horizontal direction), whereas the downward pull of the earth produced downward acceleration irrespective of the horizontal force. Though each force produces its own effect the body moves under their joint influence, unless the forces neutralize each other completely. The same motion may be produced by a single force. This last force is called the **Resultant** of the previous two or more forces. Since force is a *vector* quantity, we cannot use the ordinary arithmetical method for finding the resultant. We must use the method of parallelogram law or the triangle law. The **Parallelogram Law of Forces** is stated as follows :

(1) *If two forces acting on a particle be represented in magnitude and direction by the two sides of a parallelogram drawn from a point, the resultant is represented in magnitude and direction by the diagonal* (*of the parallelogram*) *passing through that point.*

The Triangle Law is stated as under :—

(2) *If two forces acting on a particle be represented by the two sides OA and AB of a triangle taken in order, they are equivalent to a force represented by the third side OB.*

We can also prove with the help of an experiment that two forces acting on a body are equivalent to a single force represented by the diagonal of a parallelogram constructed with the two forces as adjacent sides.

39. Experimental Proof.—*ABCD* (Fig. 31] is a wooden board held in a vertical plane by two stands. *E* and *F* are two light pulleys moving freely about their axles. *HOG* is a thread passing over the pulleys and carrying hooks at the ends. A second thread *OK* is knot-

ted at O, carrying also a hook at the other end. Slotted weights are slipped on, to each hook, and the whole system is allowed to come into equilibrium. Suppose the weights supported are P, Q, and R grams. Evidently since the three forces are in equilibrium, any one of them must be equal and opposite to the resultant of the other two. R, for instance, will be equal and opposite to the resultant of P and Q. Draw lines OG, OH, and OK parallel to the threads on the paper fixed on the board. Adopting some convenient scale (say 1 cm. to represent 10 grams), mark off OL, OM and OK to represent P,Q and R grams. Complete the parallelogram $OLNM$, ON will be found equal in length to OK, which represents R. But R is equal and opposite to the resultant of P and Q, hence we see that the resultant is represented, numerically as well as in direction, by the diagonal of the parallelogram $OLNM$, whose two adjacent sides OL and OM, represent the forces P and Q. Further, since LN is parallel and equal in length to OM, it represents the same force which OM represents, *i.e.*, Q. Moreover, $NO=OK$, therefore, ON represents R. The three forces P, Q and R which are in equilibrium are represented by the closed triangle OLN. Thus we see that, when the forces can be represented by the sides of a closed triangle, they are in equilibrium.

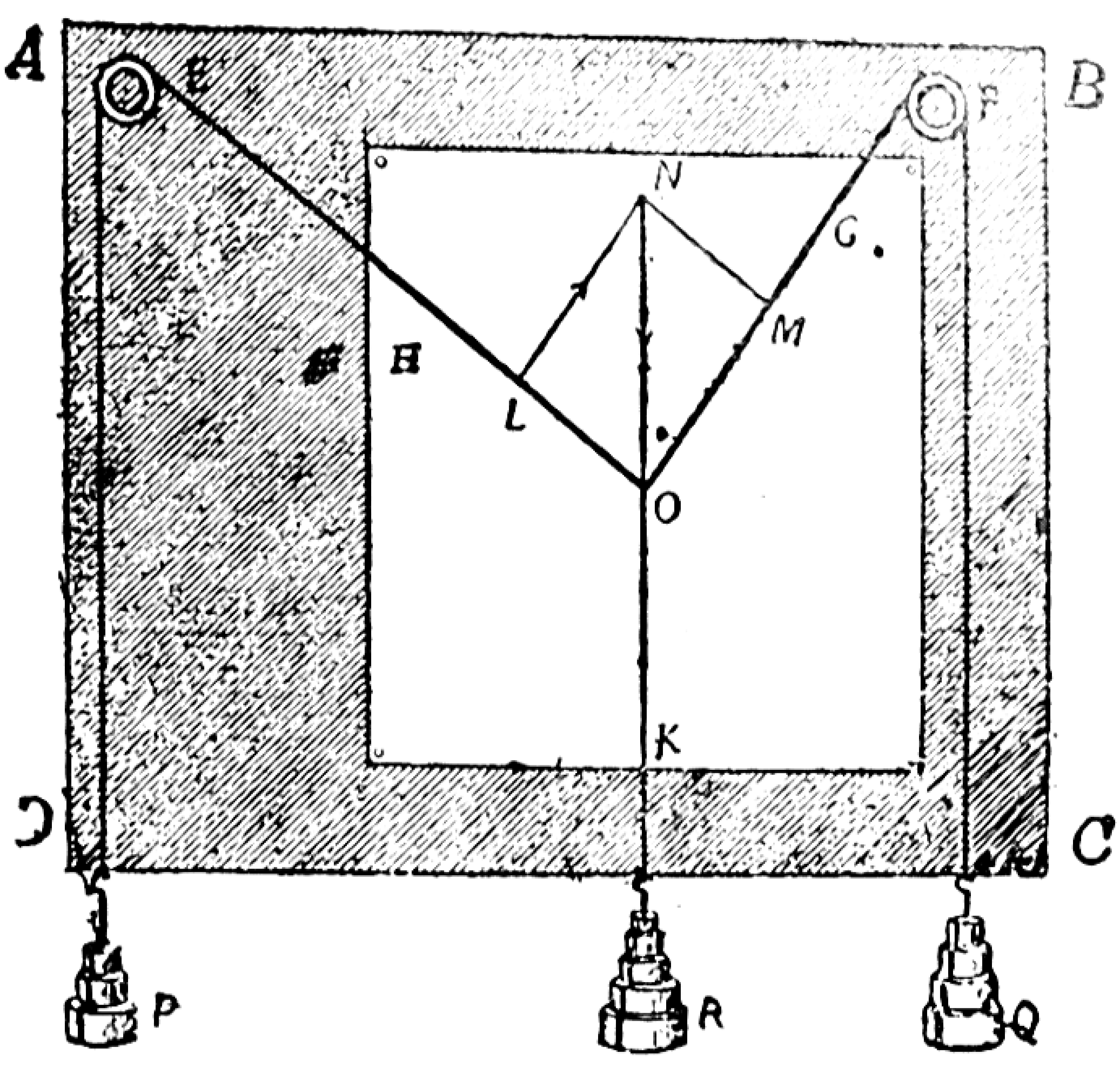


Fig. 31.

If there be more than two forces to be compounded, the **Law of Polygon of Forces** is used, which runs as follows :

If upon a particle be impressed simultaneously forces represented by the sides OA, AB, BC,...KL of a polygon, the resultant force is represented by OL the closing side of the polygon.

If the polygon is a closed one *i.e.*, if L coincides with O, the forces are in equilibrium.

40. Resolution of Forces.—A single force may be resolved like a velocity into components. We can find its component in any direction we like. Suppose OR represents a certain force F, and we want to determine its component along OX. From R draw RB, perpendicular on OX. The force OR is the resultant of the forces OB and BR. Therefore, the component along $OX=OB=F \cos \theta$, where θ is the angle which the resolved part makes with the original force F. The greater the angle the smaller the component. When the angle is 90°,

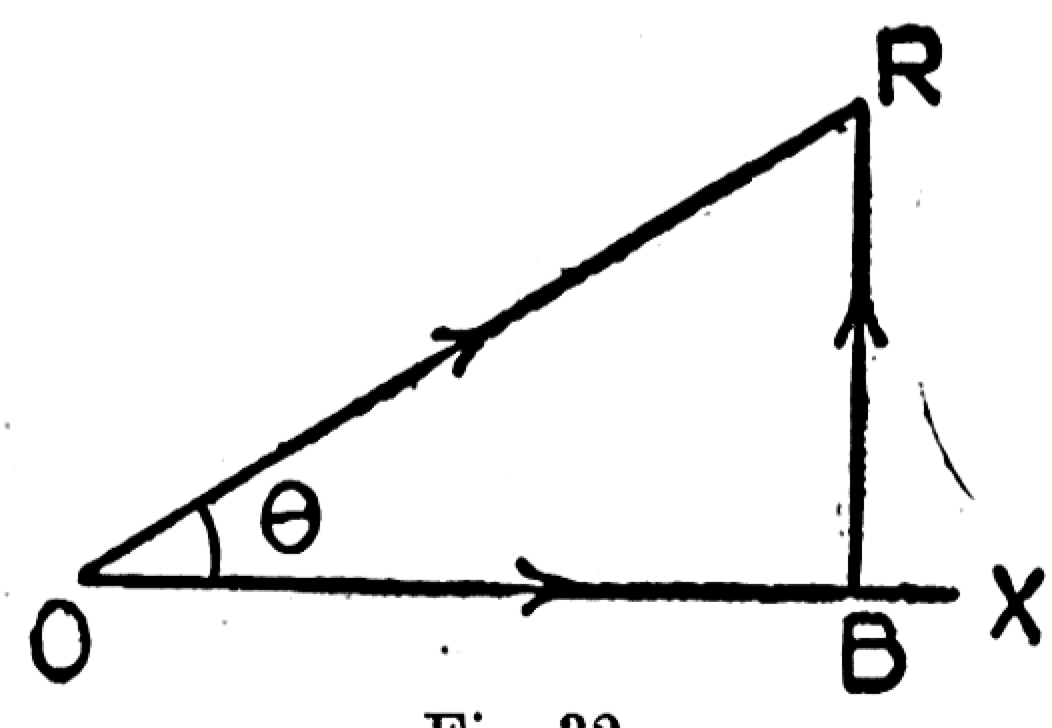


Fig. 32.

cosine is zero, hence $F \cos\theta$ component of a force in a direction perpendicular to it is zero. This shows that *a force cannot produce any effect in a direction perpendicular to itself.*

41. Uniform Motion in a Circle.—Consider the case of a particle which moves in a circle of radius r, with a uniform velocity v*. Suppose it starts its motion from P (Fig. 33), and moves from P to Q in t sec. Evidently $PQ = vt$.

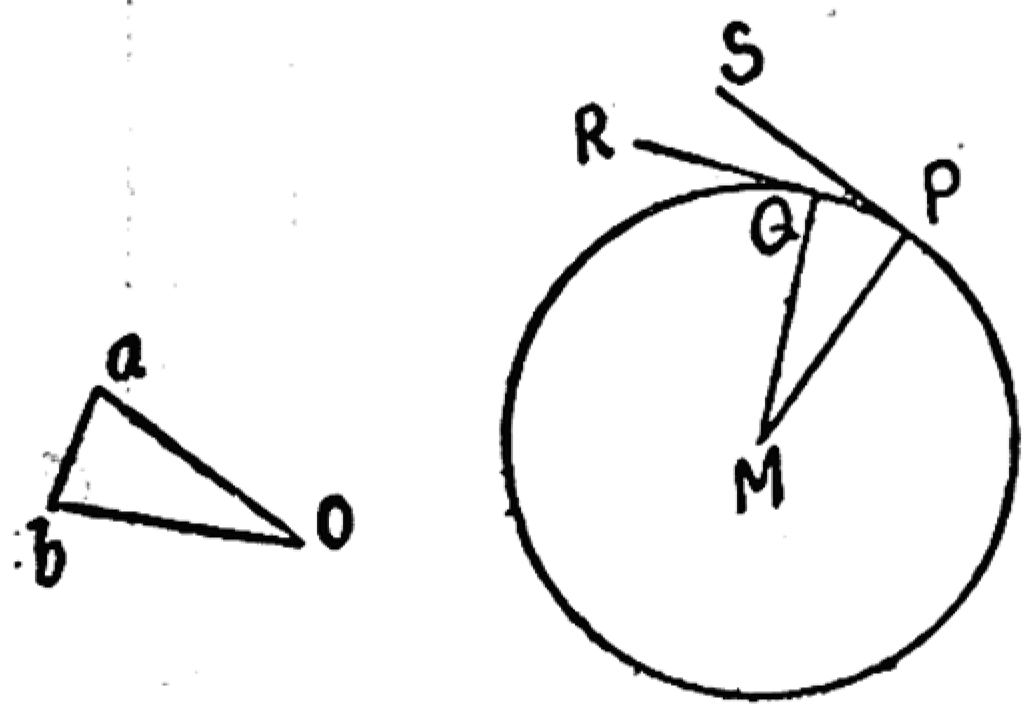


Fig. 33.

We know from Newton's first law of motion that a body would continue to move in a straight line unless some force acts on it. And since the particle moves in a circle, some force must be acting on it which pulls it away from its straight course. Further as the motion is uniform, the force must be producing no acceleration in the direction of motion, for otherwise in accordance with the second law the speed would change. Hence the acceleration must *always* be at right angles to the path of motion and so must be the force. This is possible only if the force acts along the radius of the circle, and is hence directed towards the centre.

Thus we learn that *when a particle moves in a circle with a uniform velocity, it is subject to a force directed towards the centre.* This force is called the **Centripetal Force**.

Let us find the magnitude of this force. Suppose a particle moving in a circle with a uniform velocity v starts from P, and goes over to Q in a short interval of time t (Fig. 33). As said before $PQ = vt$. At P the velocity is along PS, and at Q it is along QR. Draw oa parallel to PS to represent the velocity v at the point P. Next draw ob parallel to QR to represent the velocity v at Q. As speed is uniform, $oa = ob$.

By triangle of velocities ob, the final velocity, is equal to the resultant of initial velocity oa, and *the change in velocity*, ab, which takes place during the time t. Therefore the acceleration is $\dfrac{ab}{t}$.

Since the interval of time t is very small, we can consider PQ as a straight line and the sector MPQ as a triangle.

The two Δs, MPQ and oab, are similar.

$$\therefore \quad \frac{PQ}{PM} = \frac{ab}{ao}, \quad \text{or} \quad \frac{vt}{r} = \frac{ab}{v}, \quad \text{or} \quad \frac{ab}{t} = \frac{v^2}{r}$$

But $\dfrac{ab}{t}$ is the acceleration, $\therefore$ the acceleration $= \dfrac{v^2}{r}$.

If the mass of the particle is m,

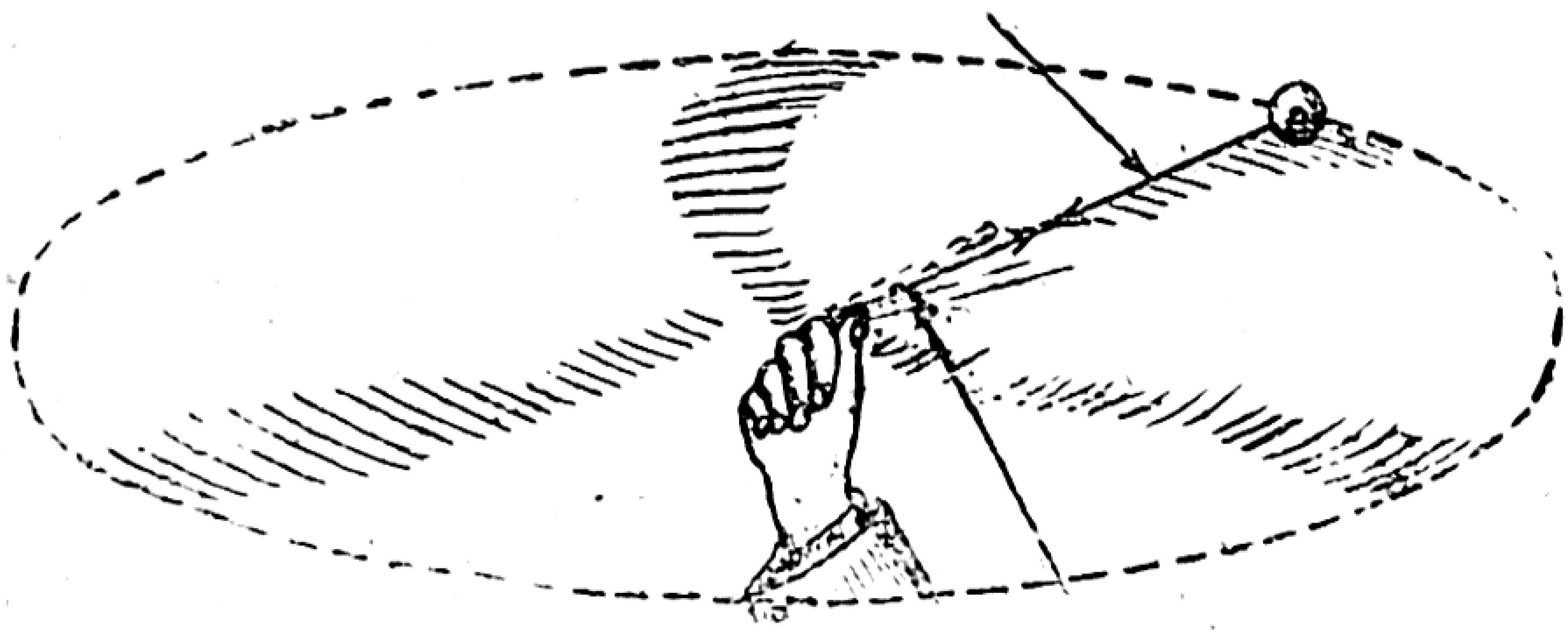
$$\textit{the centripetal force} = \frac{mv^2}{r}.$$

To experimentally verify the presence of the centripetal force in the case of a circular motion, whirl a small piece of stone tied to a

*Velocity is used here in the sense of speed.

string ; you will notice that you have to pull the string inwards in order to whirl the stone, and the faster you whirl it the harder you have to pull the string.

Centripetal Force



Centrifugal Force

Fig. 34.

An equal and opposite force is exerted by the stone on the hand. This force on the hand is called the **Centrifugal force.** Mud sticking to a bicycle tyre is pulled radially by the force of adhesion ; when this pull is less than the centripetal force required to move the mud in a circle the mud flies off tangentially.

The sparks which fly off from the grinding stone of a blacksmith are also an illustration of the same phenomenon.

Machines which depend for their working on the centrifugal force are called *centrifuges*. Such machines are found in use in sugar factories, laundries, dairies etc.

In a sugar factory, the juice of suger cane is first evaporated to the point of crystallisation and then crystals are separated from the remaining liquid by means of centrifuges.

In a laundry, wet clothes are whirled in a large vessel, the sides of which are perforated with hundreds of small holes. The water is thrown off at a tangent through these holes and the clothes thereby dry up soon.

In a dairy, cream is separated from the rest of the milk called skim-milk (consisting of water, casein, and sugar) by means of a centrifuge called cream-separator. To understand how the separation takes place we have only to remember that the heavier the body the greater the centripetal force required by it to move in a circle. Now since the skim-milk is heavier than cream its particles require a greater force to move them than the cream particles do and therefore the skim-milk remains nearer the walls of the spinning vessel whereas cream goes nearer the axis.

Example. A boy whirls in a circle of radius 40 cm. a 100 gm. stone tied to a string four times a second, show that the tension in the string is more than twenty-five times the weight of the stone. How many times to its weight would the tension be if the stone were whirled eight times a second ?

Since tension is equal to the centrifugal force, its value is given by $\frac{mv^2}{r}$. We know that $v=2\pi r\times 4=8\pi r=8\pi\times 40$ cm./sec.

$$\therefore \text{ Tension} = \frac{100 \times (8\pi \times 40)^2}{40} = 100 \times 64\pi^2 \times 40 \text{ dynes.}$$

$$= \frac{100 \times 64\pi^2 \times 40}{981} \text{ gm. wt.}$$

$$= \frac{100 \times 64 \times 9{\cdot}87 \times 40}{981}$$

$$= 100 \times 25{\cdot}75 \text{ gm. wt.}$$

If the stone is whirled eight times per second the tension will become about 103 times the weight.

If in the above example the string breaks, the stone flies off tangentially. Why ? Because as soon as the string breaks, the centripetal force ceases to act on the stone, which on account of inertia continues its motion in a straight line.

41a. Banking of Curves.—Curved tracks are banked on the outside so that a fast moving train or motor car inclines inwards to balance the centrifugal force that might throw it off the track. The sharper the curve the greater the banking required. It is interesting to note that the banking depends upon the radius of the curve and the speed of the vehicle and not on its mass.

A cyclist, while rounding a curve, leans inwards for the same reason.

In Fig. 35 (a) is represented a car rounding a curve banked at an angle θ. Two forces act upon the car; the pull of gravity mg downwards and the reaction R of the road on the car perpendicular to the road-bed in the upward direction. These forces are shown in the diagram on the right. In order that the motor car may be provided with the required centripetal force the direction of reaction must be such as to neutralise the weight mg, and supply the necessary centripetal force $\frac{mv^2}{r}$, where m is the mass, v the speed of the car and r the radius of the curve to be rounded.

Since the angle θ of banking of the road is equal to the angle between R and mg as shown in Fig. 35 (b) it follows that,

$$\frac{mv^2}{r} \cdot \frac{1}{mg} = \tan\theta$$

or

$$\frac{v^2}{rg} = \tan\theta$$

We can rewrite this relation as

$$\theta = \tan^{-1}\frac{v^2}{rg}.$$

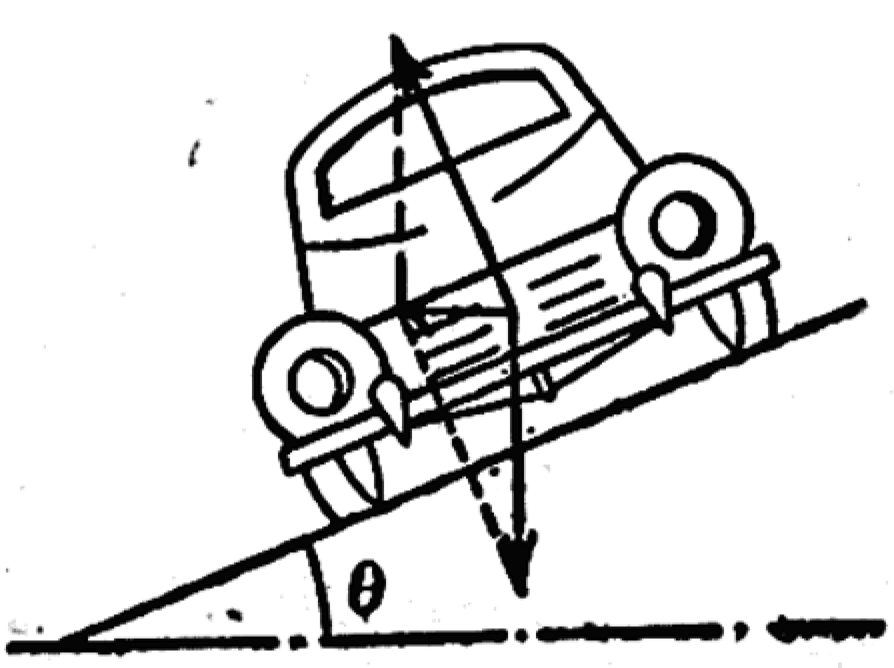


Fig. 35. (a)

Fig. 35 (b).

This relation shows that the angle of banking is independent of the mass of the vehicle. It only depends upon the speed, and radius of the curve.

If the road were not banked, the only way to get the required centripetal force is by means of friction between tyres and road-bed.

As the centripetal force supplied by friction is likely to be small and uncertain, the curve must be rounded at a low speed to avoid skidding. To overcome this difficulty the road is banked. For a given radius no one angle of banking is correct for all velocities. The roads and tracks are banked for the *average speed* of the traffic expected to use them. For slow vehicles, the angle will be too steep whereas for fast ones it will be somewhat small.

42. Third Law of Motion.—To every action there is an equal and opposite reaction. This law states that the forces must exist in pairs, that is to say, to each force in nature there corresponds an equal and opposite force. Let us take a few illustrations.

If a tea-cup is struck against the edge of a table, the table exerts a force on the cup and breaks it.

If we step out of a boat, we go in one direction and the boat goes in the other direction.

When a bullet is fired from a gun, the bullet goes forward and the gun "kicks" backward.

Unless there is this opposite force, or reaction as it is called, we cannot have an action. For instance, we cannot drive a nail into a wooden block unless it is supported against something and hence can offer reaction. Nor can we cut a piece of paper with one blade of a pair of scissors.

If all forces exist in pairs, *i.e.*, if action and reaction are equal and opposite why should an object move at all ? The reply is that action and reaction never act on the same body. They act on different bodies.

We shall consider a few illustrations to make this point clear :

(1) When a horse pulls a cart, the cart pulls the horse backwards with an equal force. You will ask, "if it be so, why should they start at all ?" Before we answer this question let us first understand what the question is : Do we want to know why the cart moves, or why the horse moves forward, instead of backward in which direction it is pulled by the cart or why they both move ?

So far as the cart goes, the problem is quite simple. There is the weight of the cart acting downwards, there is the reaction of the earth upon the cart in the upward direction ; since these forces are equal and opposite they cancel each other, and hence produce no effect. The only other force on the cart is the forward pull of the horse transmitted along the traces. On account of it, in accordance with the second law of motion, the cart moves forward.

Now let us examine the motion of the horse. The weight of the horse acts downward, and the reaction of the earth acts upwards. These forces, being equal and opposite, neutralise each other. But when the horse pulls the cart forward he pushes the earth backward and the earth pushes him forward. There is also the backward pull of the cart. Thus, there are two forces on the horse, one due to the cart in the backward direction and the other due to the earth in the forward direction ; and since the latter is greater, the horse moves forward.

Let us finally consider the motion of both the cart and the horse together. The forces acting are the weight of the cart and the horse in the downward direction, and the upward reaction of the earth ; these

forces are equal and opposite, and hence produce no effect. The forward pull on the cart and the backward pull on the horse cancel each other, so far as the system consisting of both of them is concerned. The only force on the *system* which is unbalanced is the forward thrust of the earth when the horse tries to move forward. That the motion of the system of cart and horse is due to this external force is clear from the fact that when the ground is slippery the motion becomes difficult, because slippery ground offers very little friction, and there is very little push in the horse.

It is because of the action and reaction being equal and opposite that a man cannot raise himself by pulling at his own boot-straps or when seated in a chair by lifting its arms. There is no external force acting on the system and hence there is no movement.

43. Interaction of Bodies.—We have said that unless there is an external force acting on a system it will not move, but there seems to be an apparent exception in the case of a gun which is fired. At first sight it appears that the bullet moves forward violently, without producing any reaction. On close examination, however we find that this is not so, for when the bullet goes forward, the gun "kicks" backward, the forces on the two being equal and opposite. But on account of the difference in their masses the motion produced in them is widely different. The momentum of the bullet is equal and opposite to the momentum of the gun. This principle is called the **Conservation of Momentum.** It may be stated as follows :—

The algebraical sum of the momenta of bodies in a system along any straight line remains unaltered on account of their mutual actions upon each other.

Example. With a rifle weighing 3·95 kilograms we fire a 9·7 gm. bullet with a muzzle velocity of 810 metres/sec. What will be the velocity of recoil of the gun ?

Since the momentum of the gun and the bullet before firing is zero, the total momentum after firing must also be zero. If M stands for the mass of the rifle, V for its velocity, m for the mass of the bullet and v for velocity, we have

$$MV-mv=0$$

or

$$V=\frac{mv}{M}=\frac{9{\cdot}7\times810\times100}{3{\cdot}95\times1000}$$

$$=\frac{9{\cdot}7\times81}{3{\cdot}95}\text{ cm./sec.}$$

$$=198{\cdot}9\text{ cm./sec.}$$

Let us apply this law to the motion of a piece of stone let free in air. When the earth pulls the stone downward, the stone pulls the earth upward with an equal force so that the momentum of the earth is equal and opposite to the momentum of the stone. But since the mass of the earth is infinitely greater than that of the stone the velocity produced in the earth is practically zero.

To experimentally verify this principle, place a magnet on a piece of cork floating in water. If a small piece of soft iron be floated on a

second cork piece near the magnet, it will be observed that the iron piece moves towards the magnet and the magnet moves towards the iron piece. The magnet will be seen to move through a much smaller distance, showing thereby that the velociy produced in it is much smaller. And this is exactly what we should expect because of the fact that the magnet is bigger in size than the iron piece.

43a. Rockets.—Who hasn't seen a rocket shooting into the sky and burst there in a beautiful spectacle of colours ? The going up of a rocket is another example of the principle just stated that the total momentum of the system remains unchanged so long as a force does not act from outside.

The rocket used in fireworks consists of a straight tube of paper or bamboo into which gunpowder consisting of 60% of saltpetre, 25% charcoal and 15% sulphur is hammered in small portions. Fine metal filings are added to produce a shower of pretty sparks in the exhaust. To make the rocket go up without turning round and tumbling down a guiding stick is tied to the tube.

Fire is applied to the powder by means of a fuse inserted into the bottom of the bamboo or paper tube. The moment the gases, which are produced as a result of the combustion, begin to rush out the rocket is released. It shoots upwards as a result of the reaction to the rushing outward of exhaust gases towards the earth.

The deadly *V-2* rocket used by Germans in the bombardment of London towards the end of Second World War was also based on this principle.

Jetplanes also work on this principle. Air is sucked in at the front end, is compressed by a centrifugal compressor and fuel is sprayed into it as in a diesel engine. The mixture is ignited in the combustion chamber. The gases produced create a forward thrust in the machine by exhausting at the rear end.

43b. It is clear from what is said above that on account of the effect of action and (equal and opposite) reaction, the constituents of a system may move, but that the system as a whole does not move. In other words, the C. G. of the system remains at the same place unless some external force acts on it. This point is very important, for it connects up the third law with the first law of motion. If action and reaction were not equal, the body would move *as a whole* (*i. e.*, the C. G. of the system would change its position without the application of an external force) the possibility of which the first law of motion denies. In other words, the first law of motion requires that action and reaction must be equal and opposite. This shows that the third law of motion is deducible from the first law of motion. Now let us see what is the relation between the first law and the second law of motion.

The second law says that the rate of change of momentum is proportional to the impressed force and takes place in the direction of the force. In other words, it means that when there is no force there is no change of momentum ; *i. e.*, a body will not change its state either of rest or of uniform motion unless some force acts upon it—a fact which is called the first law of motion. Thus we see that the first law is contained in the second law.

We have already seen that the 3rd law is contained in the 1st law, and now we see that the 1st law is contained in the 2nd law. From this we come to the conclusion that the 3rd law and the 1st law are both contained in the 2nd law.

Hence *the 2nd law is the law of motion.*

EXERCISES

1. A train of 120 tons mass running 45 miles an hour is pulled up in half a mile. What is the retarding force of the brakes in poundals and also in tons-weight ?

Here $u=45$ miles per hour$=66$ ft. per second,
$S=\frac{1}{2}$mile$=2640$ ft. ; $v=0$; $a=?$

Using $v^2-u^2=2aS$, we get

$$a=-\frac{u^2}{2S}$$

$$=-\frac{66\times 66}{2\times 2640} \text{ ft. per sec. per sec.}$$

But $F=ma$ poundals

$$\therefore \quad F=-\frac{120\times 2240\times 66\times 66}{2\times 2640}.$$

$$=-99\times 2240 \text{ poundals.}$$

or $$=-\frac{99\times 2240}{32\times 2240} \text{ tons-weight.}$$

$$=-\frac{99}{32}=-3.09 \text{ tons-weight.}$$

The force is $-221{,}760$ poundals or $-3{\cdot}09$ tons weight. The minus sign shows that the force is acting in a direction opposite to that of the velocity and hence produces retardation.

2. A 600-lb. cannon-ball is fired from a gun weighing 12 tons with a velocity of 2,000 ft. per second. If the gun is free to move, with what velocity will it move backward ?

By the third law, the momentum generated in the gun will be equal and opposite to that of the ball. Hence the velocity with which the gun will move is given by the following equation :—

$$V=-\frac{600}{12}\times\frac{2000}{2240}=-\frac{625}{14}=-44{\cdot}64 \text{ ft. per second,}$$

the negative sign shows that the gun will go in the opposite direction to that in which the ball moves.

3. A passenger weighing 12 stones on earth rises in a balloon with an acceleration of 4 ft./sec^2. Find his weight as recorded by a weighing machine (or spring balance).

Since there is no relative motion between the man and the balloon the reaction of the balloon must be equal to the force with which the man presses the balloon downwards. Hence the weight of the man equals the reaction of the balloon, say R.

Now this reaction of the balloon in addition to overcoming the pull of gravity has to produce in man an acceleration of 4 ft./sec.^{2}.

Therefore $R-mg=ma.$

or $$R=m(g+a)=m(32+4)$$

$$=12\times14\times36 \text{ poundals.}$$

or $$=\frac{12\times14\times36}{32\times14}=13\tfrac{1}{2} \text{ stones-weight.}$$

If the balloon were to descend with this acceleration, the reaction of the balloon would be smaller than the pull of gravity. In this case we shall have

$$mg-R=ma,$$

or $$R=m(g-a)=m\ (32-4)$$
$$=12\times14\times28 \text{ poundals,}$$
$$=\frac{12\times14\times28}{32\times14}=\frac{21}{2}=10\tfrac{1}{2} \text{ stones-weight.}$$

Note that if the balloon were moving either up or down with uniform velocity there would be no change in the weight of the man recorded by a spring balance.

4. What force is required to stop a train of 100 tons going 30 miles an hour, (*i*) in half a minute, (*ii*) in half a mile ?

Ans. $4\frac{7}{12}$ tons, $1\frac{7}{18}$ tons.

5. A force of 8 lb. wt. acting on a certain mass for 3 seconds gives it a velocity of 6 ft. per second. Find the mass in lb. *Ans.* 128 lb.

6. A mass of 10 lb. moving with a velocity of 25 ft./sec. is stopped by a uniform force in a distance of 50 ft. Find the force.

Ans. $62\frac{1}{2}$ poundals.

7. The mass of a gun together with its carriage is 7 tons. It fires 100 lb. shot with a velocity of 1000 ft. per second. What is the velocity of the recoil of the gun ? *Ans.* 6·38 ft. per second.

8. A man suddenly jumps off a table with a 10 lb. weight in his hand. What is the pressure of the weight on his hand while he is in the air ?

Ans. Zero.

9. What do you understand by the term reaction ? A lift is rising with an acceleration of 8 ft. per second per second. What pressure would a man weighing 16 stones exert on the floor of the lift ?

Ans. 20 stones wt.

10. A body whose mass is 12 lb. is moving with a velocity of 100 ft. per sec. If a constant resistance equal to a weight of 15 lb. is applied to stop it, find how far it will travel before it comes to rest.

Ans. 125 feet.

11. State the "parallelogram law of forces."

A body weighing 100 lb. sits in a swing suspended by two ropes. The swing is drawn aside and is held in equilibrium by a force of 20 lb. Find the tension in each of the two ropes of the swing. *Ans.* 51 lb.

12. A uniform rod is supported at its two ends by two strings of unequal lengths the other ends of which are tied to a nail. Show that the tensions of the two strings are proportional to their lengths.

(*P. U. 1932*)

13. A picture weighing 10 lbs. is hung by a string fastened to the two upper corners of its frame and passing over a smooth peg. The parts of the string on the two sides of the peg are inclined at an angle of 120° when the edges of the frame are horizontal. Find the tension in the two segments of the string. *Ans.* 10 lb.

14. A gramophone disc rotates at 60 revolutions per minute. When a coin of mass 13 grams is placed at a distance of 8 or less than 8 cm. from the centre, the coin remains on the disc, but as soon as it is placed at a distance greater than 8 cm. it is thrown off ; explain why ?

Calculate the centripetal force acting on the coin when it is placed at a distance of 8 cm. from the centre. *Ans.* 4106 dynes (approx.)

15. A mass of 4 oz. is attached to one end of a string 1 yd. long and revolved in a horizontal circle. Find the greatest number of revolutions per minute which the mass can make without breaking the string. Given that the string will break under a load of 10 lb. *Ans.* 197·2.

16. If action equals reaction, why is it not as dangerous to receive the "kick" of a gun as to be struck by the bullet ?

17. State the three laws of motion. Show that the first and the third laws are contained in the second.

18. In a tug-of-war the losing team exerts exactly as much force on the winning team as the winning team exerts on the losing team. How is it then that the losing team is pulled forward ?

19. Does a motor-car driver require the same force to start his car on a level road in Delhi as in Simla ?

20. Explain how the principle of inertia is made use of in the following cases :—

(*a*) Shaking a *plum* tree to bring down *plums*.
(*b*) Shovelling coal into the furnace of a boiler.

21. A rich man riding in a carriage thinks that he is helping it to move by pressing his feet forward on the floor. Is he right ?

22. Explain why it is easier to pull than to push a lawn roller.

CHAPTER IV

The Intensity of Gravity

44. It is a matter of common observation that, ordinarily, bodies, when left free in air, fall to the earth. Only a few bodies (like a balloon filled with hydrogen or hot air) rise up when released from the hand. These facts were known to the ancients as well. Before Galileo (1564-1642) these facts were explained on the hypothesis (put forward by Aristotle) that every body has its "natural place" in the world, that natural place being down on the earth for heavy bodies and high up in the atmosphere for light bodies. Another fact that some bodies fall more quickly than others, also did not escape the notice of the ancients. The difference in the times which various bodies took in falling from same height to the ground was supposed to be due to the difference in their nature and in mass. How vague these notions seem to us to-day! But all the same, they did hold the field for 2,000 years. It required a genius like that of Galileo to shatter these wrong notions. He showed clearly that there was no other natural place for bodies except the one on the ground, and that the rising up of light bodies was due to the upward thrust of the air, which is greater than their own weight.* He further showed that the time taken by a body to fall from a particular height to the ground was independent of its nature and mass, quite contrary to the view of Aristotle, who held that the time taken by a body to reach the ground depended upon its nature and was inversely proportional to its mass. According to Aristotle a chhatack should take 160 times as long to reach the ground as a 10 seer weight when dropped from the same height. Fancy, before Galileo, none thought of challenging this statement and proving it false! It was he who, first released spheres of lead, small and great, from the top of the leaning tower of Pisa† (Fig. 36) at the same time, and found that they fell on the ground simultaneously. Next he tried spheres of different materials; even then the time was approximately the same. To show that the slight difference that was observed was due to the resistance of the air, he tried bodies of different shapes, spheres and discs, made of the same material and having the same weight but different sizes. Their times of fall were widely different. On Aristotle's views this difference could not be explained; according to Galileo, however, this was simply due to the unequal resistance offered by the air to the bodies; the disc, having a

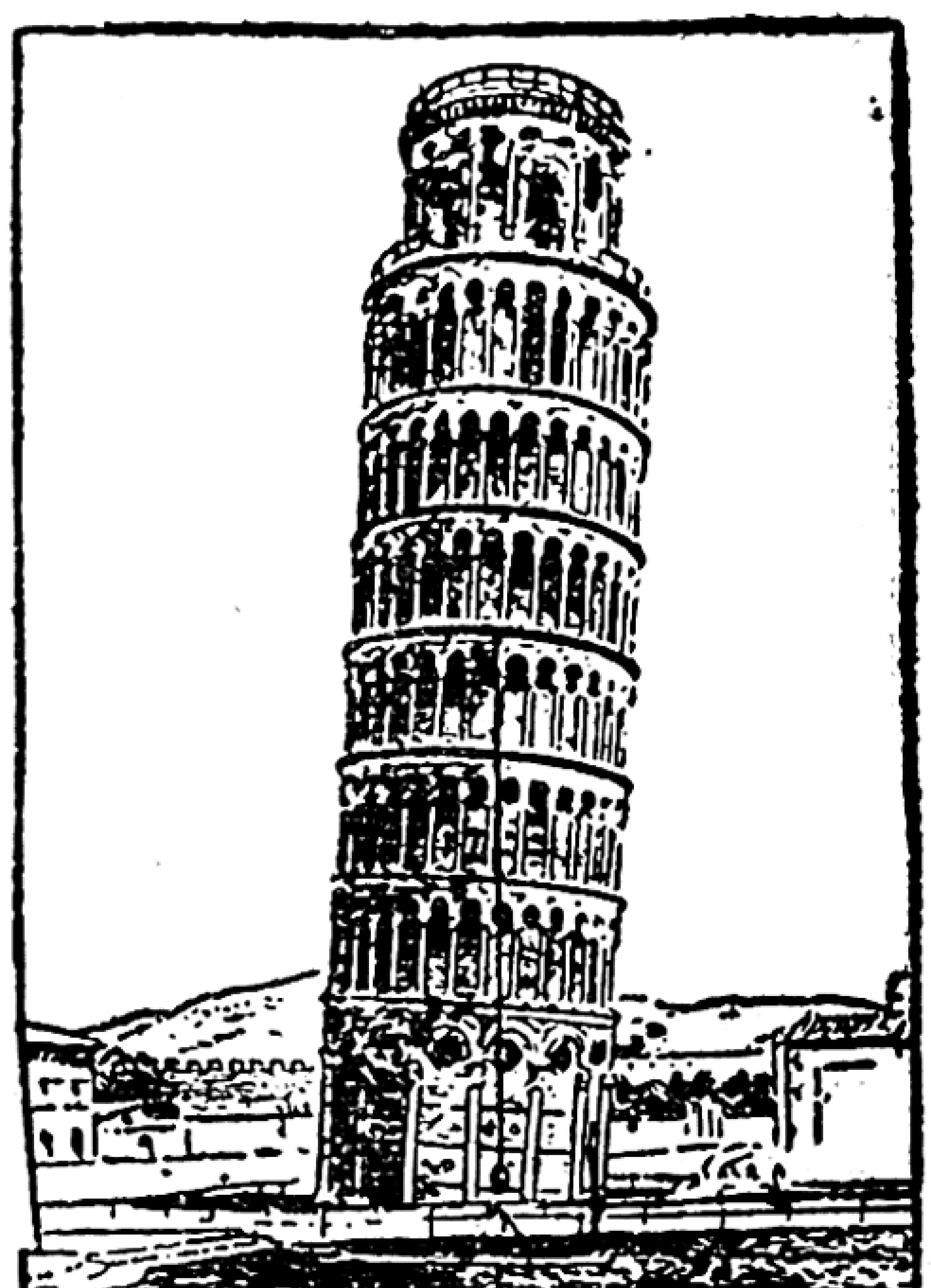


Fig. 36. Leaning Tower of Pisa.

* For a further explanation see 'Principle of Archimedes'.
† It is 179 feet high and leans 14 feet out of the vertical.

greater surface, experienced a greater resistance and therefore took a longer time to reach the ground. It was more conclusively proved a few years later by Newton with the help of the *feather and guinea experiment*. He took a glass tube about 6 ft. long and 3 inches in diameter and closed it at one end. A guinea and a feather were then inserted and the other end was closed with an air-tight cap fitted with a stop-cock. The tube was attached to an air-pump and the air was exhausted. It was then detached and inverted. The feather and guinea were seen to fall side by side, thereby showing that (in vacuum) all bodies fall towards the earth at the same rate. This proved beyond doubt that at the same place all bodies falling *freely* have the same acceleration. Note the word freely. Bodies fall freely only when the resistance of air is negligible which is never so in actual practice. To what extent the resistance of air affects the velocity of a falling body, will be understood better by a student if he is told that a hailstone 1 cm. in diameter falling from a height of 1 mile has a velocity of 35 ft./sec. whereas its velocity would be over 550 ft/sec. (a velocity equal to that of the bullet of a pistol) if it had fallen freely.

The resistance offered by air is made use of by aeronauts in descending from aeroplanes by parachutes. A parachute is an umbrella-like structure in which such a large surface is exposed to the air that even a heavy body like man is very much retarded in his fall. The speed of the man on reaching the ground when he falls from a height of 1 mile with a parachute is hardly more than 30 ft./sec.—a speed which would be attained in a free fall from a height of less than 16 feet.

45. Gravitation.—Newton was the first to realize that the attraction of the earth for bodies on its surface was not a peculiar property of the earth alone, but that it was common to all bodies; every body attracted every other body, no matter how large or how small the bodies were. The bodies considered may be two books lying on a table or two stars separated by thousands of miles. This attraction is universal. The law according to which bodies attract each other was first stated by Newton, and is called the **Law of Gravitation.** It is stated as :

Every body in the universe attracts every other body with a force which varies directly as the product of the masses of the two bodies and inversely as the square of the distance between them.

The law can be written in the form of an equation as :

$$F = G\,\frac{mm'}{r^2},$$

where m is the mass of one body, m' that of the other, r the distance between them, G a constant and F the force with which the bodies attract each other.

If F be measured in dynes, mass in grams and r in centimetres, the value of G is found to be $6{\cdot}6 \times 10^{-8}$. Since the value of G is so small the force of attraction between ordinary bodies is hardly perceptible. It is only when the bodies have huge masses that this force

becomes appreciable. How important it becomes in such cases will be clear from the fact that the force of the sun on the earth in spite of the huge distance of 93,000,000 miles, is several million billion tons. It is this force which keeps the earth moving in its orbit. The moon and the other planets also are held in space in their orbits on account of gravitation. A body, which on account of the earth's attraction, weighs 2 lb. on the earth, will weigh 27 lb. on the sun or $\frac{1}{6}$ lb. on the moon.

46. The force of attraction which the earth exerts on all bodies lying on or near its surface is only a special case of gravitation. It is called **gravity**. Since the distance of the hill-tops from the centre of the earth is greater than that of the plains, and the force is inversely proportional to the square of the distance, the force at the hill tops is less and hence gravity (generally called g) is smaller.

The difference in the value of g, however, is so small that for all ordinary purposes it can be ignored. To give an idea to the student of the actual difference in the value of g, we shall give its values at some of the places. At the equator, at the sea-level, it is 32·04 ft./sec.^{2} or 978·1 cm./sec.^{2}, and at the poles it is 32·26 ft./sec.^{2} or 983·2 cm./sec^2. At Simla (height above sea-level 7,000 ft.) the value of g is 978·29 cm./sec^2. In calculations it is usual to take the value of g as 32 ft./sec.^{2}. or 981 cm./sec^2. The acceleration of 32 ft./sec.^{2} is a tremendous acceleration. To verify experimentally the laws of falling bodies, it is usual to dilute the value of g, because otherwise the bodies fall much too quickly to be conveniently observed. The methods employed to dilute the value of g can be divided into two classes. (1) by making the body fall down an inclined plane, and (2) by making the falling body move another body in addition to itself. We shall consider these methods one by one.

47. Motion of a Body along an Inclined Plane.—An inclined plane is a plane surface inclined at an angle to the horizontal. We can represent it diagrammatically by a right-angled triangle whose each side stands for a surface. Let AC be the horizontal surface, and AB the surface inclined at an angle BAC or θ to AC. BC is the height of the plane.

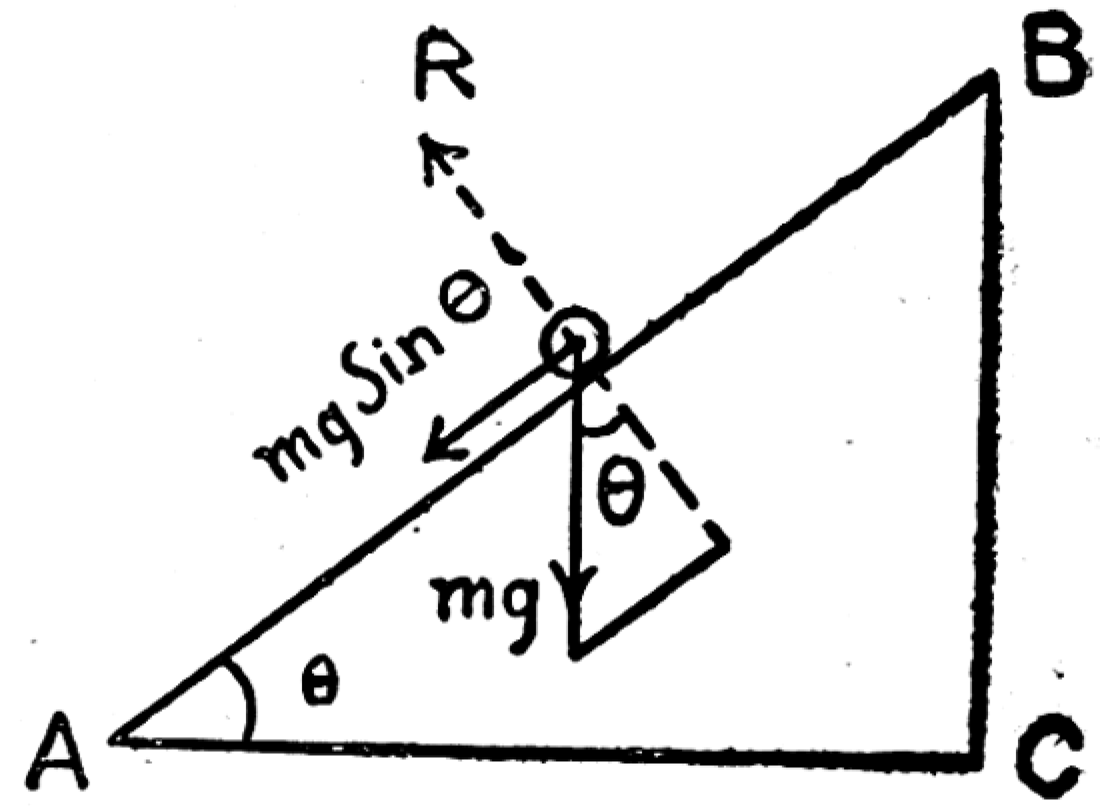


Fig. 37.

Let us consider the motion of a body along such a plane. We shall suppose that the plane is smooth and hence offers no resistance parallel to its surface to a body moving along it ; the reaction offered by such a plane is perpendicular to its surface.

Suppose m is the mass of the body moving down the inclined plane, and R, the reaction of the inclined plane acting on the body. The weight mg (dynes, or poundals) of the body acts vertically downwards. Resolve it into two components, one along the plane and the other perpendicular to it. The component along the plane is $mg \sin \theta$,

and perpendicular to the plane is $mg \cos \theta$. Since there is no motion perpendicular to the plane, the forces in this direction, *i.e.* R and $mg \cos \theta$, must be equal and opposite. We can therefore write

$$R = mg \cos \theta \qquad \text{..................} \qquad (i)$$

The only downward force acting along the plane is $mg \sin \theta$; let us suppose it produces an acceleration a in the body. By the second law of motion,

$$mg \sin \theta = ma,$$

or

$$a = g \sin \theta \qquad \text{..................} \qquad (ii)$$

The maximum value of $\sin \theta$ is 1, and that is the case when θ is 90°, *viz.*, when the body falls vertically ; for all other angles $\sin \theta$ is less than 1 and hence a is less than g. Thus we see that when a body moves down an inclined plane the acceleration is reduced to a value depending upon the inclination of the plane.

Let us now see what will be the velocity of the body when it reaches the bottom of the plane l ft. long starting from rest at the top. Using $v^2 - u^2 = 2as$ and remembering that here $u = O$, $a = g \sin \theta$ and $s = l$, we get

$$v^2 = 2g \sin\theta \times l = 2g \times l \sin \theta = 2gh$$

where $l \sin \theta = h$, the height of the plane.

But if the body were to fall freely through a height h even then the velocity acquired would have been $\sqrt{2gh}$. Hence we learn that

The velocity acquired by a body in falling down along an inclined plane is equal to that which it would acquire if it were to fall vertically through the height of the plane.

48. Let us now consider an arrangement in which the falling body moves another body in addition to itself. Let the two bodies P and Q be connected together by a string passing over a frictionless pulley. If Q moves downwards with an acceleration of a ft./sec², P moves upwards with the same acceleration. On Q are acting two forces, Mg downwards and T, the tension in the string, upwards. Since Q moves downwards with an acceleration a ft./sec², fro the second law of motion we get,

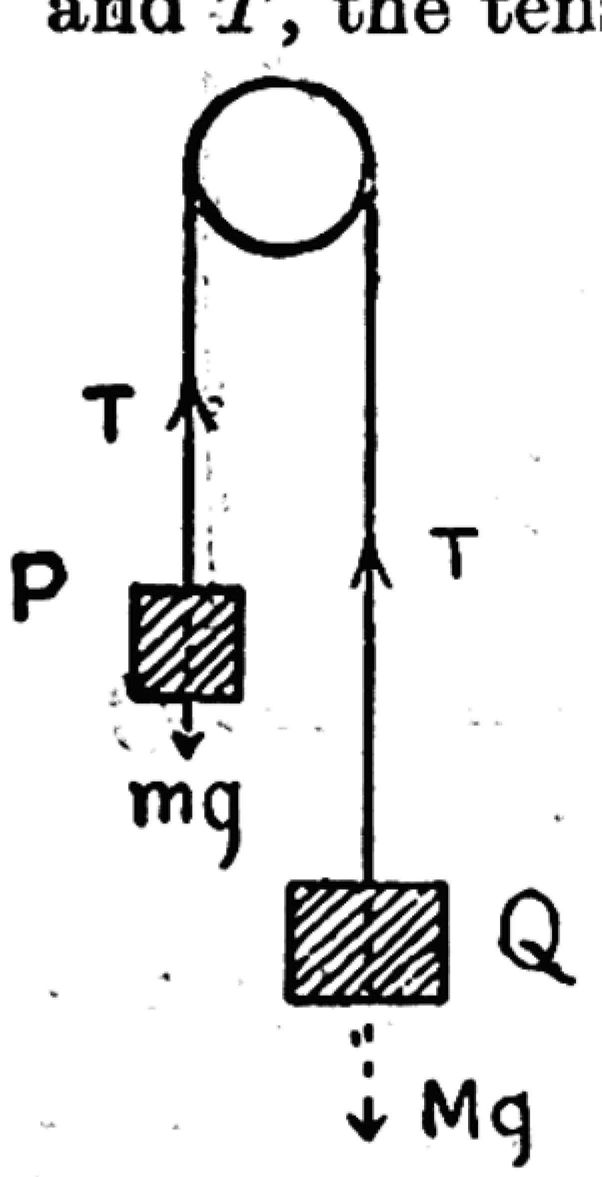


Fig. 38.

$$Mg - T = ma \qquad \text{...} \qquad (iii)$$

The forces acting on P are mg downwards and T upwards. As under the action of these forces it moves upwards with an acceleration a ft./sec². we can write

$$T - mg = ma \qquad \text{..................} \qquad (iv)$$

Adding equation (*iii*) and (*iv*) we get

$$Mg - mg = (M + m)a,$$

or

$$a = \frac{M - m}{M + m} g. \qquad \text{..................} \qquad (v)$$

Now $\frac{M-m}{M+m}$ is necessarily smaller than 1, hence a is smaller than g. For instance if M be 17 lb. and m 15 lb. $a = \frac{2}{32} \times 32 = 2$ ft. per sec. per sec. By properly selecting the values of M and m we can make the acceleration of P or Q as small as we like.

Let us see what is the value of tension in the string. Instead of adding equations (*iii*) and (*iv*) divide one by the other.

$$\frac{Mg-T}{T-mg}=\frac{M}{m},$$

or
$$mMg-Tm=MT-Mmg,$$

or
$$(M+m)T=2Mmg,$$

Therefore
$$T=\frac{2Mm}{M+m}g \qquad \ldots \qquad (vi)$$

The *pressure on the axle of the pulley is twice the tension and* not the sum of the weights of bodies, except when the masses are equal.

49. In Art. 47 we have seen that when a body moves down an inclined plane, only the component, $mg \sin \theta$ of the weight mg is effective in producing motion.

Obviously, any force greater than $mg \sin \theta$ would drag the body up along the plane. It would require a force greater than mg if the body were to be raised vertically upwards. It is clear, therefore, that we can raise a body through a given height with a much smaller force if we use an inclined plane.

50. Measurement of g.—Any one of the relations proved above can be used to find the value of g. For instance, when a body moves down an inclined plane, its acceleration is $g \sin \theta$. To find the value of g note the time that a body takes in going down a plane from the top to the bottom ; and since it starts with zero velocity

$$l=\tfrac{1}{2}at^2=\tfrac{1}{2}g \sin \theta \times t^2.$$

In this equation l, t, and θ are known, hence we can find g.

If we hang two masses M and m connected by a string passing over a pulley, we know that the acceleration produced in the masses is given by the relation

$$a=\frac{M-m}{M+m}g.$$

If we know a, M and m we can find the value of g. To do it, we note the time which one of the masses takes in travelling a certain distance, and therefrom calculate a (using the equation $l=\frac{1}{2}at^2$). Substituting this value of a in the above equation we get g.

This is the principle of **Atwood's Machine**, which consists of a frictionless pulley, over which passes a string carrying two equal masses P and Q. The weight Q is placed on the clamp A (Fig. 39). A small slotted weight is slipped on to it. As soon as the clamp is removed from underneath Q, due to unequal weights on the two sides, Q moves down and P moves up. The time which Q takes to cover a certain distance is noted, and therefrom a is calculated. When a is known, g can be determined.

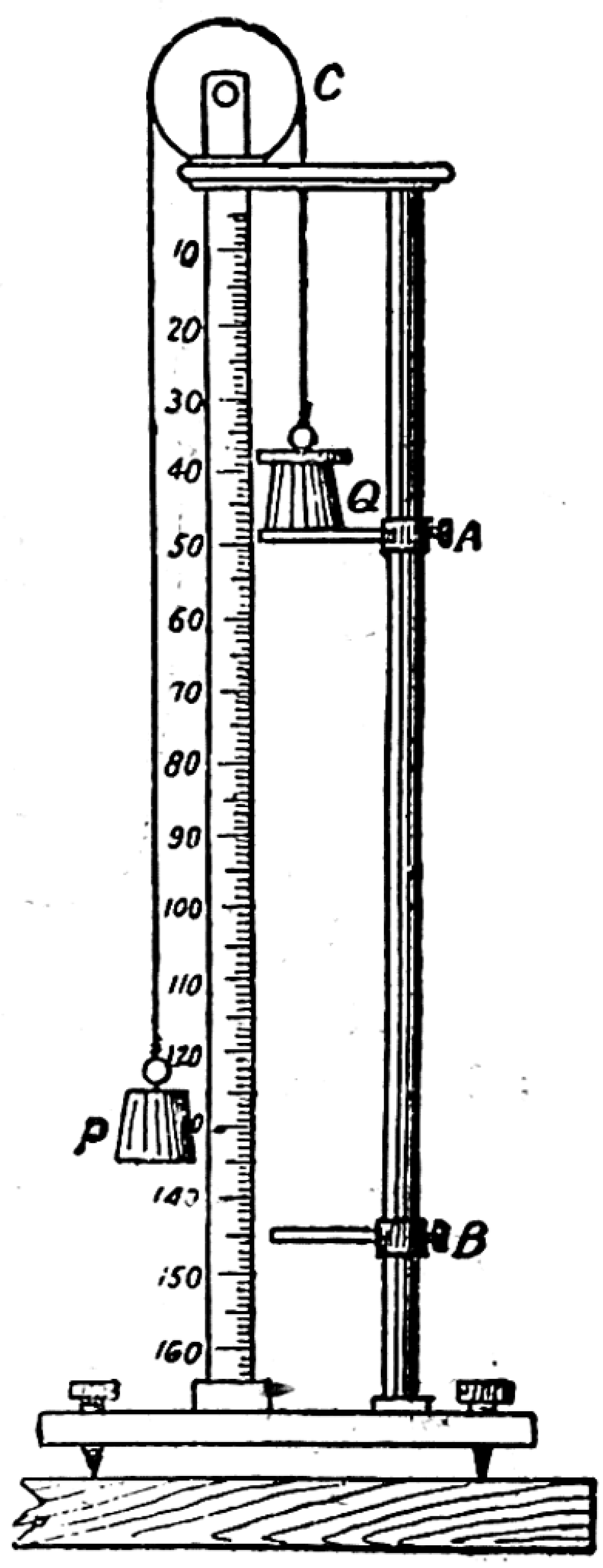


Fig. 39.

The Atwood's machine as outlined above gives very approximate results.

By using an improved form Cusson and Johnson have recently obtained better results. Their machine consists of a stout metal rod A fixed vertically to a support and is fitted with a frictionless pulley B at the upper end. A paper ribbon P runs over the pulley and is joined at its two ends to the masses m_1 and m_2, where m_2 is slightly heavier than m_1. The ribbon at the top of the pulley moves past the end of a marking point D attached to a vibrating arm V making definite number of vibrations per second.

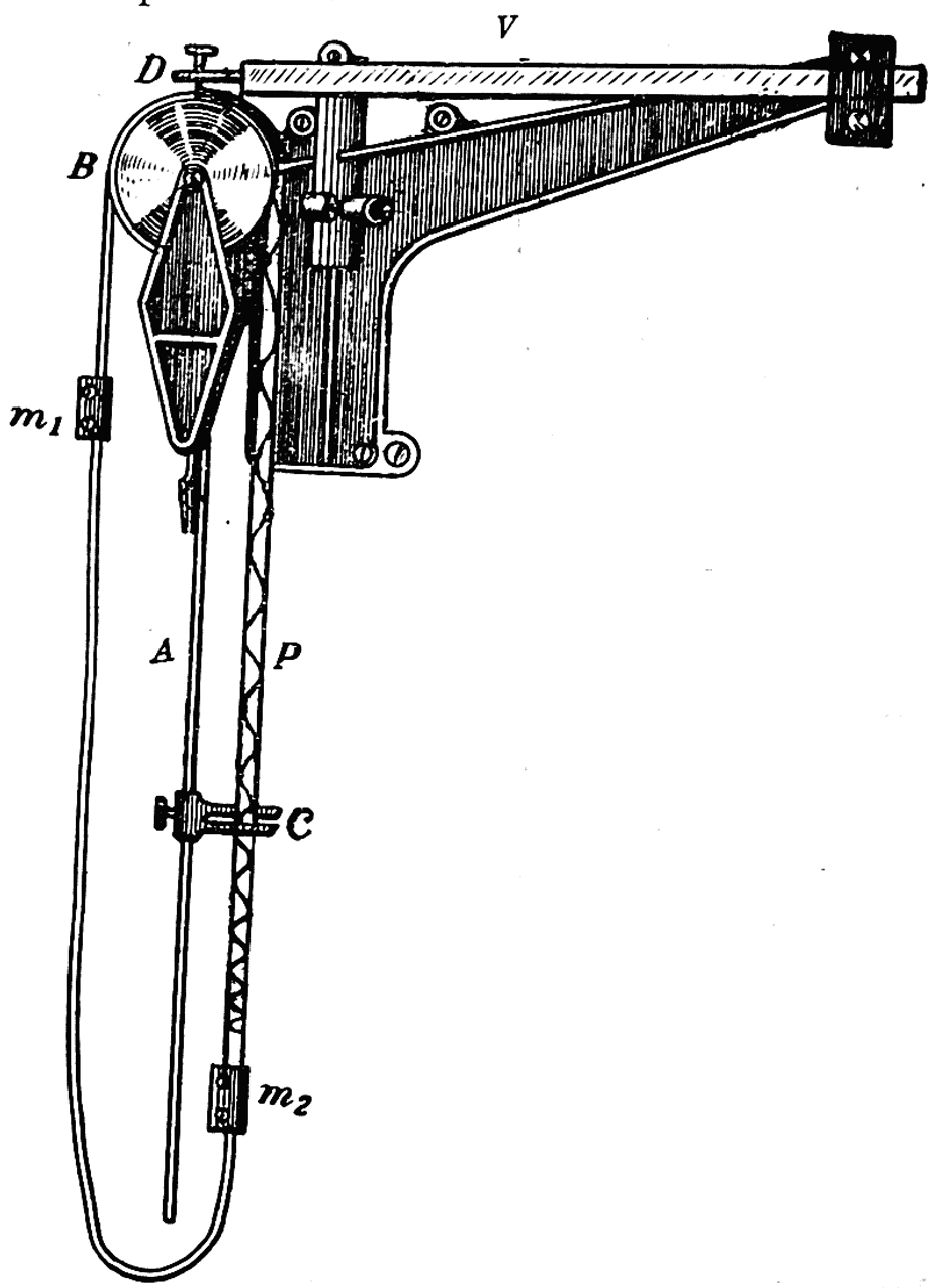


Fig. 40 (*a*).

To use the machine the clamp C is adjusted at a suitable height and the mass m_2 supported on it. The vibrating arm is set into motion

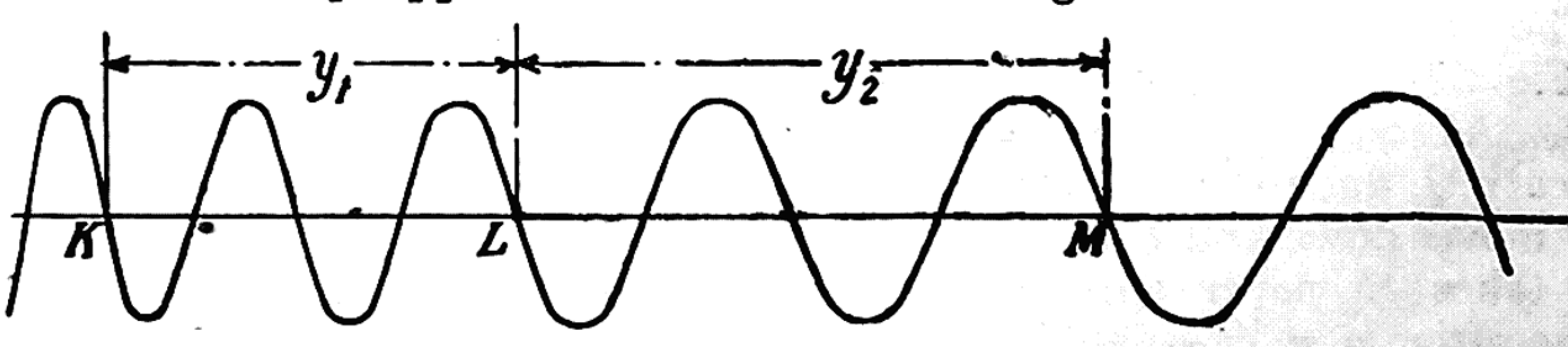


Fig. 40 (*b*).

and the clamp C is released allowing the mass m_2 to move downwards.

A wavy curve like that shown in Fig. 40 (*b*). is traced on the ribbon by the point *D*.

Let *K*, *L*, and *M* be three points at which a straight line passing along the middle point of the ribbon cuts the curve. Obviously the intervals *KL*, and *LM* are traversed by the same number of vibrations say *n*, and hence in the same time.

Let the velocity of the ribbon at *K* be u_1, the acceleration of the system be *a* and the time taken to make *n* vibrations be *t* seconds.

We can write

$$y_1=u_1t+\tfrac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (i)$$

Similarly we can write $y_2=u_2t+\tfrac{1}{2}at^2$

But $u_2=u_1+at$

$$\therefore \quad y_2=(u_1+at)t+\tfrac{1}{2}at^2$$
$$=u_1t+at^2+\tfrac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (ii)$$

Substituting equation (*i*) from equation (*ii*) we get

$$y_2-y_1=at^2.$$

Since the frequency of the vibrating arm is known, *t* is known. Subtracting the value of *t* and of y_2 and y_1 we can find the value of *a* and hence of *g*.

An extremely simple and yet an accurate method of finding *g* is the method of *Pendulum* (explained below).

51. Pendulum.--The pendulum has been in use for nearly three hundred years to work clocks. Nothing better has so far been found. We shall describe later on the construction of a pendulum as actually used in clocks but at this stage we shall talk of a simple pendulum only. *A simple pendulum consists of a heavy particle suspended by a weightless, inextensible, and perfectly flexible string.* In practice it is difficult to realise completely all these conditions, but a small lead sphere suspended by a fine, strong cotton thread meets the description with a fair degree of accuracy. The distance between the point of support and the centre of the lead sphere is called the *length of the pendulum*. If such a pendulum is drawn aside and then released, it begins to oscillate to and fro in a vertical plane, the ball describing an arc of a circle.

Suppose the ball is left free at *B*, (Fig 41.). It swings to a point *C* at an equal distance on the other side of *A* (the position in which the pendulum was originally at rest). It goes on oscillating between these two extreme positions (*B* and *C*). *The to and fro motion of the pendulum from one extreme position to the opposite and back is called a Vibration.* Some people define a vibration as the motion of a body from one extreme position to the other ; we shall call it oscillation.

The greatest distance travelled by the ball from its mean position is called the amplitude of the vibration.

The motion of a pendulum is periodic and the time taken to complete one vibration depends upon the length of the pendulum and the intensity of gravity. Although the amplitude* goes on decreasing gradually, yet the time of one complete vibration remains the

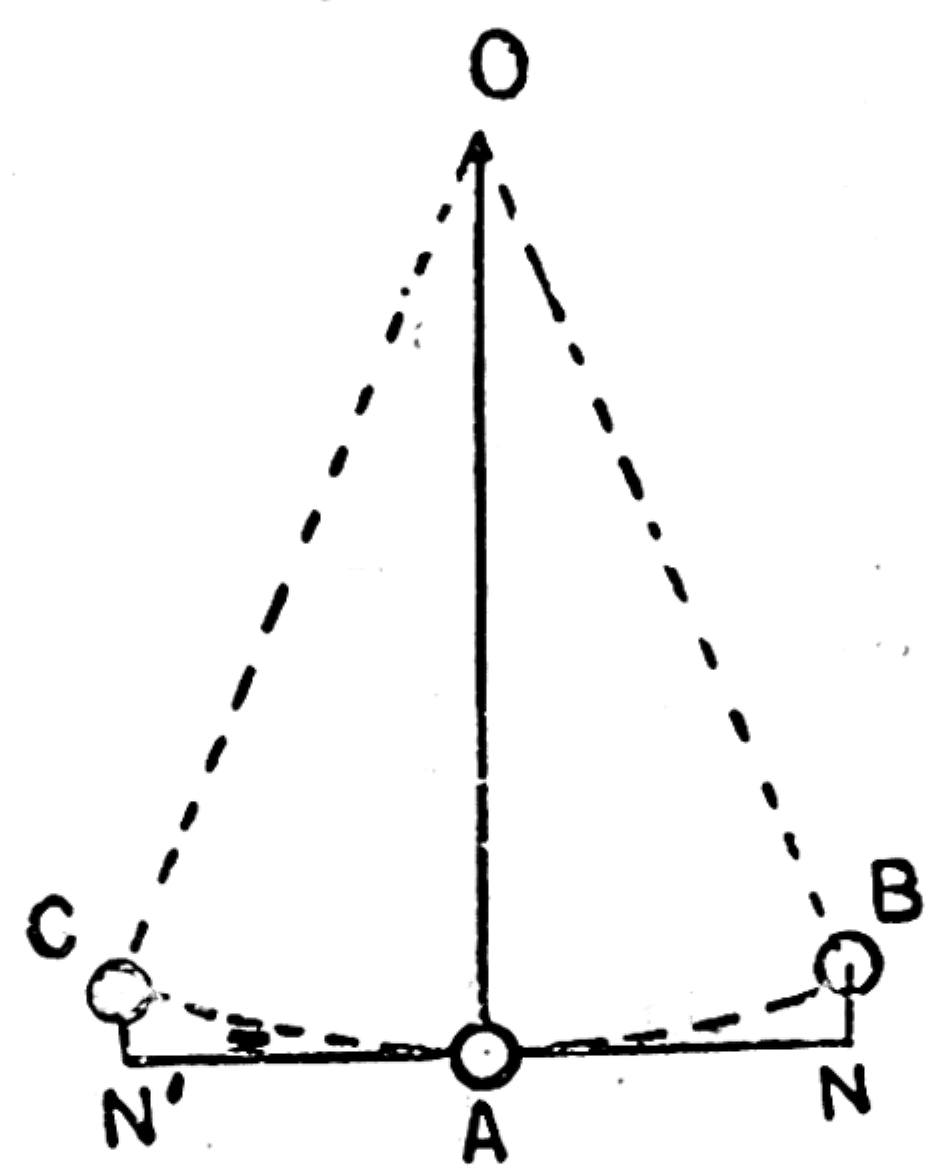


Fig 41.

* On account of the resistance of air and the friction at the point of suspension etc.

same.* This fact is often expressed by saying that the vibrations of a pendulum are **isochronous.** *The time of one complete vibration,* i.e., *the time between two passages through the same point in the same direction, is called the Time Period or the Period of the Pendulum.*

The time-period is independent of the nature of the material of the ball. The time t of one complete vibration can be proved theoretically to be equal to

$$2\pi\sqrt{\frac{l}{g}},$$

where l is the length of the pendulum, and g the intensity of gravity. Knowing t and l we can find the value of g. To find the time-period it is advisable to note with a stop watch the time for 40 or 50 vibrations, and therefrom to find the time of one vibration. Let it be equal to t seconds. Measure the length from the point of suspension to the centre of gravity of the ball. Making use of the relation

$$t=2\pi\sqrt{\frac{l}{g}},$$

or

$$g=\frac{4\pi^2 l}{t^2},$$

find the value of g. It may be pointed out here that, when we say a seconds-pendulum, we mean a pendulum which makes half a vibration in one second.

52. Work.—Ordinarily speaking, any physical effort is work; for instance, when we support a heavy beam on our shoulder or try to lift from the floor a heavy stone slab which we cannot move, we think we have done work. But from the standpoint of Physics no work is done in both the above cases, for in Physics work is said to be done only if the force moves the body on which it acts. This shows that in Physics the word "work" stands for accomplishment, and not for effort or fatigue. Suppose a building is under construction and a labourer carries 8 bricks at a time (each weighing 5 lb.) up a ladder to the top of the first storey (15 ft. high). He is doing work in the scientific sense, for the force that he exerts moves the bricks through 15 ft. in the upward direction. Now suppose he carries 16 bricks at a time to the top of the first storey. It is evident that in the second case he does double the work, for he moves twice the material through 15 ft. and therefore exerts double the force. If he were to carry 8 bricks to the top of the second storey (30 ft. high) even then he would do twice as much work as in the first case. This shows that by doubling the distance or the force, the work done is doubled. If we double both, the force, and the distance, the work done is four times.

These results easily follow from the expression

$$Work = Force \times Distance$$

Writing W for work, F for force and S for distance we can express this relation as

$$W=F\times S$$

* Provided the angle AOB does not exceed 5°.

It should be noted that the work done does not depend on the path by which the bricks are raised so long as the height is the same. For instance, the work done is the same whether the labourer goes up a vertical spiral stair-case or a slanting ladder Let us see why ? When the labourer goes up a slanting ladder which is a form of an inclined plane (Fig. 42), the force needed to raise the bricks up is $mg \sin \theta$, and it acts through length S of the ladder. The work W done is given by the expression

$$mg \sin \theta \times S.$$

But $S \times \sin \theta = h$, the height through which the weight rises, *i.e.*, the height of the house, hence

$$W = mg \sin \theta \times S = mgh$$

and is the same as would be done if the bricks were carried up a vertical spiral stair case.

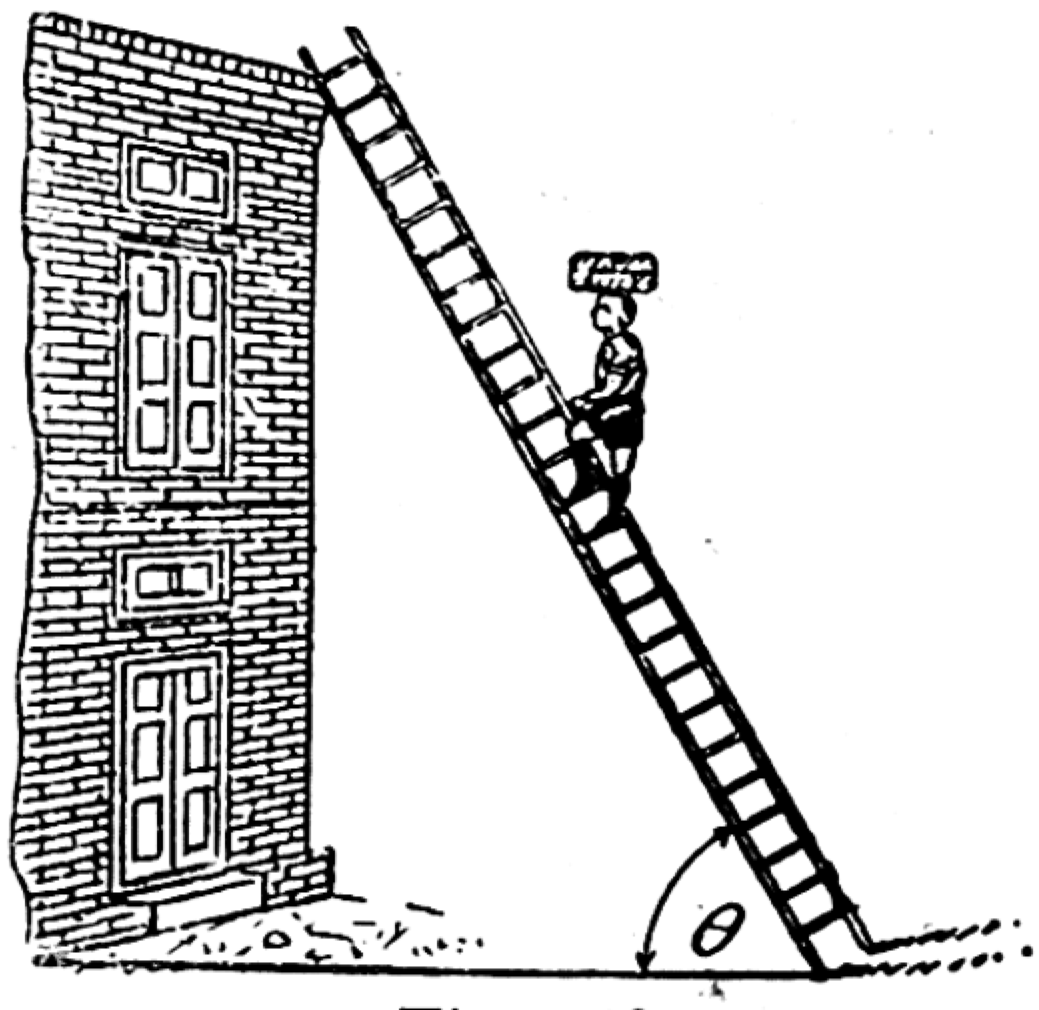


Fig. 42.

From the expression $W = mg \sin\theta \times S$, where $mg \sin \theta$ is the component of the force in the direction of motion, we learn that when the force makes an angle with the direction of motion

$$\text{The work done} = \begin{Bmatrix} \text{component of the force along} \\ \text{the direction of motion} \end{Bmatrix} \times \text{distance},$$

$$\text{or the work done} = \text{the force} \times \begin{Bmatrix} \text{distance moved along} \\ \text{the direction of force.} \end{Bmatrix}$$

Keeping this in view it is obvious that the work done by a man carrying a suit case on a level road from one place to another is zero, for the force is vertical and the distance moved is horizontal.

Note, the work done does not depend on the time taken to move the body. For instance, in our example of the labourer carrying the bricks, one labourer may work steadily but slowly, and another may take rest at intervals and work vigorously, yet the work done by both will be the same if the number of bricks raised by them through a given height is the same during the day.

53. Units of Work.—In the *F.P.S.* or British system, the unit of work is the **foot-poundal**; *it is equal to the work done by a force of 1 poundal in moving a body through 1 ft. in the direction of the force.*

In the *C.G.S.* system, the unit of work is called the **erg** ; *it is the amount of work done when a force of 1 dyne moves a body through 1 cm. in the direction of the force.*

These two units are the absolute units of work.

In practical life the units adopted are different. In the *F.P.S.* system, the practical unit is called the **Foot-pound**, which is the amount of work done by a force of 1 lb. wt. acting through a distance of 1 ft. In the *C.G.S.* system, the practical unit is called the **Kilogram-metre**, which is equal to the amount of work done when a kilogram moves through a metre against the force of gravity.

54. Rate of Performing Work.— So far we have considered the amount of work done by an agent, not caring to know in how much time it is done. But frequently we want that a given amount of work should be done within a certain time, and in order to do that we must

employ an agent who can finish it within the required period. This necessity introduces us to a second idea about work *i.e., the rate of doing work* or **power** as it is generally called. It is measured by work done in a second.

The practical unit of power, adopted by the British engineers, is called the **Horse Power**. It was introduced by James Watt, who supposed that a horse could raise 33,000 pounds *one foot high* in one minute. It is now known that very few horses are capable of working continuously at this rate for any length of time. However, this unit, *i.e.*, the horse power or 33,000 foot pounds a minute or 550 foot-pounds a second, is the one that is generally used.

The unit of power used in the *C.G.S.* system is called a **Watt**. It is the power of doing 10^7 ergs of work in one second. It is useful to remember the relation between the two units :

$$\text{One } H.P. = 746 \textit{ watts}.$$

55. Energy.—When work is done upon a body, it is found to have an increased power of doing work itself. It can produce physical changes in other bodies. It is, therefore, said to possess more energy than before. **Energy** is defined as *the power* or *capacity to do work.* It may take any one of the two forms : energy due to the motion of the body, or energy due to position. The first kind is called the **Kinetic Energy,** and the second kind, the **Potential Energy.**

56. Kinetic Energy.—As an example of kinetic energy, let us consider a stream of water. The water flowing down-stream on account of gravity is, by virtue of its motion, capable of doing more work than an equal mass of water at rest. For instance, it sets the water-mill in motion if made to fall upon the vanes of the mill. Similarly a bullet shot from a gun pierces through objects and overcomes resistance offered by them.

57. How to Measure it.—When once a body is set in motion, it continues to move uniformly in a straight line until a new force acts upon it. If the new force opposes its motion, its velocity will gradually decrease till at last it comes to rest. Suppose a body moving with a velocity of v ft. per second, is brought to rest by a force F in a distance S. The work done against the force is FS units. Since the initial velocity of the body is v and the final velocity zero, the distance passed through by it before coming to rest is given by the relation $v^2=2aS$, where a is the retardation produced by force F and is equal to $\frac{F}{m}$. Substituting this value for a we get

$$v^2=2\frac{F}{m}S$$

or

$$\tfrac{1}{2}mv^2=FS.$$

But FS is the work done before the body stops, hence it is the measure of the energy possessed by the body due to its motion. Thus we see that

$$\textit{Kinetic Energy}=\tfrac{1}{2}mv^2.$$

58. Potential Energy.—As has been said above, the potential energy of a body is due to its position. Now either a body may change its position as a whole with respect to its surroundings, as for instance, when a weight is moved from the ground to the roof a house, or, one part of it may move with respect to the other parts, as in the case of a stretched or wound-up spring or a bent bow. In both the cases the body possesses greater energy after the change of position is brought about. The ball lying on the roof of a house, for instance, is capable of driving a nail into the ground, if allowed to fall on it and, a watch-spring, when wound up possesses enough energy to keep the wheels of the watch in motion for a day or even for a week.

59. How to Measure it.—No formula can be given to express the potential energy of a body, so universally applicable as $\frac{1}{2}mv^2$ in the case of kinetic energy. But in certain cases the formula is very simple. For instance, in the case of a ball lying on a roof, the potential energy is mgh, where mg is the weight of the ball and h is the height of the roof above the ground. It should be noted that when the ball is lying on the ground, we suppose it possesses no energy, but it is not so in reality, for even then it possesses potential energy—equal to the amount of work done in raising it from the centre of the earth to the surface, but we cannot make use of this energy. Hence when we say that a body possesses no energy, we mean that it possesses no *available* energy.

60. Conservation of Energy.—Since we have to deal frequently with bodies raised to certain heights we shall consider this case in greater detail. When the ball is on the roof of a house, it possesses potential energy equal to mgh. When it is half-way down half of its energy is kinetic and half potential. So far as the potential energy is concerned, it is clear that it is equal to $\frac{1}{2}mgh$, because the height now becomes $h/2$; but what about the kinetic energy? The velocity at half the height will be given by the relation

$$v^2=2gS=2g\times\tfrac{1}{2}h=gh,$$

and the kinetic energy of the ball will be $\frac{1}{2}mv^2$ or $\frac{1}{2}mgh$, which is half the initial potential energy. The total energy is $\frac{1}{2}mgh+\frac{1}{2}mgh=mgh$, the same which the ball possesses when on the roof. When it is at $\frac{1}{4}$th the height, the potential energy is only $\frac{1}{4}mgh$, and the kinetic energy is $\frac{3}{4}mgh$*. The total energy again is the same as at the top. When the ball reaches the ground, the (available) potential energy is zero, and the kinetic energy is equal to the initial potential energy. The velocity with which the ball falls on the ground is given by

$$v^2=2gS=2gh,$$

because $S=h$, the height of the roof. The $K.\ E.=\frac{1}{2}mv^2=\frac{1}{2}m.2gh.=mgh$. This example shows that the total energy remains the same. This fact is very important, and it forms the basis of one of the most important laws—the law of **Conservation of Energy** *which states that the sum-total of energy in this world is unalterable.*

---

* Let the kinetic energy be $\frac{1}{2}mv'^2$, where $v'^2=2g\times\frac{3}{4}h=\frac{3}{2}gh$.

$\therefore\quad K.\ E.=\frac{1}{2}m\times\frac{3}{2}gh$ or $\frac{3}{4}mgh$.

A word must be said about the case in which energy seems to be lost. For instance, when we haul a body up along a rough inclined plane the potential energy that it possesses at the top is found to be less than the work done upon it. It is on account of the fact that a part of the energy spent in overcoming friction is changed into heat and is hence irrecoverable. Almost in every operation, a part of energy is lost to us in the form of heat. In the example of the ball, the energy spent in overcoming the friction of air is lost. This loss of energy is called **Dissipation of Energy**. As a consequence of this dissipation, the availability of energy tends to become less and less, but the sum-total remains the same.

61. An interesting example of mutual conversion of potential into kinetic energy and kinetic into potential energy is afforded by the pendulum. It also explains beautifully what we mean by the dissipation of energy. The ball of the pendulum is at rest when it is at the lowest position A, Fig. 41, the string is vertical in this position. When it is displaced to B, it not only moves to the right but also rises upwards, for it moves in an arc of a circle with O as centre and OA as radius. In order to raise it through the height NB work must be done. Thus we see that when the ball is at B, work is done upon it which is stored in it as potential energy. The ball will do an equal amount of work in coming back to its original position. As soon as it is released at B, it begins to move with a gradually increasing velocity, which becomes greatest when the ball is at the lowest position. As a matter of fact in this position the whole of the potential energy is changed into kinetic energy. Since a moving body cannot come to rest unless some external force acts on it, the ball cannot come to rest in the lowest position by itself. It continues the motion on the opposite side to an almost equidistant point C, where the whole of the kinetic energy is converted into potential energy once more. It starts coming back towards the mean position and at the lowest position the entire potential energy is once again changed into kinetic energy. In each swing the available energy decreases on account of the resistance of air etc. and hence the amplitude goes an decreasing. The energy spent in overcoming the resistance of air and friction at the support etc. is converted into heat and is hence dissipated. This dissipation of energy goes on till the whole of the available energy which was originally stored in the pendulum is converted into heat.

EXERCISES

1. In an Atwood's Machine the weights P and Q are 500 gm. each and the rider weighs 50 gm. The distance moved by one weight in 3 seconds is 210 cm. Find the value of a and hence of g.

In order to find a, use the relation $S=\frac{1}{2}at^2$.

Substituting 210 cm. for S and 3 sec. for t we get

$$210=\tfrac{1}{2}a.9$$

or

$$a=\frac{420}{9}=\frac{140}{3}$$

$$=46\tfrac{2}{3}\text{ cm./sec.}^2$$

To find the value of g, use the relation

$$a=\frac{M-m}{M+m}g.$$

Here $a=46\frac{2}{3}$ cm./sec.^{2}, $M=550$ and $m=500$. Substituting the values we get

$$46\tfrac{2}{3}=\frac{50}{1050}g=\frac{1}{21}g$$

or

$$\frac{140}{3}=\frac{1}{21}g$$

or

$$g=\frac{21\times140}{3}$$

$$=980 \text{ cm./sec.}^2.$$

2. A monkey of mass M climbs up a rope which passes over a frictionless pulley and has fastened on the other end a counterweight W of the same mass as the monkey. Discuss the motion of the monkey and the counterweight.

Let us suppose that the monkey climbs with an acceleration a_1.

The equation of motion of the monkey from the Second Law of Motion is

$$Ma_1=T_1-Mg \qquad \dots \text{(i)}$$

Since tension in the rope at all points is the same hence $T_2=T_1$, and the equation of motion for counter weight is

$$Ma_2=T_2-Mg=T_1-Mg \qquad \dots \text{(ii)}$$

Comparing equations (i) and (ii) we find that

$$a_1=a_2$$

Thus we learn that the monkey and the counter-weight will have the same upward acceleration, and hence both will ascend at the same rate.

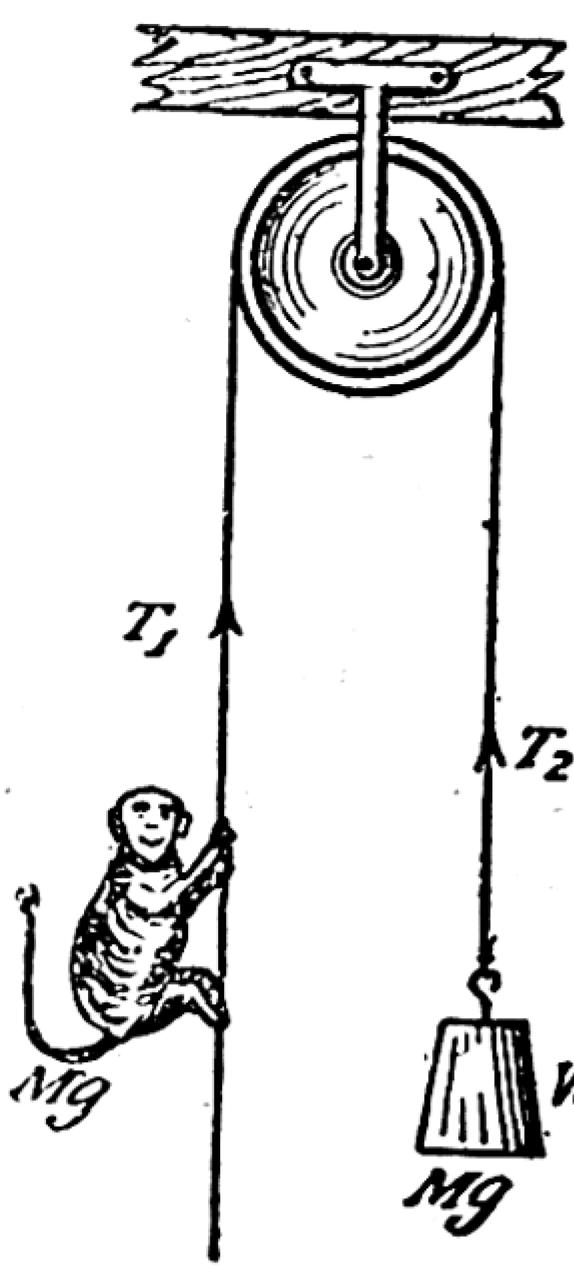


Fig. 43.

3. A man cycles up a hill, the slope of which is 1 in 10, at the rate of 8 miles an hour. The weight of the man and the machine is $187\frac{1}{2}$ lb. Find his horse power ?

In one minute the man covers $\frac{2}{15}$ of a mile or 704 ft. The work that he does is FS where $S=704$ ft. and $F=mg\sin\theta$*.

or

$$F=\frac{375}{2}\times\frac{1}{10}=\frac{75}{4} \text{ lb. wt.}$$

$\therefore$ Work done per minute neglecting the work done against friction

$$=\frac{75}{4}\times704=13{,}200 \text{ ft. pounds.}$$

$$\text{His H. P.}=\frac{13{,}200}{33{,}000}=0{\cdot}4.$$

4. The mass of a train is 250 tons, and the resistance to its motion due to friction on a level line amounts to 15 lb wt. per ton.

*Since the man is moving with no acceleration, $F=mg\sin\theta$

Find the horse power of the engine which can maintain a speed of 40 miles per hour on a level track.

Since the train is running on a level track, the engine has to do no work against gravity, it has to work against friction only in order to maintain the speed. The frictional force is 15×250 lb. wt.

In one minute the engine travels $\dfrac{40 \times 1760 \times 3}{60}$ or 3520 ft.

Hence the work done per minute $= 15 \times 250 \times 3520$ foot lb.

Therefore H. P. required $= \dfrac{15 \times 250 \times 3520}{33000} = 400.$

5. A bullet of mass one ounce strikes a target with a velocity of 400 ft. per sec. and is brought to rest after piercing 3 inches into it. Calculate the average resistance offered by the target.

The $K.\ E.$ of the bullet $= \frac{1}{2}Mv^2 = \frac{1}{2} \times \frac{1}{16} \times (400)^2$
$= 5000$ foot poundals.

If F be the resistance offered by the target the work done by the target on the bullet $= F \times S = F \times \frac{3}{12}$

Since kinetic energy $=$ work done by the retarding force

$$F \times \tfrac{3}{12} = 5000$$

or

$$F = 5000 \times 4$$
$$= 20000 \text{ poundals}$$
$$= 625 \text{ lbs. wt.}$$

6. Two bodies of mass 17 and 15 lb. respectively are connected by a string which passes over a smooth pulley. Find the acceleration with which they move, and the tension of the string.

Ans. 2 ft./sec.^{2}, 510 poundals.

7. Two weights each of 5 lb. are tied to the two ends of a long cord which passes over a pulley in a vertical plane. An additional weight of 2 lb. is suddenly placed on one of them. Find the acceleration of the weights and also the velocity and displacement after 3 seconds. *Ans.* $5\frac{1}{3}$ ft./sec.^{2}, 16 ft. per second ; 24 ft.

8. A body of 8 lb. mass is placed on a smooth plane inclined at 30° to the horizon, and is connected by a string passing over a smooth pulley at the top of the plane with a mass of 4 lb. Find the acceleration. *Ans.* 0.

9. Find the length of a seconds-pendulum at a place where $g=$ 981 cm. per sec. per sec. *Ans.* 99·38 cm.

10. How long will a man whose weight is 11 stones take in getting from the ground to the top of a steeple 400 ft. high by means of a ladder if he works at the rate of $\frac{4}{15}$ H. P. *Ans.* 7 minutes.

11. A train of 10 tons moves up a rough incline of 1 in 100 at the rate of 25 miles per hour, the resistance offered by friction being 10 lb. wt. per ton. Find the H. P. of the engine. *Ans.* 21·6 H.P.

12. A train of mass 200 tons is travelling with a velocity of 30 miles per hour. Calculate its (*i*) momentum and (*ii*) kinetic energy. State the units in which you express your results.

Ans. $19{\cdot}712 \times 10^6$ F. P. S. units ; 6050 ft. tons.

13. If in the last exercise the velocity of the train increases to 45 miles per hour in 5 minutes, calculate the additional horse power put in by the engine. *Ans.* $123\frac{1}{3}$ H. P.

14. A bullet of mass 10 grams moving with a velocity of 200 cm./sec. just comes to rest after piercing a target 8 cm. thick. What is the average resistance offered by the target? *Ans.* 25,000 dynes.

15. A shell weighing 260 lb. was found to have penetrated the ground a distance of 8 ft. in the direction of impact. The striking velocity was 820 ft./sec. What was the average resistance offered by the ground ?

If the velocity of the shell had been 1000 ft./sec. at striking, how far would it have penetrated ?

Ans. (*i*) 153 tons weight ; (*ii*) 11·9 feet.

16. Discuss the scientific knowledge of the labourer who liked his job because he thought that he had only to carry the bricks to the second storey whereas the mason up there had to do all the work.

17. A man weighing 9 stones 6 pounds climbs to the top of a tower 100 feet high in 2 minutes. At what horse power does he work ? *Ans.* 0·2 H. P.

18. Do objects all over the earth fall in the same direction ?

628
5000
20000
82
8

CHAPTER V

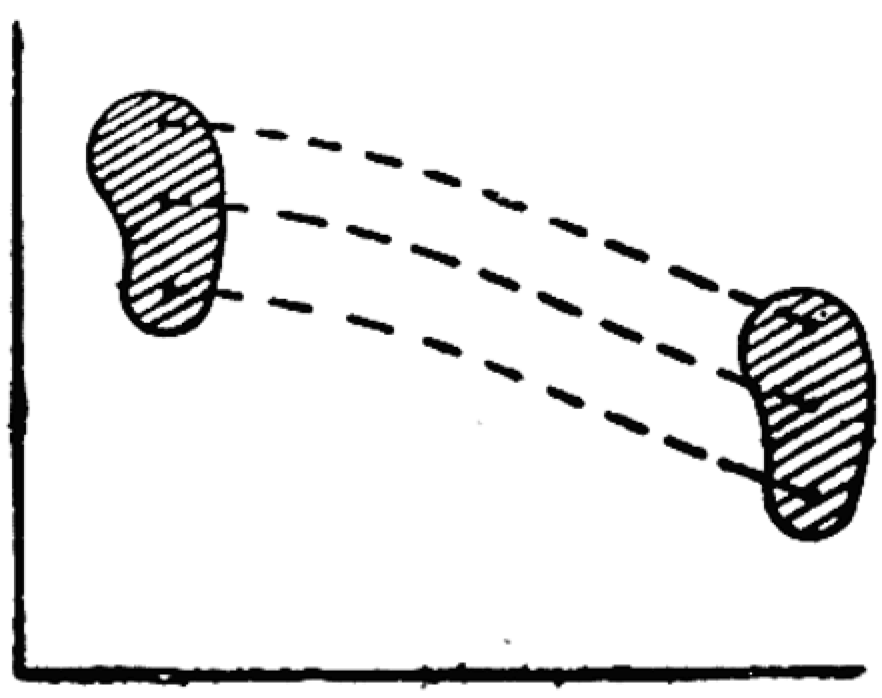
Forces on Rigid Bodies

62. So far we have been dealing with forces acting on particles or bodies so small that they could be regarded as such. We shall now consider the effect of forces on bodies whose size is not so small. We shall suppose that the bodies are rigid, *viz.*, they do not undergo any change in shape under the action of forces. A perfectly rigid body is defined as follows :

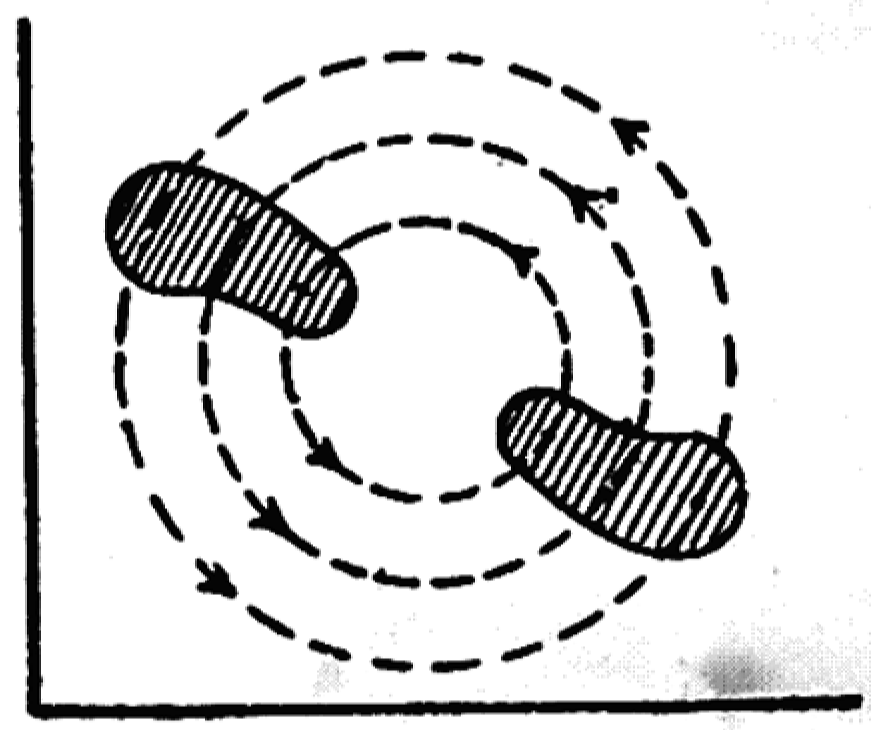
A perfectly rigid body is one, the distance between any pair of particles of which remains unaltered, no matter what forces act on it.

It is important to remember that in this world no body is perfectly rigid, because all bodies yield more or less to the action of forces, but the yielding is so slight that in practice almost all solids can be regarded as perfectly rigid.

A rigid body may have two kinds of motion : it may move bodily from one place to another, with all its particles moving along parallel lines and with the same speed, or it may rotate about a fixed line with its particles describing concentric circles with different



(i)



(ii)

Fig. 44.

speeds. The first type of motion is called **translatory** and the second type **rotatory**. In Fig 44(*i*) is shown a body moving with a translatory motion. Notice that the paths of all particles are equal in length and are parallel to each other. Fig. 44(*ii*) represents a body moving with a rotatory motion. A body may possess both these motions simultaneously. In order to know which type of motion will be produced in the rigid body by a force, we must know the point of application of the force. We did not care for this point so far because there could not be any rotation in a particle, but the case of rigid bodies is different. To know the full effect of a force in their case we must know its point of application.

Let us consider a concrete case. Suspend a football from the ceiling of your room by a string tied to the leather strap. Strike the football with a thin stick, taking care that the line of action of the force passes through the centre. The football will be seen to move to

one side. Now strike it in such a manner that the force is directed towards a point lying on one side of the centre. This time the football will not only move to one side but will also rotate. Thiss hows that the type of motion produced in a body depends upon the point of application of the force.

63. Transmissibility of Force.—Let a force P act upon a body at A along AN. Consider any other point B lying on the line of action of the force (*i.e.*. on AN produced backwards). Impress upon the body at the point B (Fig. 45) two opposite forces Q and R, each equal to P. Since they are equal and opposite they do not produce any effect. The effect of the three forces P, Q and R is the same as the effect of the single force P. As the forces P and Q are equal and opposite, they may be considered to cancel each other, and the effect produced may be considered to be due to R alone. This shows that we can replace the force P acting at A by an equal force R acting at another point in the same line of action. This is the *principle of transmissibility of forces*. It is stated as follows :

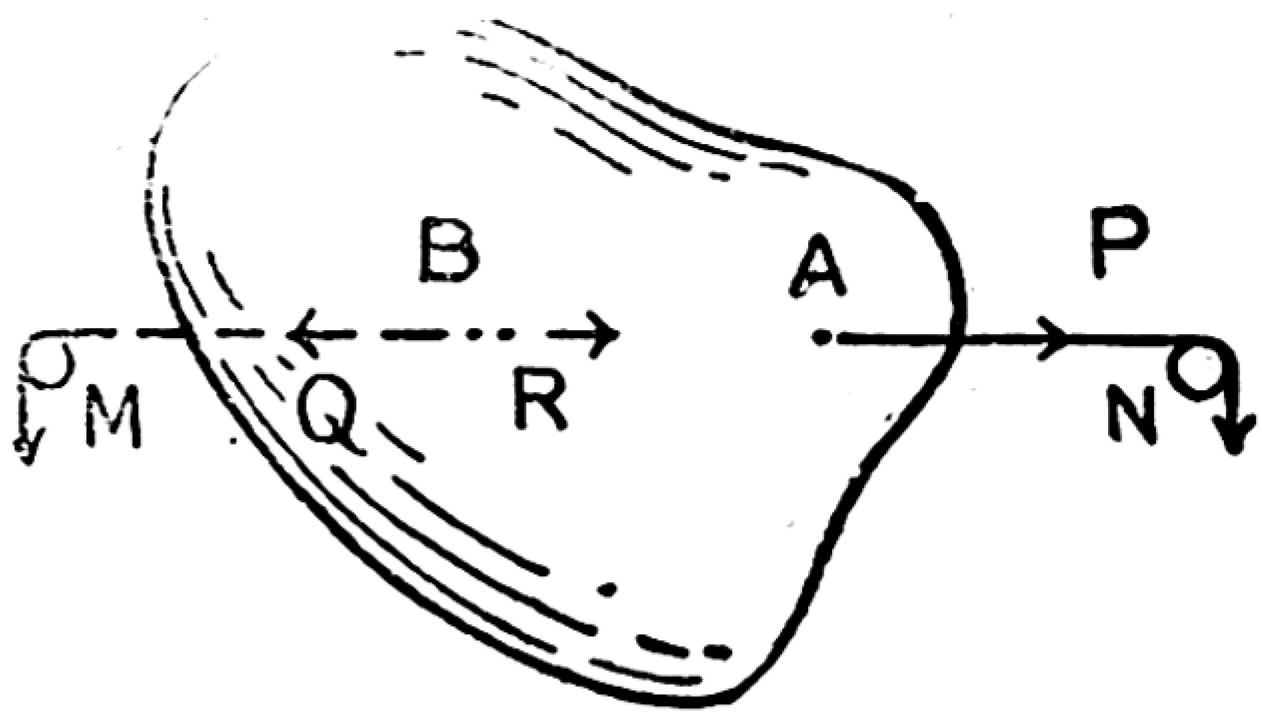


Fig. 45.

The effect of a force upon a rigid body depends upon its magnitude and on the line along which it acts, and remains unaltered by changing its point of application along the line of action.

64. Composition of Forces in the Same Plane.—If the forces are in the same plane, we can use the parallelogram, the triangle or the polygon law of forces.

Suppose A and B are two points at which the forces P and Q act in the directions AN and BM (Fig. 46). Produce these two lines to meet at O. By the principle of the transmissibility of forces, P can be supposed to act at O, and so also the force Q. This means that we have to deal with forces P and Q acting on the same particle O. Represent the two forces by the sides of a parallelogram and complete it. The diagonal passing through the point O, will represent the resultant in magnitude and direction. If the number of forces be greater than two, we can use the polygon law of forces.

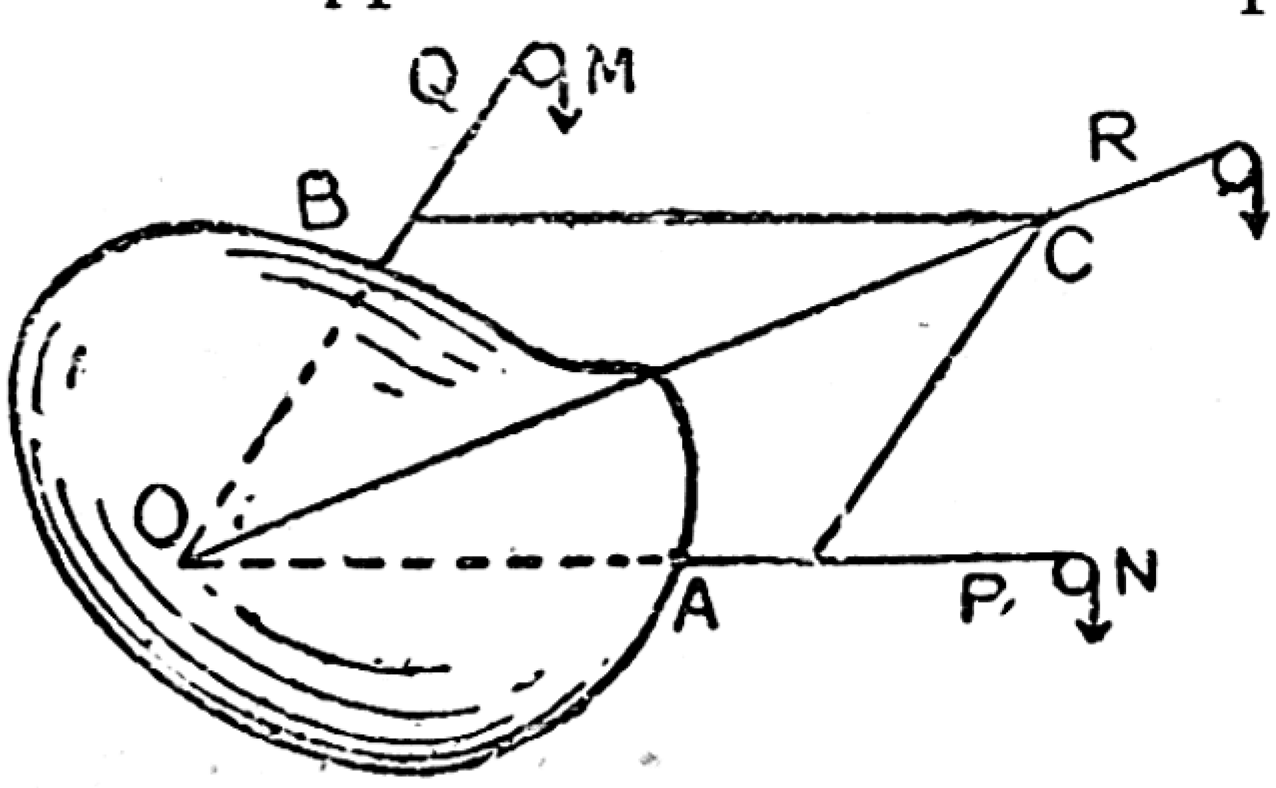


Fig. 46.

But if the forces are parallel to each other, their directions will not meet at a common point, and therefore, the above construction will fail. How to compound such forces ? Before we explain how to find their resultant it is well to remember that either all the parallel forces may act in the same direction in which case they are said to be **like parallel forces**, or some of them may be in one direction and others in the other direction, in which case the forces are said to be **unlike parallel forces**.

65. Composition of Parallel Forces.—Suspend a metre rod (Fig. 47) from two spring balances in such a manner that the rod is horizontal, and the balances vertical. If the spring balances are equally distant from the middle point C at which the weight, w, of the rod can be supposed to act, they will both indicate the same weight, *i.e.*, $\frac{w}{2}$. The weight of the rod acts in the downward direction whereas the forces, P and Q, exerted by the spring balances act in the upward direction. And since each spring balance reads $\frac{w}{2}$, the sum of the readings on the two balances is equal to w, the weight of the rod.

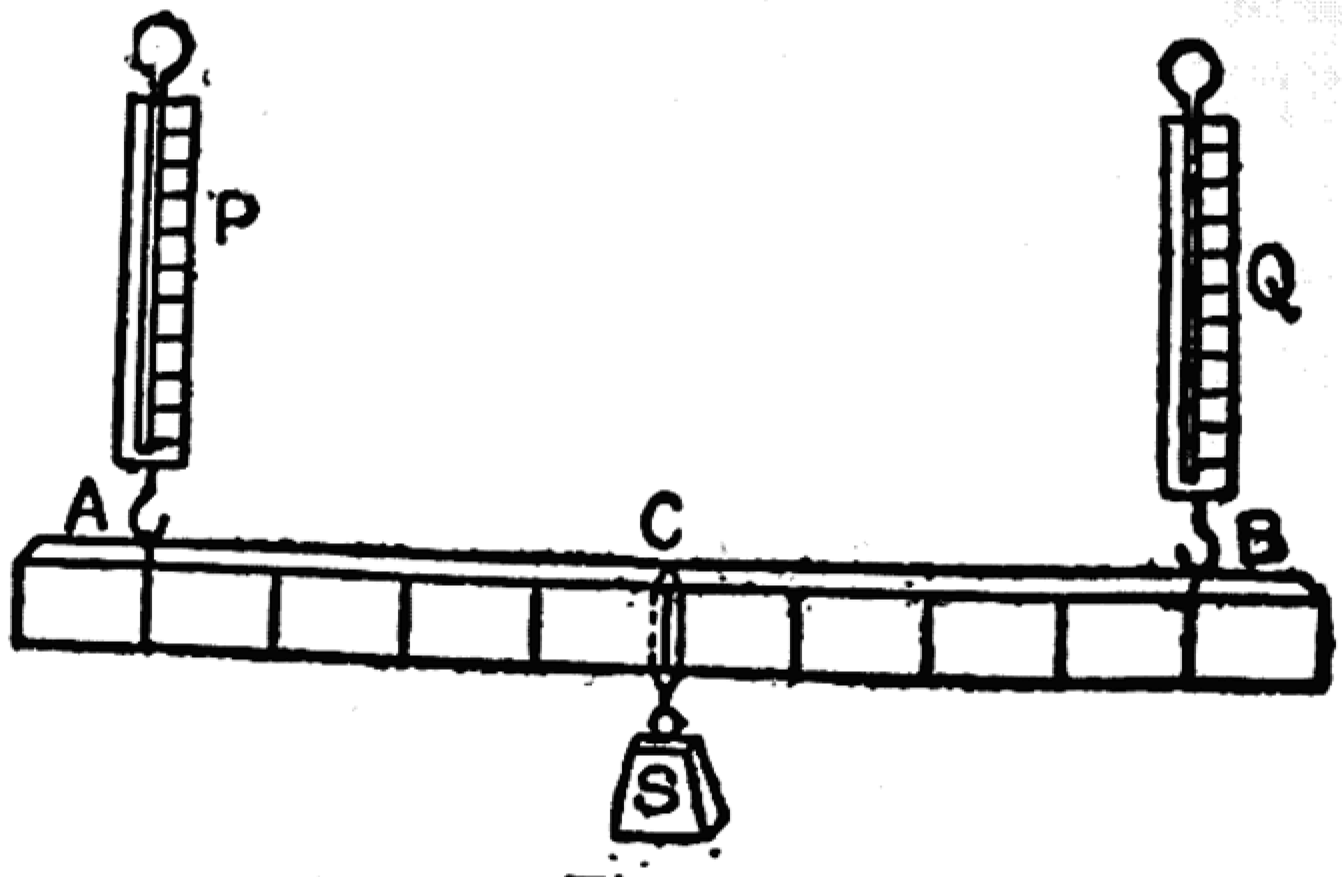


Fig. 47.

Now suspend a weight, S, at the middle point, C, the reading of each balance will increase by $\frac{S}{2}$. The sum of the readings will be $S+w$. If the spring balances are at different distances from the middle point, the nearer balance will read more and the distant one will read less, but the sum will be the same, *i.e.*, $S+w$. Thus we see that in each case the upward forces are equal to the downward forces. It is clear from what is said above that

$$P+Q=S+w$$

Since the forces P, Q and $S+w$ are in equilibrium, the resultant (call it R) of P and Q must be equal to $S+w$ and act upwards at the point C. As $S+w$ is vertical, R must also be vertical which means that it will be parallel to P and Q, for they are both vertical. Next let us see what is the relation between the reading of a spring balance and its distance from the resultant.

It will be seen that the readings are such that

$$P.AC=Q.BC$$

where P is the reading and hence the force exerted by one balance, AC its distance from the point C and Q the reading on the second balance and hence the force exerted by it and BC, its distance from C. The above result can also be expressed as

$$\frac{P}{Q}=\frac{BC}{AC} \quad \ldots\ldots \quad (a)$$

This enables us to find the position of C, if we know P, Q and AB. The relation (a) can be expressed in words by saying that the resultant acts at a point which divides the distance between the points of application of the forces internally in the inverse ratio of the forces. To sum up, we have come to the following conclusions :

(1) *The resultant is parallel to each of the forces, and is equal to the sum of the two.*

(2) *The point at which the resultant acts divides internally the distance between the two forces in their inverse ratio.*

Since the three forces P, Q and $S+w$ are in equilibrium, any one of them can be considered as equal and opposite to the resultant of the other two. Suppose we want to compound the forces Q and $S+w$ acting at B and C respectively. The resultant must be equal and opposite to P, in order that the rod may be in equilibrium. But the readings are such that $P+Q=S+w$, therefore $P=S+w-Q$. We can express it by saying that when the forces are unlike, the resultant is equal to their difference, and is parallel to them in direction.

It will be seen that the distances AB and AC are such that

$$Q.AB=(S+w).CA$$

or

$$\frac{Q}{S+w}=\frac{CA}{AB} \qquad \ldots\ldots\ldots (b)$$

The relation (b) can be expressed in words by saying that the resultant divides externally the distance between the two forces in their inverse ratio. Thus for unlike forces we come to the following conclusions :—

(1) *The resultant is parallel to each of the two forces, and is equal to the difference of the two forces. It acts in the direction of the greater force.*

(2) *It divides externally the distance between the two forces in their inverse ratio.*

If we have more than two parallel forces to compound we shall first find the resultant of any two of them, and then compound this resultant with the third force, and so on. By repeating this process, we can find the resultant of any number of parallel forces acting on a rigid body.

Example.—If there be two forces, 5 lb. and 3 lb., acting in opposite directions, at the ends of a rod AB 4 ft. long, find the magnitude and position of the resultant.

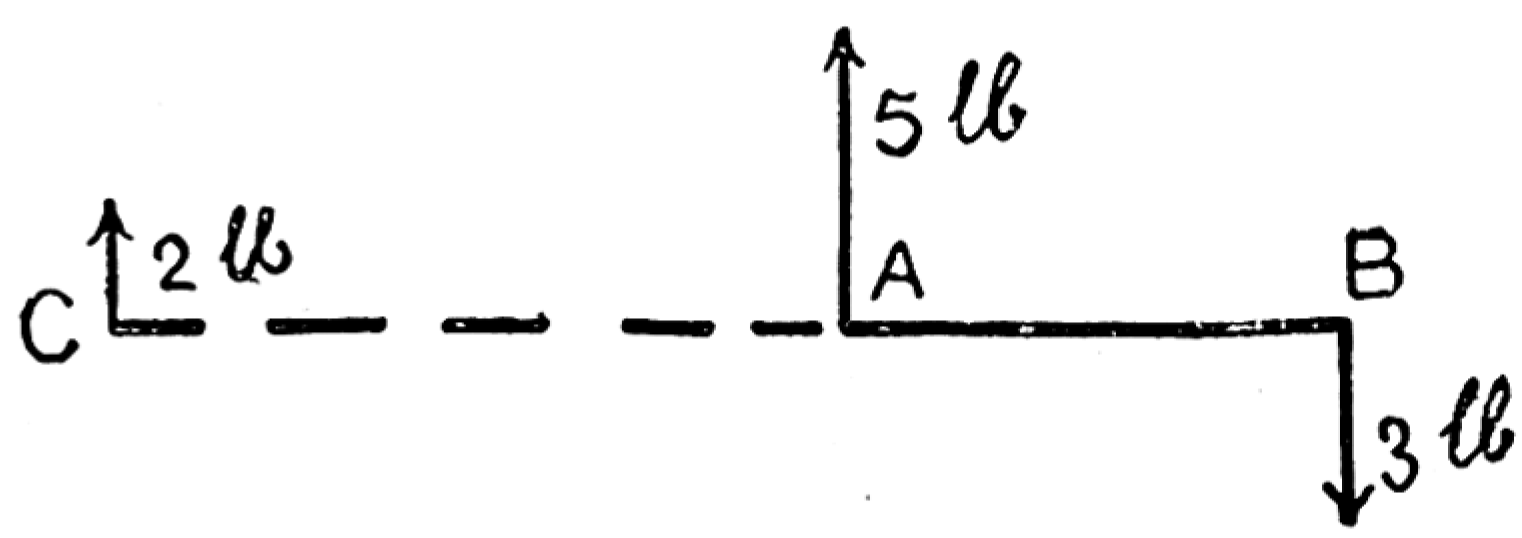


Fig. 48.

Obviously the resultant R will be equal to $5-3=2$ lb. It will act in the direction of the greater force, *viz.* 5 lb. and will divide externally the distance between the forces in their inverse ratio, *i.e.*

$$5AC=3BC,$$

or

$$AC=\tfrac{3}{5}BC.$$

To know the position of the point C, we have to remember simply that

$$BC=AC+AB=\tfrac{3}{5}BC+4\ ;$$

therefore

$$\tfrac{2}{5}BC=4$$

or $BC=10$ ft. and $AC=6$ ft.

66. Centre of Parallel Forces.—Let us suppose we have a metre rod AB on which two vertically downward forces $P(=3$ lb.) and $Q(=5$ lb.) are acting at A and B. If we neglect the weight of the metre rod, the resultant will be 8 lb. and act in the downward direction at the

point C, dividing the distance AB internally in the inverse ratio of the forces P and Q. Now suppose P and Q, instead of acting vertically downward, act along the dotted lines as shown in Fig. 49. Even now the resultant will be 8 lb. and the point at which it will act will divide

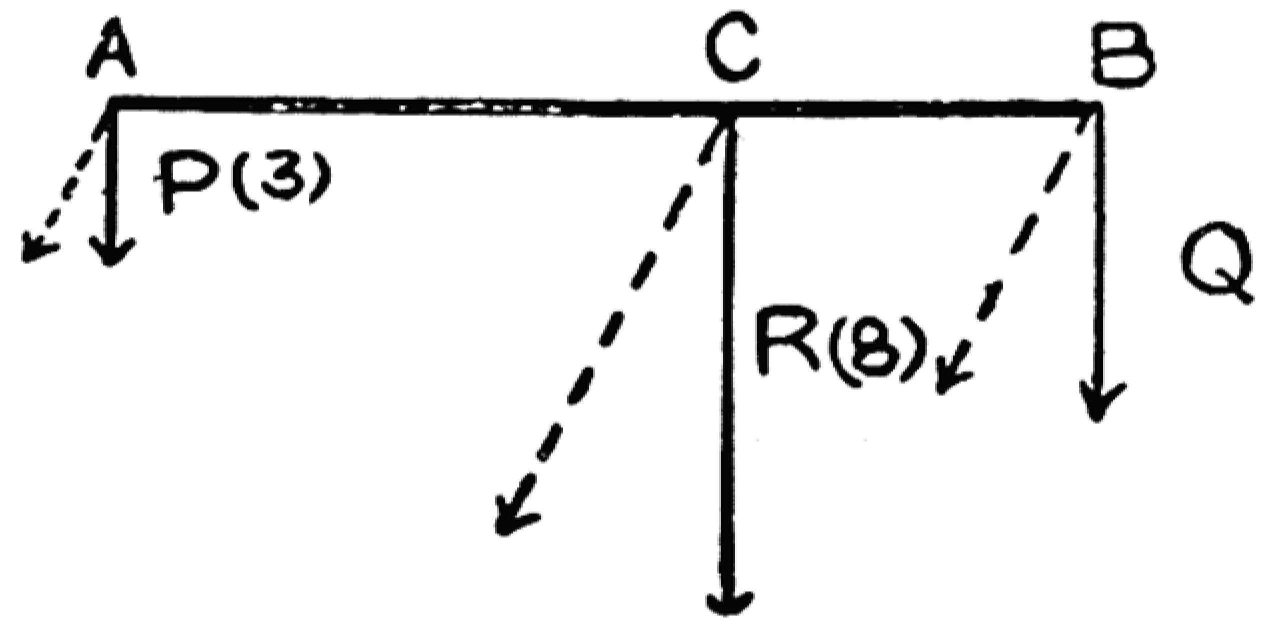


Fig. 49.

the line AB internally in the inverse ratio of the forces. In other words, the point where the resultant acts is independent of the inclination of the parallel forces P and Q. This point, which is fixed for a system of parallel forces, is called the *centre of parallel forces.*

Let us consider a body under the action of gravity. Each particle of it is being attracted by the earth, and consequently *there is a very large number of parallel forces* acting upon it.*

The centre of all these parallel forces, which is a fixed point in a body, is called the Centre of Gravity. This is defined as follows : *The resultant of all the parallel forces acting on the various particles of a body passes through a fixed point, whatever be the position of the body. This point is called the Centre of Gravity of the body.* In simpler words we can say that the centre of gravity of a body is a point at which its entire weight may be supposed to act. Centre of gravity is, for brevity, written as *C. G.*

67. C. G. of a Body lies below the Point of Suspension.—When a body is freely suspended the only forces acting on it are its weight and the tension of the string by which it is suspended. Since the body is at rest, the two forces acting on it must be in equilibrium and have no resultant, which means that the forces must act in opposite directions along the same line. Now since the weight acts vertically downwards at *C. G.*, the force of tension must act vertically upwards and pass through the *C. G.* of the body. This shows that the centre of gravity of the body must lie vertically below the point of suspension.

A body suspended freely on a string is called a *plumb line*. It is used to find whether a wall is vertical or not.

68. To determine Experimentally the C. G. of a Body.—To find the position of the C. G. of a body, say a triangular piece of cardboard, suspend it freely from one corner. The centre of gravity lies vertically below the point of suspension. Draw with the help of a plumb-line the vertical line passing through the point of suspension on the cardboard piece. Next suspend it from another corner. The centre of gravity must lie below this point also. Draw the vertical line as before. Now since the *C. G.* lies in the first line as well as in the second it

*We consider the forces to be parallel, for the lines joining the various particles of the body to the centre of the earth meet at a distance of 4,000 miles which distance when compared to a few inches is infinite.

must lie at the point of intersection of the two. Suspend it now from a third corner, draw the vertical line as before passing through the point of intersection of the first two lines. Hence remember that the point where all the three lines meet is the centre of gravity. In the same way the *C. G.* of any other body may be determined.

69. Moment.—Have you ever played at a see-saw ? If you have you must have noticed that a small boy by moving away from the centre of the plank can balance a heavy boy (See Fig. 50). The necessary condition for the balancing of a see-saw is not that the weights shall be equal but that their turning effects shall be equal. When the small boy moves away from the centre he increases thereby the turning effect of his weight. The turning effect depends both on the weight of the boy as well as on his distance from the centre or axis of rotation. In Physics we call the turning effect of a force as the moment of a force. It is measured by *the product of the force and the shortest distance between the line of action of the force and the axis of rotation*. Mark the words, "the shortest distance". They imply that in finding the moment of a force we must take into account the perpendicular distance between the axis of rotation and the line of action of the force, and not the ordinary distance.

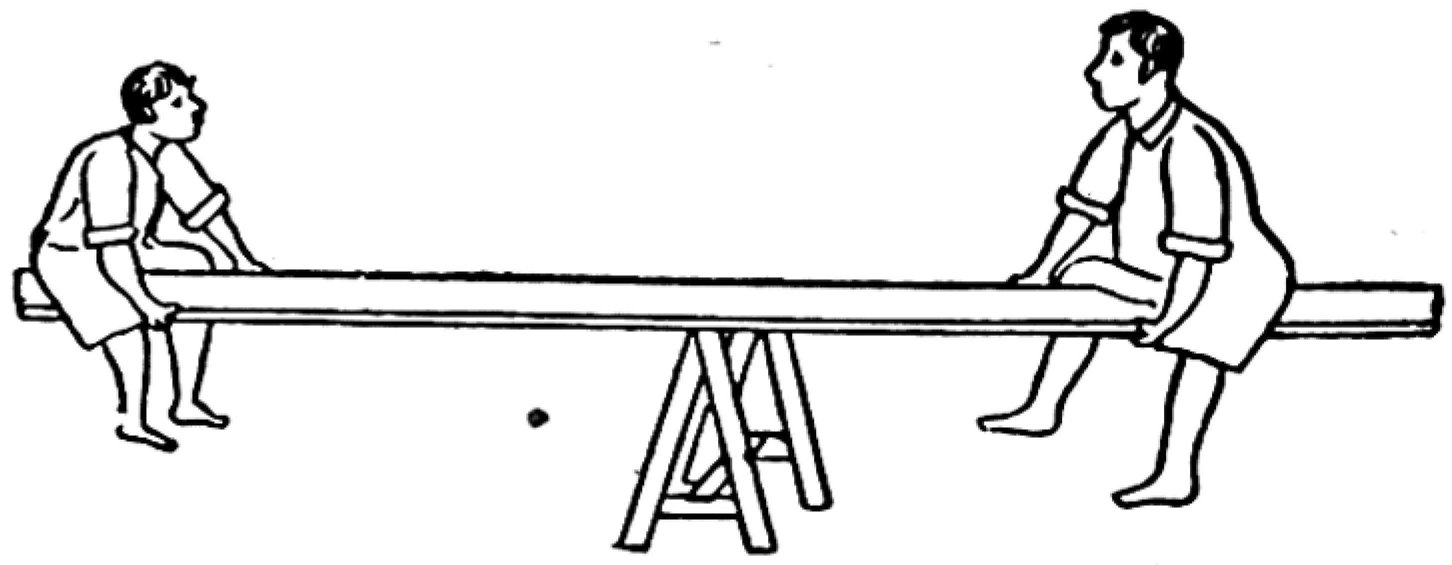


Fig. 50.

To verify this fact take a half-metre rod and bore holes at *A*, *B* and *C* where $AC=40$ cm. and $AB=20$ cm. Suspend the rod from a nail about the end *A* so that it may move freely. Attach a string at *B* and pass it over a pulley *D* fixed a little above *B* and hang a weight of say 5 lb. at the end of the string. Fix a convenient stop at *F* to prevent the rod from being moved out of the vertical position by the pull of the string *BD*. At the point *C* attach another string and pass it over a second pulley *E* fixed at the same height as point *C* so that the string *EC* is horizontal. Slip weights on the hook as shown in Fig. 51 until the rod *just* begins to come away from the stop. Suppose it happens when the weight is equal to 2 lb.

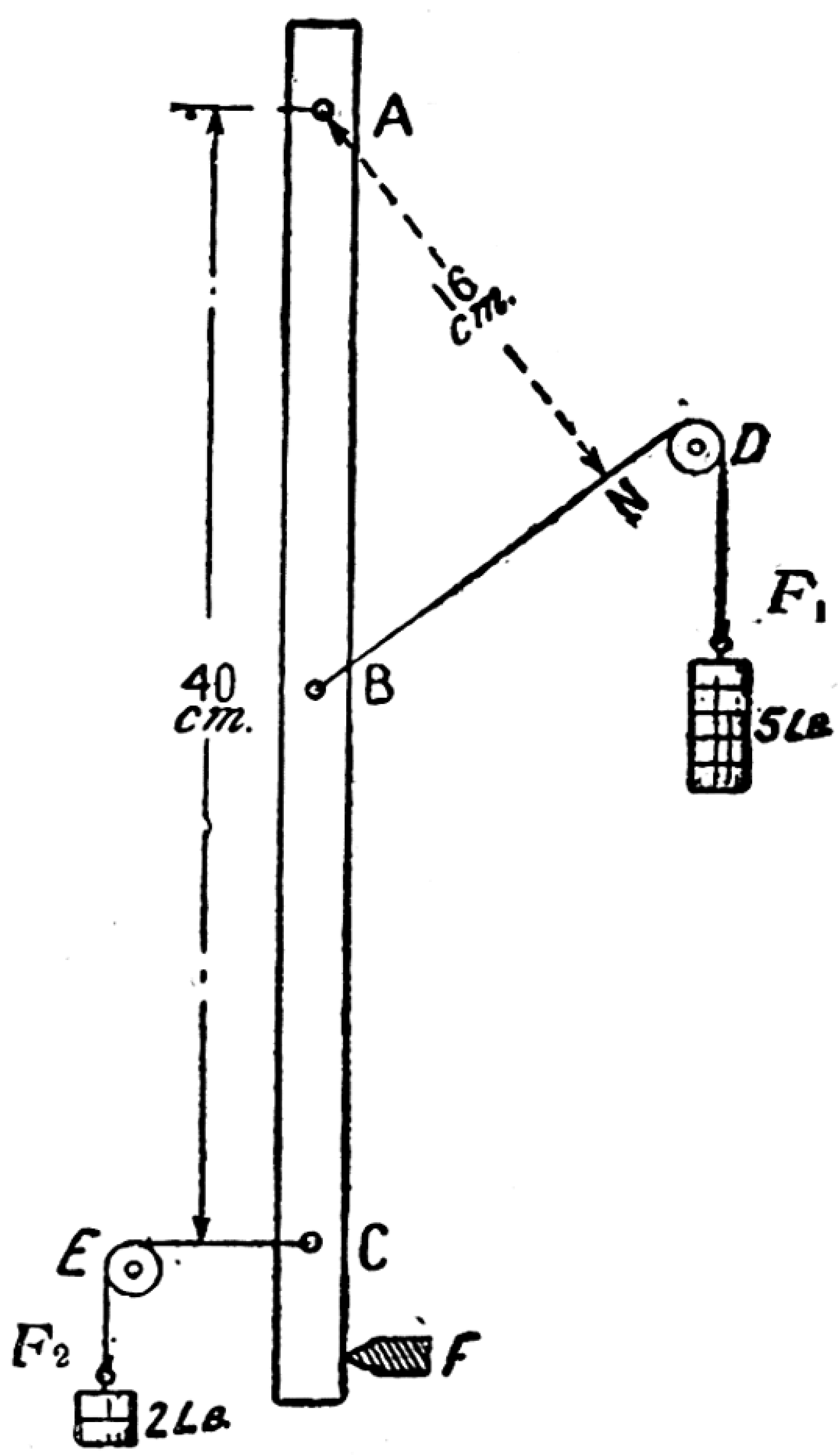


Fig. 51.

New since the weight of 2 lb. tends to rotate the rod away from the stop *F* and the weight of 5 lb. tends to rotate it towards the stop, the turning effects or moments of these forces must be equal. Due to the two weights the rod is in equilibrium.

The moment of force F_2 about *A*

$$=2\times40=80 \qquad \ldots\ldots\ldots \quad (a)$$

The moment of force $F_1 = 5\times y$ . . . . (b)
where y is the perpendicular distance (AN) between axis of rotation and the line of action of the force F_1.

Since the rod is in equilibrium, the two moments are equal, *i.e.*, $80=5y$ or $y=16$ cm.

On measuring the perpendicular distance AN it will be seen that it is 16 cm. This proves that the moment of a force depends upon the perpendicular distance and not on the ordinary distance.

We cannot express a moment completely, unless we take into consideration the direction in which the force tends to rotate the body. The rotation may be either in the clockwise or in the anti-clockwise direction. In order to state fully the effect of a force we must state in what direction the force tends to turn the body. It is usual to take the moments in the anti-clockwise direction as positive and those in the clockwise direction as negative.

If a rigid body, fixed about an axis, be acted upon by a number of forces, the moment of the resultant will be equal to the algebraic sum of the moments of the various forces.

But if the algebraic sum of the moments be zero, that is to say, the sum of the moments in the clockwise direction be equal to the sum of the moments in the anti-clockwise direction, the body will be in equilibrium. This brings us to the **principle of moments**. It is stated as follows: *When a body is in equilibrium the sum of the clockwise moments is equal to the sum of the anti-clockwise moments.* This principle may be used to find the magnitude and direction of an unknown force acting on a body in equilibrium, or it may be used to find the resultant of a number of forces. To illustrate the application of this principle let us consider the following cases :—

Example 1.—A lamp weighing 10 lb. hangs from the end of a horizontal rod 10 ft. long, sticking out perpendicularly from a wall. The other end of the rod is hinged to allow motion in a vertical plane. A string is attached to the middle of the rod and to a hook in the wall 5 ft. above the hinge. Find the tension in the string, if the rod weighs 5 lb. (*P.U. 1931*)

Let AB (Fig. 52) be the rod hinged at A, carrying a lamp weighing 10 lb. at the end B. Let one end of the string CD be attached to the rod at C and the other end to a hook in the wall D, where $AD=5$ ft.$=CA$. From A drop AE perpendicular on CD.

$$AE=\frac{5}{\sqrt{2}}\text{ft.}$$

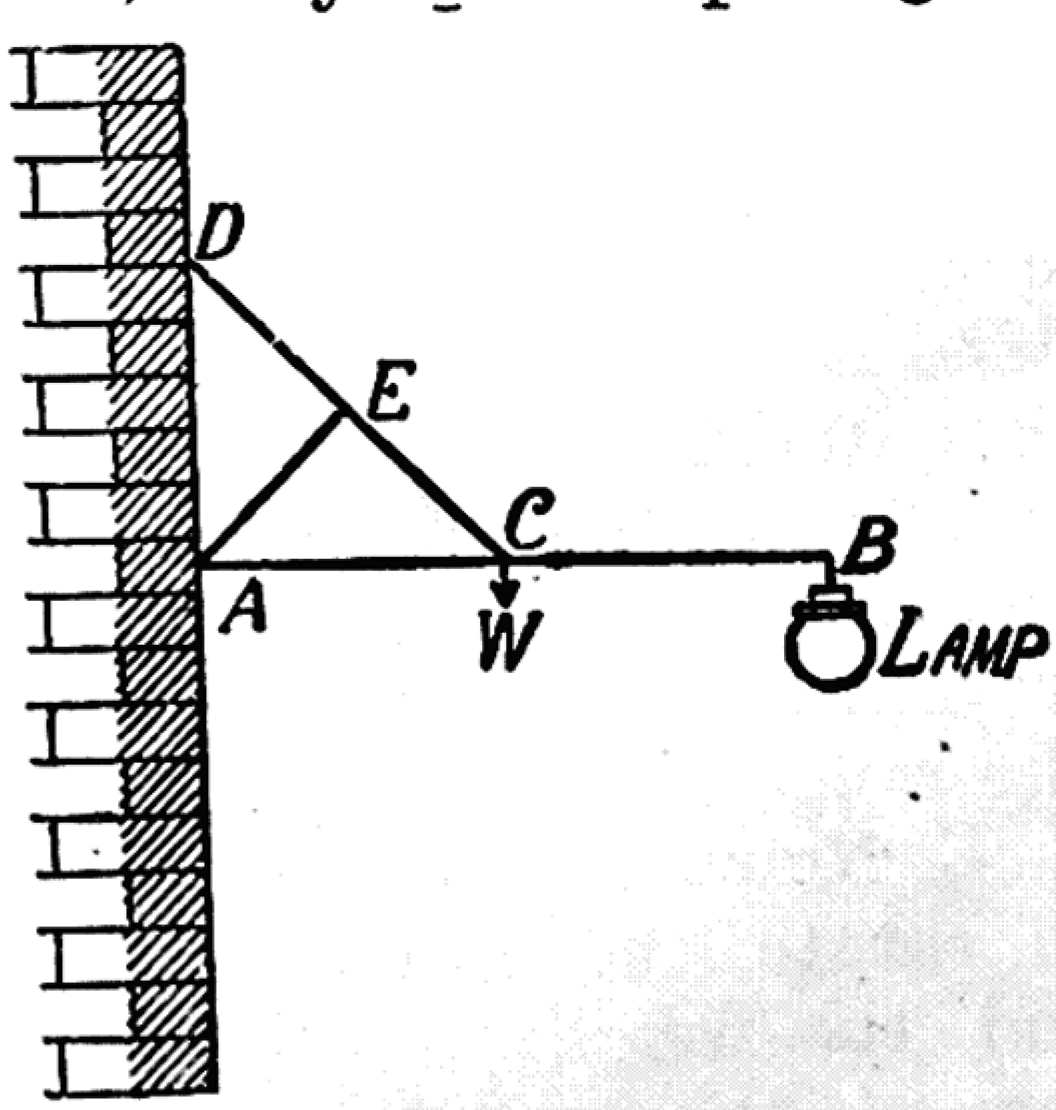


Fig. 52.

The rod is in equilibrium under the action of the following forces : (*i*) the weight of the rod acting at C, (*ii*) the weight of the lamp at B, (*iii*) the tension in the string at C along CD, and (*iv*) the force exerted by the hinge. Since these forces are in equilibrium, it is clear from the principle of moments that the algebraic sum of the moments about a point must be zero. As we do not know the force exerted by the

hinge, to eliminate it we would take the moments about the point A. Doing that we get

$$T.AE = 10 \times 10 + 5 \times 5$$

or
$$T.\frac{5}{\sqrt{2}} = 125$$

or
$$T = 25\sqrt{2} \text{ lb.}$$

Example 2.—Find the point of application of the resultant of a number of parallel forces P, Q, R, acting on a rod AB.

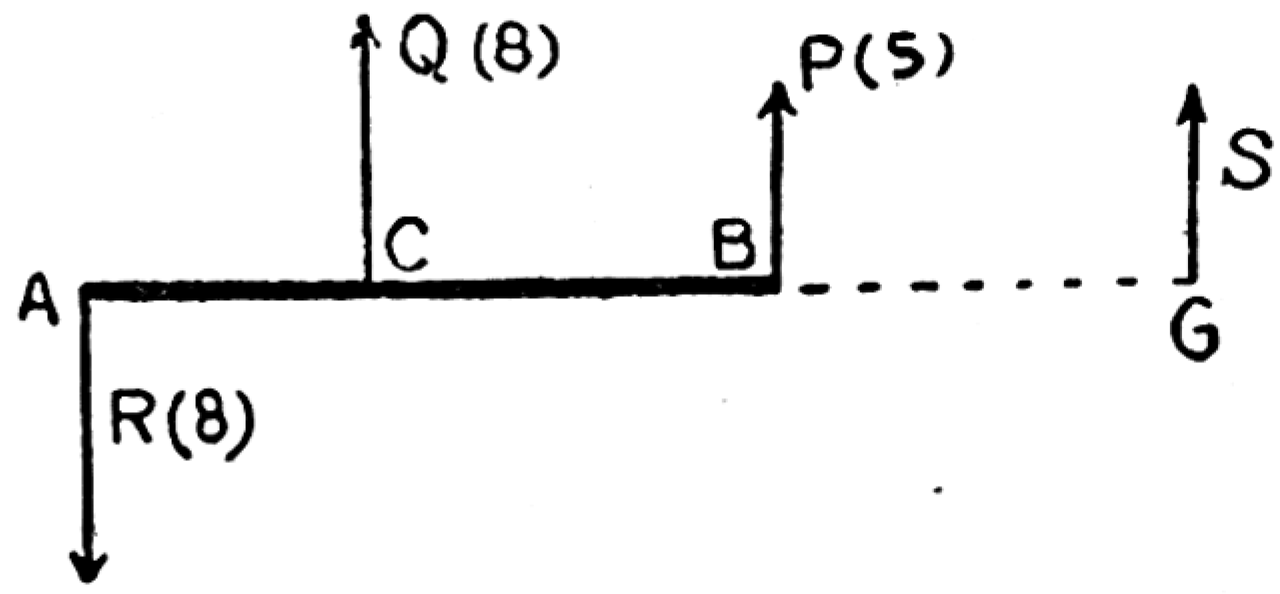


Fig. 53.

In this example AB is at right angles to the forces; if it be not the case actually, instead of AB take into consideration a line at right angles to the forces. The magnitude of the resultant,

$$S = P + Q - R$$

To find the line of action of the resultant we shall make use of the principle of moments. Suppose the resultant S passes through G. The moment of the resultant about the point A will be equal to $S.AG$. Hence the sum of the moments of the components also must be equal to $S.AG$.

Therefore
$$P.AB + Q.AC = S.AG$$
$$= (P + Q - R)\ AG$$

or
$$AG = \frac{P.AB + Q.AC}{P + Q - R}.$$

This gives us the distance of the line of action of the resultant from the end A. Suppose $P = 5$ lb., $Q = 8$ lb., $R = 8$ lb., and $AC = 2$ ft. and $AB = 5$ ft.

then
$$AG = \frac{25 + 16}{5} = \frac{41}{5} = 8\tfrac{1}{5} \text{ ft.}$$

70. Couple.—Let us now consider a special case in which two equal and unlike parallel forces act on a rigid body at different points. Their resultant is evidently zero. Suppose in the example given above, there is no force like P, and the only forces are Q and R so that S = zero. Although the resultant vanishes, its moment does not, for the forces S and Q tend to rotate the body in the same direction. Such a pair of forces constitutes a couple. We define a *couple as a pair of equal, parallel and unlike forces having different lines of action.*

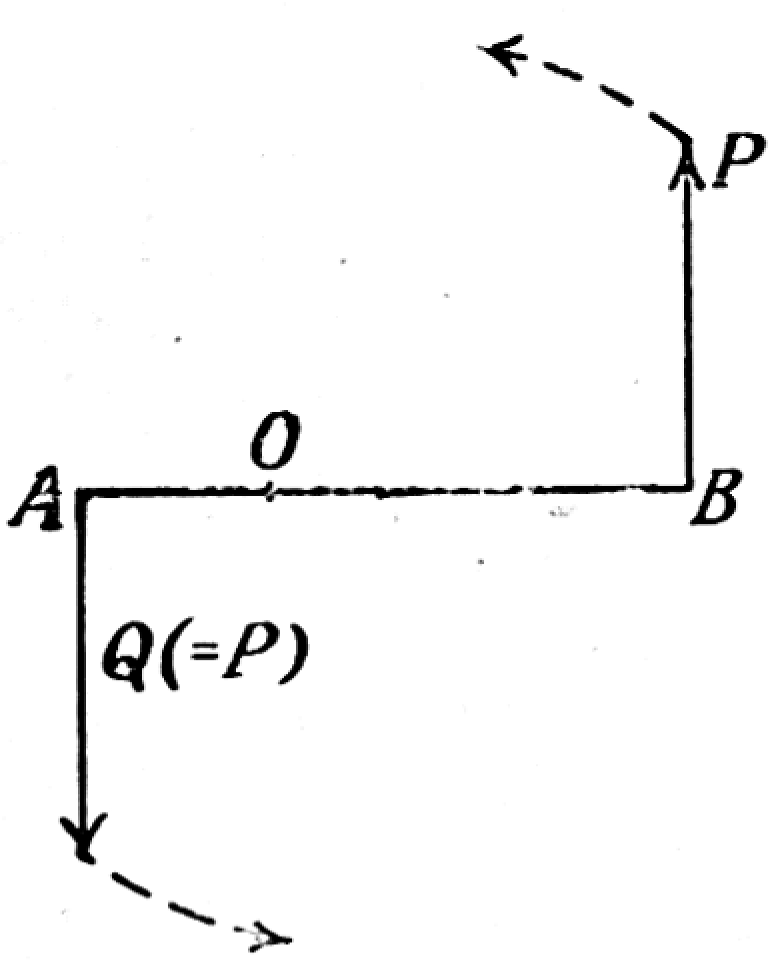


Fig. 54.

The perpendicular distance between the two forces is called the arm of the couple.

Now let us find the moment of a couple. Suppose P and Q are the forces and AB is the arm (Fig. 54). The moment about the point A is $P.AB$: Q produces no effect because it passes through A. Similarly the moment about B is $Q.AB$ which is equal to $P.AB$, for $P = Q$.

The moment will be the same about any other point also. Let us consider the moment

about the point O. The moment of Q about O is $Q.OA$, and the moment of P about O is $P.OB$. Since $P=Q$ and the moments are in the same direction, the moment, $P.OB+Q.OA=P.OB+P.OA$ or $P(OB+OA)=P.AB$. Thus we see that *the moment of a couple is equal to either force multiplied by the arm.*

Remember the effect of a couple acting on a body is to turn it round without moving it bodily as a whole.

71. Equilibrium.—When all the forces acting on a body produce no change in its state, they are said to be in equilibrium. It is important to note the words "no change in the state of a body" ; they imply that the forces do not produce any acceleration. They do not mean that the body is necessarily at rest. All that is meant is that if the body is originally at rest it will continue to do so under the action of the forces, otherwise it will continue to move with uniform velocity. Thus it is clear that **Equilibrium is zero acceleration**.

Now let us consider the **conditions of equilibrium** of a body when it is acted upon by a number of forces *all acting in one plane.* Either all the forces may be parallel, or they may meet in a point, or some of them may be parallel, and others may meet in a point. So that a body may be at rest under their influence the conditions will be different in each case. Let us take these cases one by one.

72. When all the forces are parallel.—If they are to be in equilibrium, naturally their resultant must be zero, *i.e.*, the algebraic sum of the forces, $P+Q+R$, etc., must be zero. When it is so, the body cannot have a translatory motion, but it *may* rotate. It will be so if the forces acting on the body constitute a couple. Now so that the body may not rotate the sum of the moments of the various forces must be zero. We learn, therefore, that for the forces to be in equilibrium the conditions are *that the resultant should be zero, and further the sum of their moments about any point should be zero.* If these two conditions are satisfied, the body will be in equilibrium.

73. (*a*) **When all the forces meet in a point.**—So that the forces meeting in a point may be in equilibrium *they must be capable of being represented by the sides of a closed polygon* (triangle, if there are only three forces). When this is so, they have a zero resultant, and the body will have neither any tendency to move bodily nor to rotate about any fixed **line.**

(*b*) **When some forces are parallel, and others meet in a point.**—The forces which meet in a point can be reduced to a resultant, and so also the parallel forces ; if the combined resultant of both is zero, the body will be in equilibrium unless the forces are reduced to a couple. So that the body may be in equilibrium even in this case, the sum of moments must be zero.

Hence the conditions of equilibrium are (*i*) *the resultant should be zero*, and (*ii*) *the sum of the moments of the forces about any point should be zero.* The last two conditions are quite general. If these are satisfied, the body must be in equilibrium.

74. Stable, Unstable and Neutral Equilibrium.—The equilibrium of a body is not always of the same kind. Consider, for instance

three bodies lying on a smooth table ; a cube, an ellipsoid and a sphere (Fig. 55). When the cube is raised from one side and is left free, it returns to its former state of rest. Such an equilibrium is called **Stable**. But when the ellipsoid standing upon its longer axis is slightly displaced it moves further off from its original position, such an equilibrium is said to be **Unstable.** When, however, the sphere is slightly displaced, it neither comes back nor topples over, but simply rolls ; equilibrium of this kind is said to be **Neutral**. Why these three bodies behave differently will be clear from the following discussion. When the cube

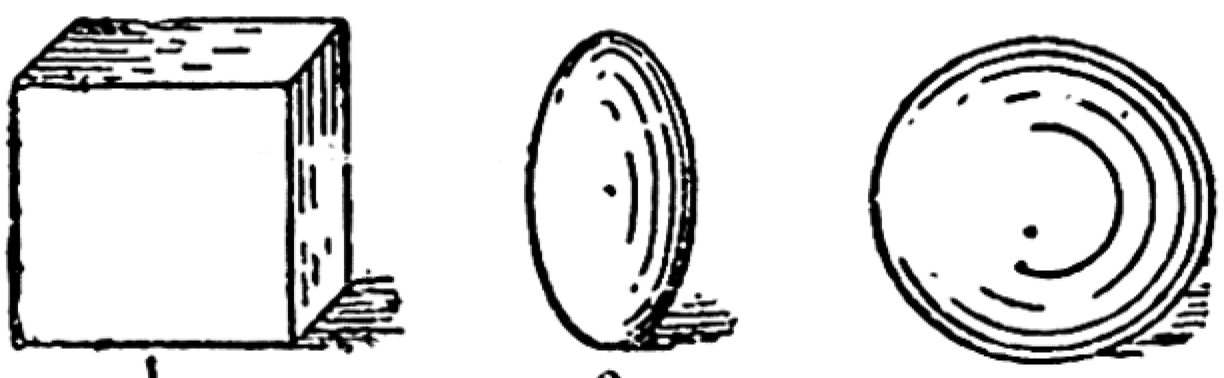


Fig. 55.

is tilted up, its centre of gravity is raised, and the moment of the weight tends to bring the centre of gravity down to the lowest position. Hence the cube, when free, comes back to its original position. When on the other hand, the ellipsoid is displaced, its centre of gravity begins to descend and the moment of the weight tends to bring the *C.G.* lower down, and hence the ellipsoid moves away from its original position. In the case of a sphere, since the *C.G.* of the body remains at the same height, it neither tends to come back to the original position nor topples over.

This tells us that *if a body is to be in stable equilibrium, its C.G. must be as low as possible.* A boat would capsize more easily if the men were standing in it than it would, if they were all sitting.

To know whether a body is in stable, unstable or neutral equilibrium, we have simply to note whether the displacement tends to raise its *C.G.*, to lower it or to leave it unaffected.

There is, however, another important point to remember in this connection. A body may be in stable equilibrium for slight displacements, and in unstable equilibrium for violent displacements. For

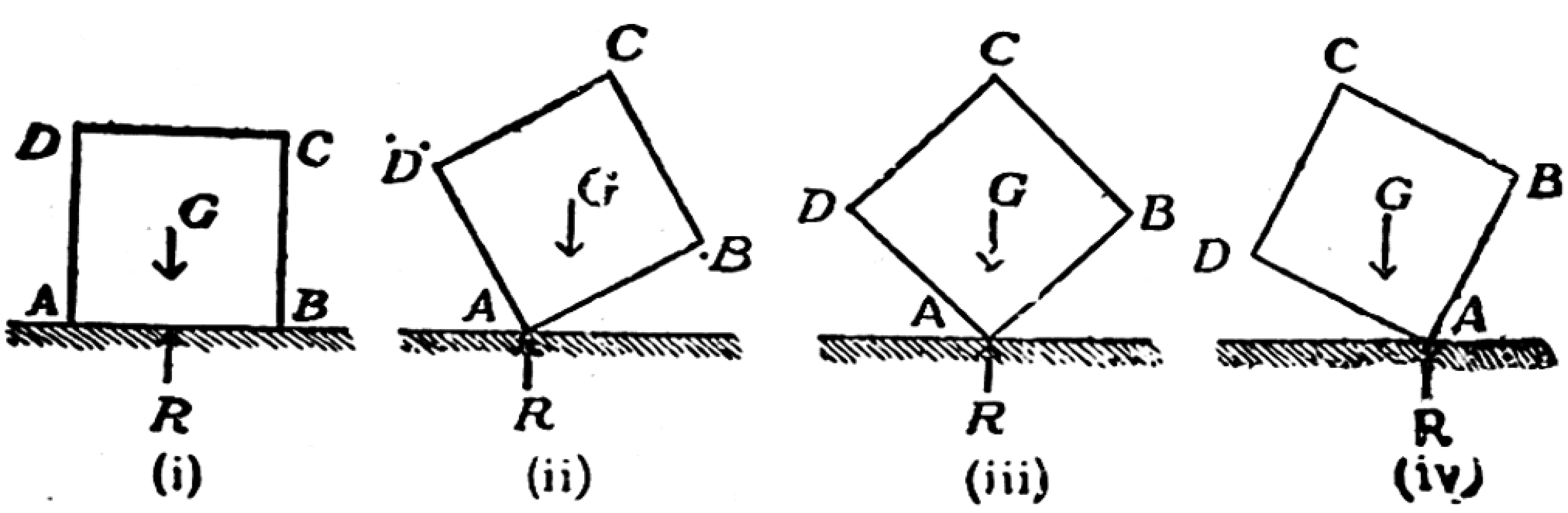


Fig. 56.

instance, suppose that we slightly displace the wooden cube *ABCD* [Fig, 56 (i)] resting on a smooth table. Its centre of gravity is raised [see Fig. 56 (ii)]. The couple due to the reaction of the table and the weight tends to bring the cube back to rest on the face *AB*. This is, therefore, a case of stable equilibrium. But when it is so much displaced that its *C.G.* comes directly over its line of contact [Fig. 56 (iii)] it is in unstable equilibrium. If it is displaced so violently that its

C.G., goes over to the other side of the line of contact, the couple acting on it will bring it to rest on the face AD and not on the face AB [Fig. 56 (iv)]. This shows that the cube remains in stable equilibrium so long as the vertical line through its *C.G.* falls within the base. This also shows that the stability of an object may be increased by enlarging the base and by having the *C.G.* as low as possible. A remarkable illustration of this fact is met with in the leaning tower of Pisa (Fig. 36, page 53), which is 179 ft. high and leans 14 ft. out of the vertical and is still in stable equilibrium. It has stood in this position for centuries. If you carefully see the figure, you will notice that the vertical line through the *C.G.* falls within the base.

EXERCISES

1. A pole 10 ft. long, whose centre of gravity is 4 ft. from one end, is carried horizontally by two men, each carrying the same weight. One of them is at the heavier end. At what distance must the second man be from the other end ?

Let us suppose AB represents the pole whose *C.G.* is at G (4 ft. from B). Suppose w is the weight of the pole Let one of the men be at B and the other at C where $CG=x$. Since the load is to be equally divided between the two men, the moments of the forces applied by them about G must be equal and opposite.

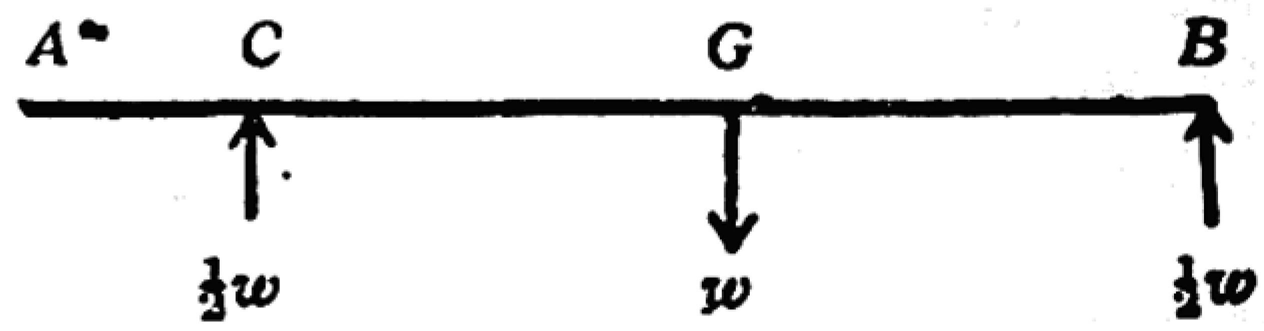


Fig. 57.

Hence $\quad \frac{1}{2}w \times 4 = \frac{1}{2}wx,$

Therefore $\quad x=4 \text{ ft} = CG.$

Hence the distance of the man from A is 2 ft.

2. One end of a uniform ladder weighing 60 lb. rests against a smooth vertical wall at a height of 15 ft. above the ground, the foot of the ladder being 12 ft. from the wall. Find the pressure on the ground.

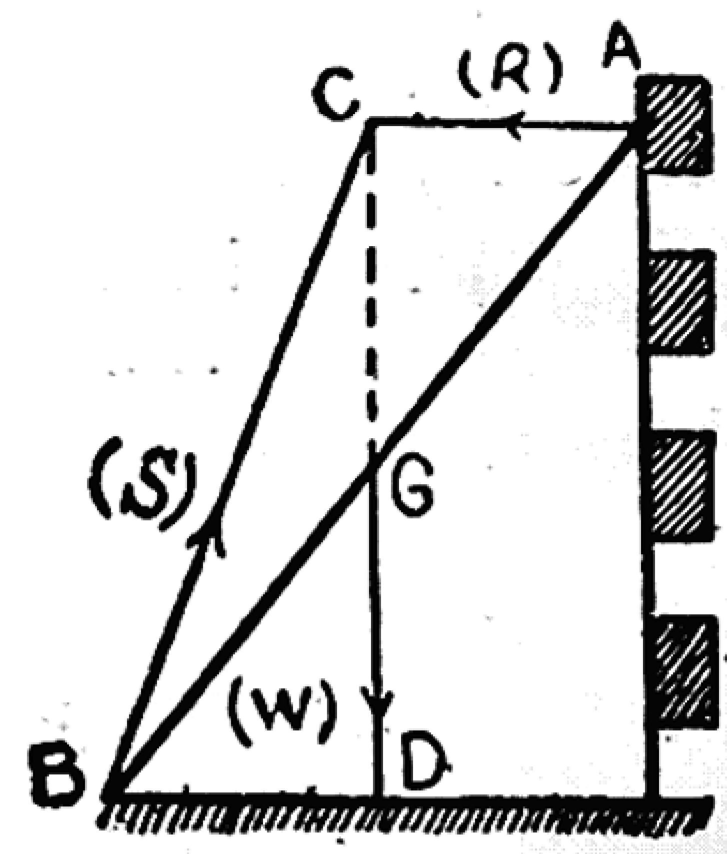


Fig. 58.

The ladder is in equilibrium under the action of three forces, its weight W acting at the middle of the ladder, R the reaction of the wall perpendicular to it and S the reaction of the ground. The weight W and reaction R, both of them, meet at C, hence the third force, *i.e.*, the reaction of the ground, must also pass through C. Since the forces are proportional to the sides

of the $\quad \triangle BCD, \quad \frac{S}{W}=\frac{BC}{CD}.$

Since $CD=15$ ft. and $BD=6$ ft.

$\therefore \; BC=\sqrt{225+36}=\sqrt{261}$ ft.

Hence $\quad S=\frac{60}{15}\times\sqrt{261}=4\sqrt{261}=64{\cdot}62$ lb.

3. A rod 10 ft. long, and weighing 100 lb. is supported at its two ends. A weight of 150 lb. is placed 3 ft. from one end. Find the pressure on the supports. *Ans.* 155 lb. and 95 lb.

4. Two men A and B carry a weight slung from a pole resting on their shoulders. If the maximum weight A can carry is 120 lb. and the maximum weight B can carry is 90 lb. what is the maximum load they can carry between them, and where must it be slung on the pole ? The weight of the pole is to be neglected. *Ans.* 210 lb. , 4 of AB from B.

5. A rod AB 4 ft. long, weighing 10 lb. is hinged to a wall at A, and is kept at right angles to the wall by a string fastened to B and to a point in the wall 3 ft. vertically above A. Find the tension in the string. *Ans.* $8\frac{1}{3}$ lb.

6. A rod AB of weight w can turn freely about a pivot fixed in a wall at A and is supported at B by a horizontal string which is fastened to a point (vertically above A) in the wall at C. Find the tension of the string and the reaction of the pivot if the distance AC is 4 inches and BC 6 inches. *Ans.* $T=\frac{3}{4}w$; $R=\frac{5}{4}w$.

7. A uniform bar of length $3\frac{1}{2}$ ft. and weighing 4 lb. is supported on a smooth peg at one end by a vertical string distant 6 inches from the other end. Find the tension of the string. *Ans.* 2·33 lb. wt.

8. A ladder weighing 56 lb. and 30 ft. long rests against a smooth wall with its foot 15 feet from the bottom of the wall. Find the pressure on the ground and on the wall if the $C.G.$ of the ladder is $\frac{1}{3}$ of its length up from the bottom. *Ans.* Pressure on the wall 10·77 lb.
,, on the ground about 57 lb.

9. The mass of the moon is $\frac{1}{80}$ of the mass of the earth ; if the distance between the two be 240,000 miles, where would the $C.G.$ of the system be ? *Ans.* 2963 miles from the centre of the earth.

10. What is the moment of a couple ? Why it is that a couple cannot be balanced by a single force ?

CHAPTER VI

Machines

75. In our daily life we come across several devices which enable us to multiply our feeble force. Consider for example, a screw jack by means of which we lift a motor car weighing about one ton or so in order that a wheel or tyre can be removed ; or an inclined plane, by which heavy barrels can be loaded into or unloaded from a truck; or a combination of pulleys, by which we raise heavy girders of steel to the roof levels while building houses.

We also come across devices like a bicycle to gain speed or a fixed pulley to merely change the direction of the force applied. In the last case neither the force is multiplied nor is there a gain in speed and yet it is a useful device; for it enables us to apply force in a convenient direction. To raise water from a deep well we have to pull a rope down if it is made to pass over a fixed pulley.

Devices, such as these, which enable us to multiply force or gain in speed or change direction of the force applied are known in Physics as *machines.*

Of course the term, machine, includes a numberless variety—from the knife used to mend a pencil or a pair of scissors used to cut a piece of cloth, to the mighty engine which propels a ship, weighing thousands of tons, through the pathless sea. None of these machines, whether big or small, simple or complex enables us to increase the energy ; at best we can get out of it as much energy as we put into it. In other words, even in the perfect machine in which there is no loss of energy on account of friction etc., the work done *on* it is at best equal to the work done *by* it. To put it in commercial language,

Input=Output.

This statement is generally called the **principle of work.**

If P denotes the power applied to a machine, D, the distance through which the power acts, W the resistance overcome (usually called the load), and d the distance through which the resistance acts or the load is moved, the principle of work can be expressed as

$$PD=Wd.$$

It shows clearly that when we overcome a great resistance with a small force, we exert the force through a greater distance whereas the resistance rises through only a small distance. This point should never be lost sight of. This principle is often stated as *whatever is gained in power is lost in speed or distance.* The multiplication of force which a machine produces is called *Mechanical Advantage.* It is equal to *the ratio of the load moved or the resistance overcome to the effort applied. It is usual to express it as*

$$M\,A.=\frac{W}{P}$$

The ratio of the distance through which the power moves to that through which the load or resistance moves is termed the velocity ratio $\left(=\frac{D}{d}\right)$. If the various parts of a machine have no weight and there is no friction, in other words in a *perfect machine*,

Mechanical Advantage = Velocity Ratio.

Writing this result as

$$M.A. = \frac{D}{d}$$

This value of the mechanical advantage is sometimes called Theoretical Mechanical Advantage.

By making D large in comparison with d the applied force can be multiplied. On the other hand when d is made large as compared with D the speed is increased. In this case the $M.A.$ is less than one.

In actual machines, however, we have always frictional and other losses to overcome, and therefore a part of the energy is spent in overcoming them. This part is wasted in the sense that it cannot be recovered. Hence output is less than input, *i.e*, Wd is less than PD. The equation of work for actual machines is

Input = Output + Work done against friction etc.

The ratio of output to input is called the efficiency of a machine. If we denote **efficiency** *of a machine by* η

then
$$\eta = \frac{\textit{useful work done}}{\textit{whole work done}} = \frac{Wd}{PD}$$

or
$$\frac{W}{P} = \eta\frac{D}{d},$$

viz.,
$$\text{Efficiency} = \frac{\text{Practical Mechanical Advantage}}{\text{Velocity Ratio}}$$

Sometimes this result is expressed as

$$\text{Efficiency} = \frac{\text{Practical Mechanical Advantage}}{\text{Theoretical Mechanical Advantage}}$$

Since for all actual machines efficiency is less than 1, velocity ratio is greater than 1.

Machines are divided into two types, simple and compound. We shall first briefly deal with simple machines, because compound machines, however complex, when analysed, are found to be made up of simple ones ; and if the student understands the principle of the simple machines, he will have no difficulty in understanding the principle of any compound machine. The simple machines are six in number, (1) **the lever,** (2) **the wheel and axle**, (3) **the pulley**, (4) **the inclined plane**, (5) **the wedge,** and (6) **the screw.**

It may be pointed out here that the wheel and axle and the pulley are modifications of a lever whereas the wedge and screw are modifications of an inclined plane.

1. The Lever.

76. A lever is a rigid bar which is capable of motion of rotation about a fixed point called the **fulcrum**. Since levers are used to overcome resistances, soft, flexible substances should not be used, for otherwise part of the power is lost in bending them. While dealing with levers, we have to consider three things, the **power** applied, the **weight** overcome and the **reaction** at the fulcrum. Since the lever is in equilibrium under the action of these three forces, they must either all meet in a point or be parallel to each other. To find the relation between the power and the weight remember that since the lever is in equilibrium the moment of the power about the fulcrum must be equal and opposite to the moment of the weight.* If a and b denote the perpendicular distances of the direction of P (power), and of W (weight) from the fulcrum, [Fig. 59 (i)],

$$Pa = Wb\,,$$

or

$$\frac{W}{P} = \frac{a}{b}\,.$$

The perpendicular distances a and b are called the **arms** of the lever. The relation obtained above shows that if the arms are equal, $P=W$; *i.e., the power is equal to the weight* but if the two arms are unequal, *the ratio of weight to power is inversely as the ratio of the arms of the lever.* For instance, if the arm a is 5 times the arm b, $W=5P$, and so on. This shows that the resistance overcome may be made as great as we like by simply increasing the power arm of the lever in comparison to the weight arm. It was this fact which led Archimedes to remark, "Give me a lever long enough and strong enough and something to rest it on, and I shall move the world." The ratio $\frac{W}{P}=\frac{a}{b}$ gives the **mechanical advantage** of the lever.

The student should find the value of the mechanical advantage of a lever by applying the principle of work. He will find that the value is the same as given above.

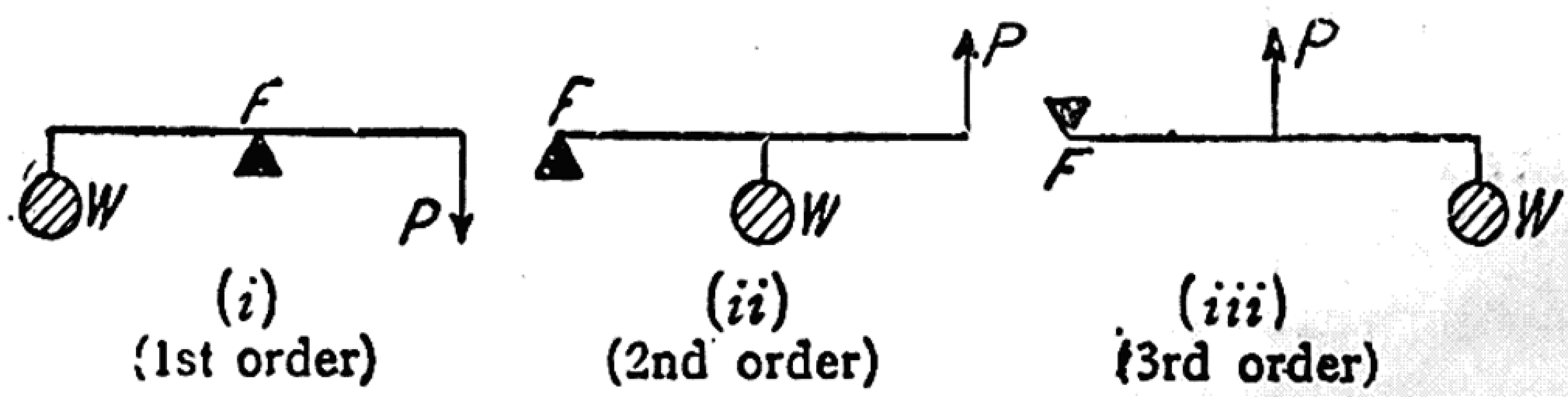


Fig. 59.

The fulcrum may be either, between the power and the weight, when the lever is said to be of the *first order* [Fig. 59 (i)], or on one side of the power and the weight as in levers of 2nd and 3rd order. If W (the weight) is nearer F (the fulcrum), than P (the power), the lever is of the *second order* [Fig. 59 (ii)] ; if, on the other hand, P is nearer F than W, the lever is of the *third order* [Fig. 59 (iii)].

* The reaction at the fulcrum is not considered, for its moment about the fulcrum is zero.

The examples of the first order are afforded by a balance, a pair of scissors (two levers are combined together in this case), a pair of pliers, etc. The examples of the second order are met with in an oar*, a nut-cracker, a wheel-barrow, etc. The examples of the third order are forearm† used in lifting weights, fire-tongs, knife, and fork, etc.

It is not necessary that a lever should always be straight. When a hammer, for instance, is used to draw a nail it is a lever of the first order but it is a *bent* lever. The law of moments holds for it. The dotted lines a and b represent the length of power and weight arms.

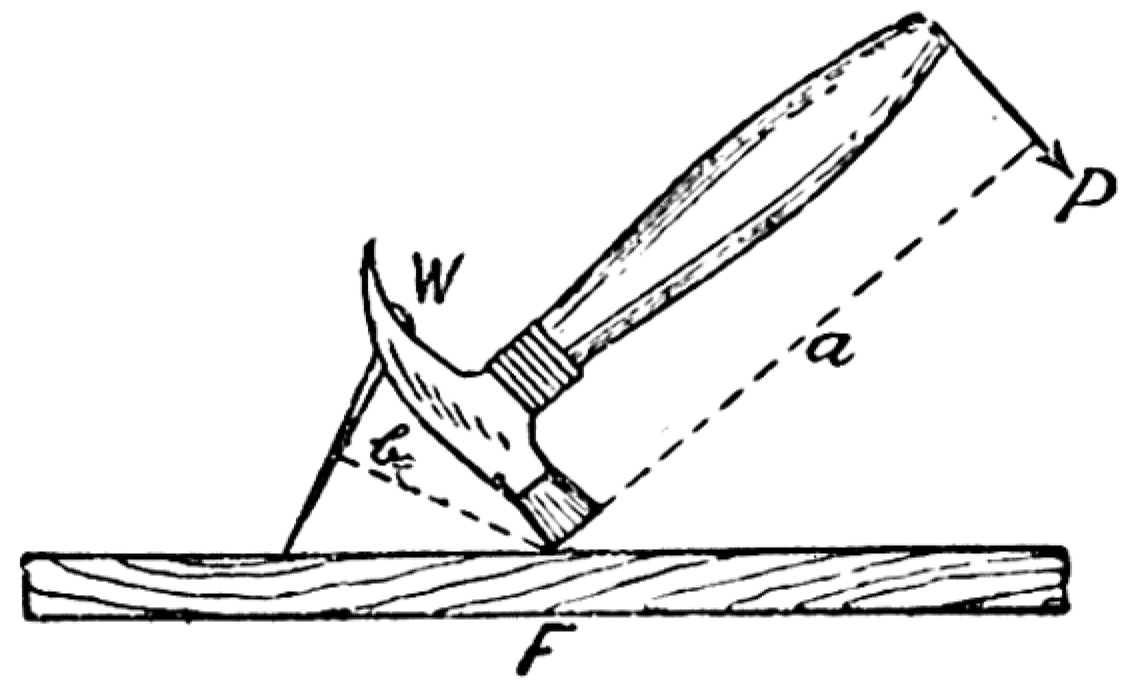


Fig. 60.

2. Wheel and Axle.

77. We can raise a heavy body with the help of a lever, but the height to which it can be raised is limited, for necessarily the height must be less than the weight-arm. If the body is to be raised to very great heights, the fulcrum on which the lever rests must be moved continuously. In order to do this conveniently, the lever is modified to what is called the *wheel and axle*.

It consists of two cylinders of different diameters turning about a common axis (Fig. 61). The cylinder of bigger diameter is called the **wheel**, and of smaller diameter is called the **axle**. The weight W is hung from the rope coiled round the axle. The power P is applied to the end of the second rope coiled round the wheel in the opposite direction, so that when rope No. 2 is uncoiled the rope No. 1 is coiled round, raising thereby weight (W). Frequently instead of wheel and rope No. 2, handles projecting from the rim of the axle are employed, as, for instance, in the case of the familiar device used for raising water from the wells.

To find the mechanical advantage suppose the radii of the wheel and axle are a and b respectively. The moment of the force P about the point O will be $P.AO$ or Pa, and of W will be $W.OB$ or Wb. So that there may be equilibrium

$$Pa = Wb, \qquad \ldots \ldots \quad (i)$$

or

$$\frac{W}{P} = \frac{a}{b} = \frac{\textit{radius of the wheel}}{\textit{radius of the axle}} \qquad \ldots \ldots \quad (ii)$$

Equation (i) shows that the wheel and axle has the same principle as a straight lever. As a matter of fact, at any given instant, AOB forms a straight lever; but at the next moment the radius AO moves off and is replaced by another radius, and so on. Thus we see *that the wheel and axle is simply a continuous or perpetual lever.*

---

*The fulcrum being at the end of the blade which is kept at rest by the pressure of the water, the weight at the row-lock, and the power at the end of the oar.

†The weight being on the palm, the power at the middle of the arm, and the fulcrum at the elbow.

We can also derive the expression for the mechanical advantage by applying the principle of work. When the wheel completes one

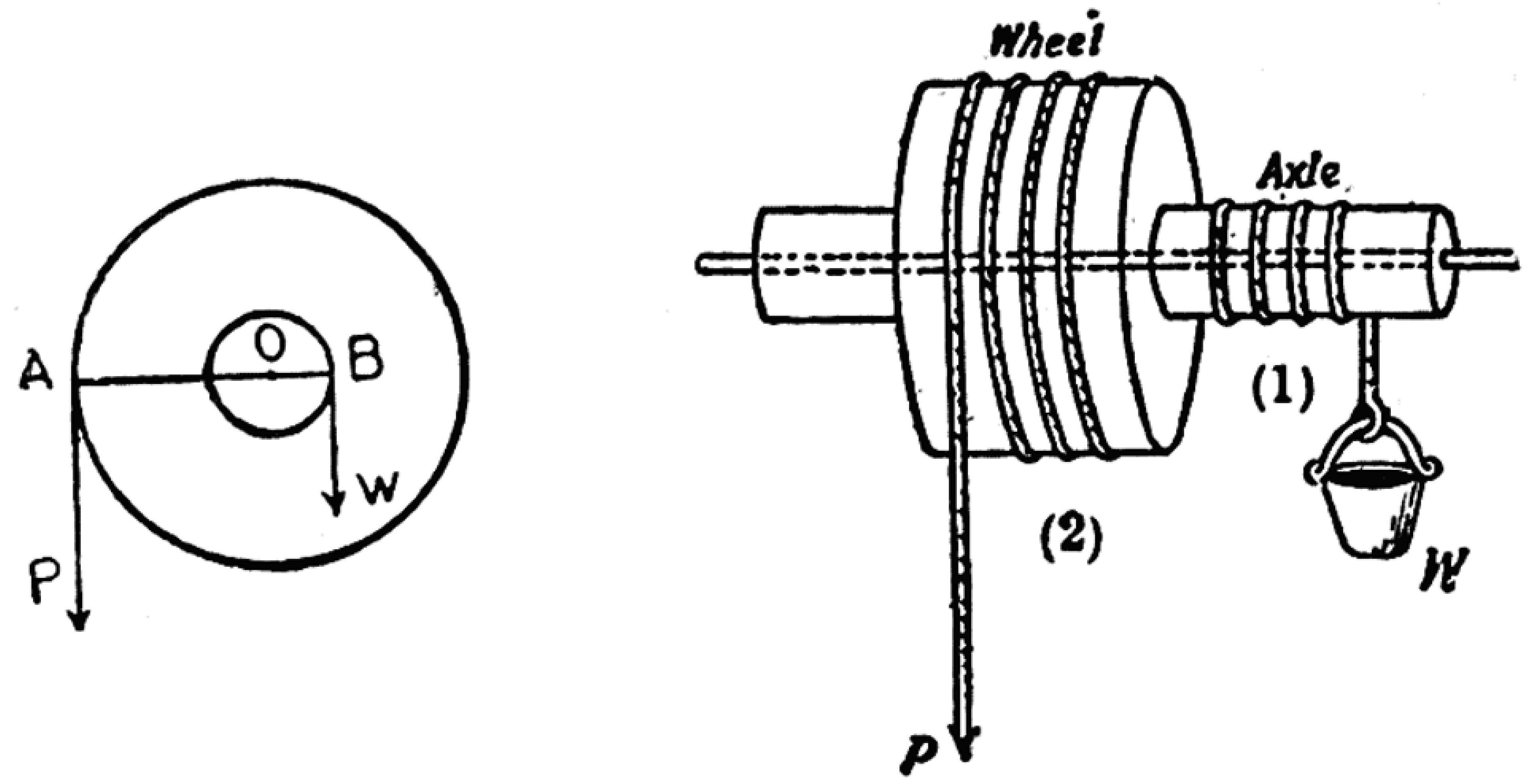


Fig. 61.

revolution, the distance through which the weight rises is equal to the circumference of axle *i.e.*, $2\pi b$ and the distance through which the power acts is equal to the circumference of the wheel, *i.e.*, $2\pi a$. The input$=P\times 2\pi a$ and the output$=W\times 2\pi b$.

Since Input$=$Output, we get

$$P\times 2\pi a=W\times 2\pi b \qquad \ldots\ldots\ldots \quad (iii)$$

or

$$\frac{W}{P}=\frac{a}{b} \qquad \ldots\ldots\ldots \quad (iv)$$

3. Pulley.

78. The pulley consists of two parts, the sheave and the block. The **sheave** is a wheel with a groove cut in its circumference round which a string can pass. It is mounted on an axle which is fixed to a frame-work called the **block**. The sheave rotates freely in the block. The block may be fixed or movable. If the block is fixed the pulley is called a fixed pulley. In this case the pulley can only turn on its axle ; it cannot move up or down. But if the block is movable, the pulley can turn as well as move up and down.

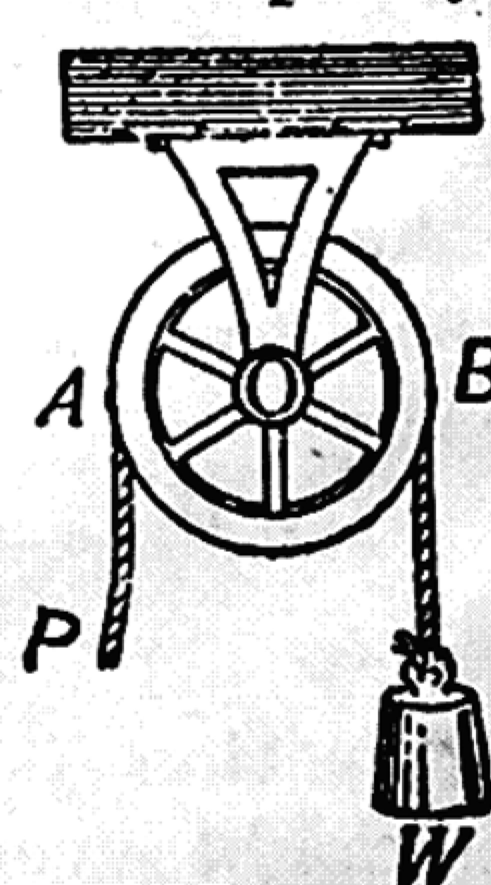


Fig. 62.

(1) **Single Fixed Pulley**.—When the pulley is fixed, no mechanical advantage is obtained, for it simply works like a lever with equal arms. So that there may be equilibrium the moments of P and W about O must be equal and opposite, or mathematically

$$P\times AO=W\times OB \qquad \ldots\ldots \quad (v)$$

But

$$AO=OB \text{ [being the radii]},$$

Hence

$$P=W \qquad \ldots\ldots\ldots \quad (vi)$$

A fixed pulley is used *to change the direction of the force*, for instance to raise a weight *up* we are to pull the rope *down*, which is more convenient.

(2) **Movable Pulley.**—In this case the weight is suspended from the pulley block. One end of the string passing round the pulley is attached to a fixed support as at C (Fig. 63), whereas the other end is free. It is at this free end that the power is applied. We shall suppose that there is no friction*, and therefore the tension is the same at all parts of the string. When the segments of the string are vertical it is clear that the total upward force is $2P$, and if the weight of the pulley is neglected, the downward force is W. When the pulley is in equilibrium we have

$$2P = W,$$

or

$$\frac{W}{P} = 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (vii)$$

Thus *the mechanical advantage of a movable pulley is 2.*

It is clear from the diagram that when W moves through 20 cm. P moves through 40 cm., *i.e.*, double the distance.

If the weight of the pulley cannot be neglected, and is equal to w, the condition of equilibrium will be

$$2P = W + w$$

$$\text{Mechanical Advantage} = \frac{W}{P}$$

$$= 2 - \frac{w}{P} \quad . \quad . \quad (viii)$$

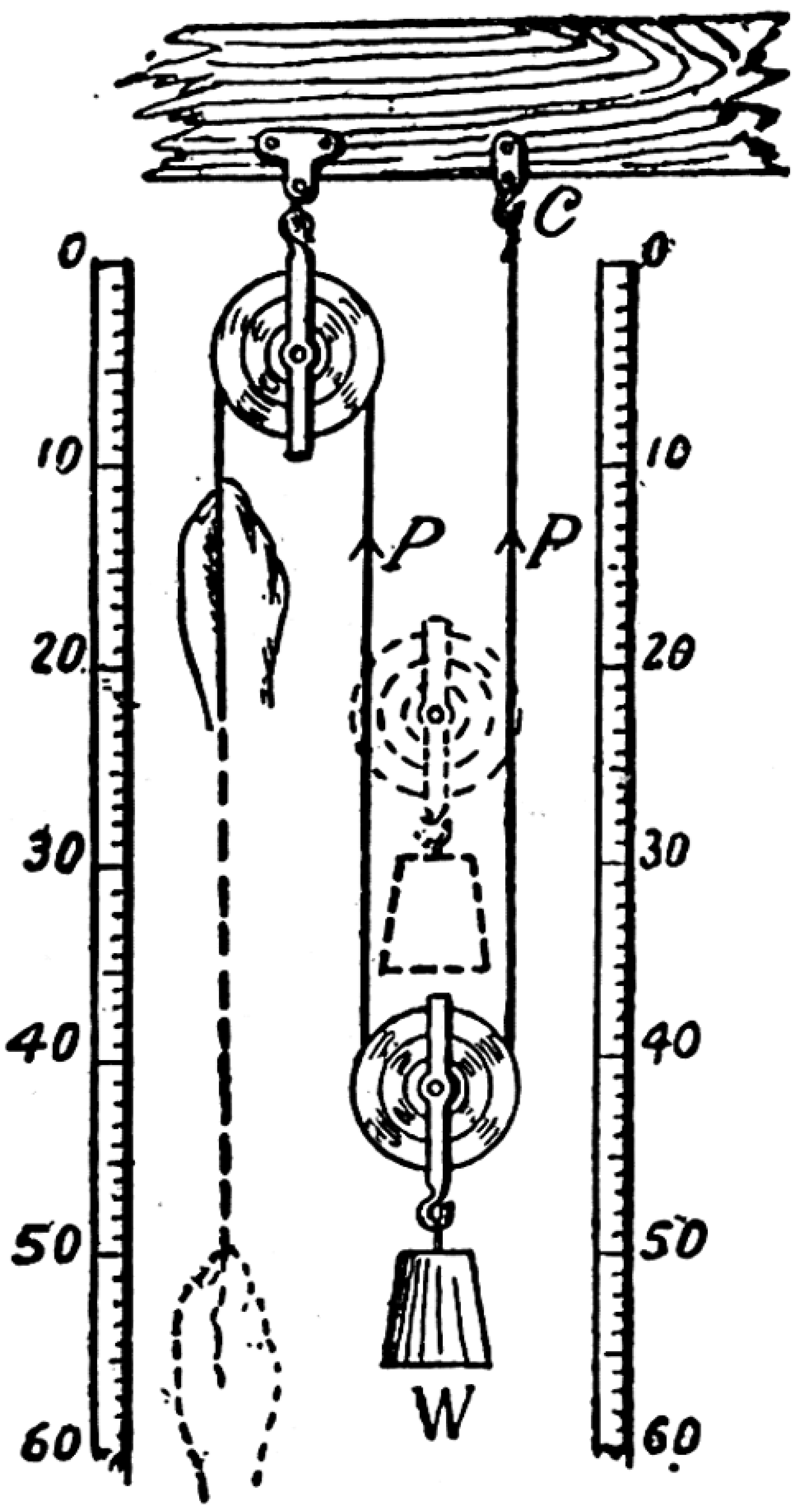


Fig. 63.

This shows that in actual practice the mechanical advantage is slightly less than 2.

If the direction of the force is to be changed, another fixed pulley (see Fig. 63) is used. The use of this extra pulley does not in any way affect the mechanical advantage.

In order to secure a mechanical advantage greater than two, a combination of pulleys is used. Several combinations are possible, but we shall discuss only those types which are in common use.

79. Pulley Block or Block and Tackle.—This combination consists of two blocks of pulleys. The upper block is fixed and the lower is movable. The pulleys in each block are generally mounted on a common axle as shown in Fig. 64 (a). But in order to make clear how the string goes round them the pulleys are usually shown of different sizes and are arranged below one another as in Fig. 64 (b).

The string is fastened either to the upper or to the lower block and is then passed round a movable and a fixed pulley in turn so that finally

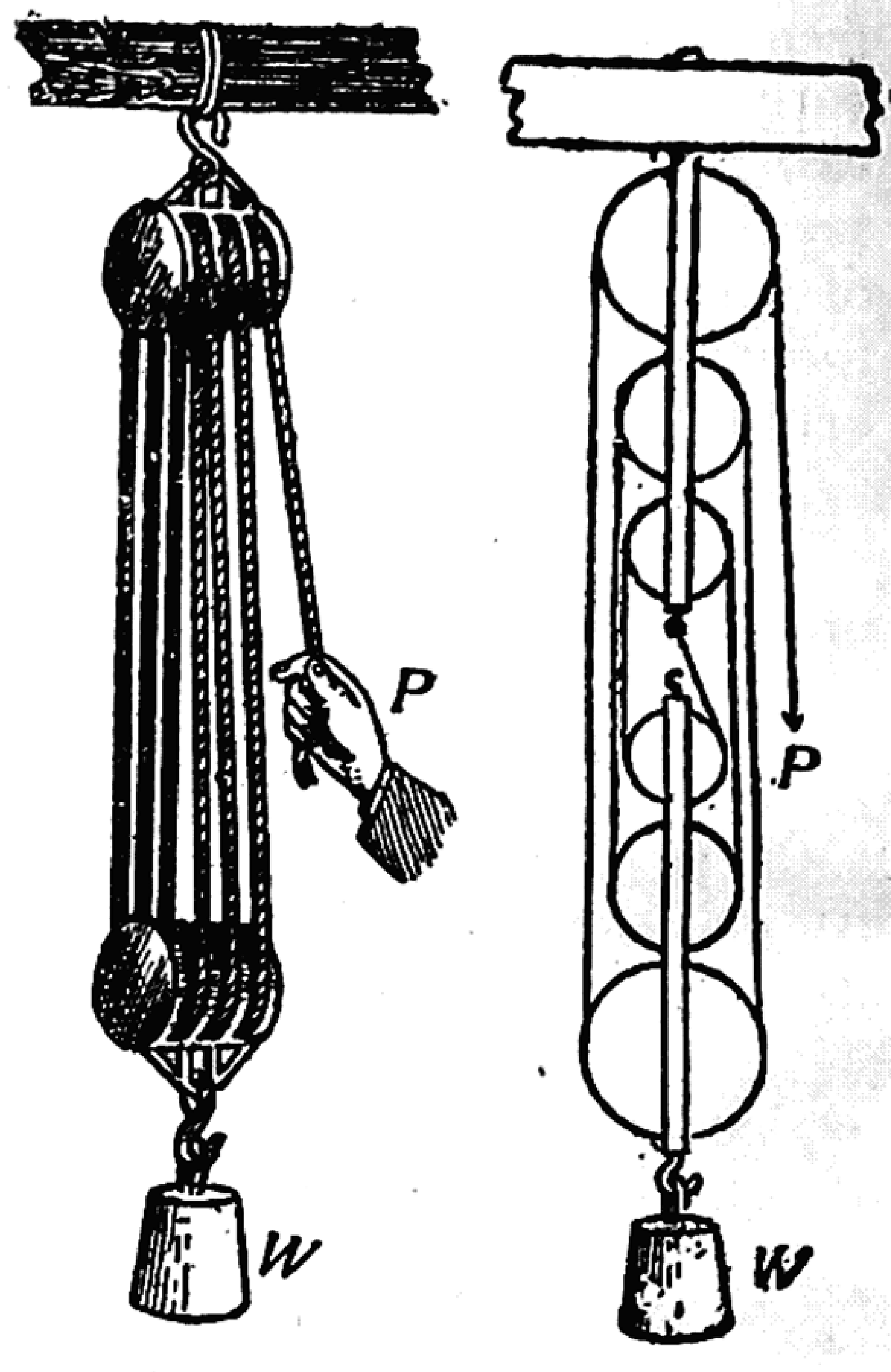
* It may, however, be remarked that the greater the size of the pulley, the less the force required to overcome friction, so that while dealing with big pulleys the friction is almost negligible.

it passes over a fixed pulley and is pulled downwards. The string is fastened to the upper block if the number of sheaves is the same in each block and to the lower block if the number of sheaves in the lower block is one less than that in the upper block.

Let us consider the condition of equilibrium. The tension everywhere along the string is P, and if there are n segments of the string which support the lower block, the upward force on it is nP; this must be equal to the weight supported, if we neglect the weight of the lower block. The mechanical advantage

$$\frac{W}{P}=n. \quad . \quad . \quad . \quad . \quad (ix)$$

It is important to remember that *n means the number of segments of the string which support the lower block.*



(a) (b)

Fig. 64.

80. If the weight of the lower block cannot be neglected, let it be w. For equilibrium the total upward force must be equal to the downward force, *i.e.*,

$$nP=W+w,$$

or

$$\frac{W}{P}=n-\frac{w}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (x)$$

i.e., the Mechanical Advantage is less than n. If friction be taken into account the M.A. will be still smaller. The efficiency of an actual block and tackle is about 60%.

81. Differential Pulley.—It consists of two blocks—the upper block having two sheaves of different diameters cast together so as to form one piece and the lower block containing one sheave only [Fig. 65 (*a*)]. The upper block is fixed and the lower is movable. In place of a string an endless chain is used, the links of which fit into the projections on the rims of the sheaves. This arrangement prevents the slipping of the chain which passes over the pulley A, and then goes round the movable pulley C and fixed pulley B as shown in Fig. 65 (*b*). In raising a load the loose chain passing over the pulley A is pulled down, and when this pulley makes one complete revolution, a length of the chain equal to the circumference of A (*i.e.*, $2\pi R$, where R is the radius of A) is drawn over by the effort from chain 1 and at the same time a length equal to the circumference of the smaller pulley B (*i.e.*, $2\pi r$, where r is the radius of B) is let out on chain 2. Hence the chain passing round C shortens by a length $2\pi(R-r)$. But the load is lifted through half this length because the chain shortens equally on two sides of C.

The input $=2\pi R \times P$

and output $= W \dfrac{2\pi(R-r)}{2}$

$= W\,\pi(R-r)$

From the Principle of Work, *i.e.*, input $=$ output, we get

$$2\pi R \times P = W\,\pi(R-r)$$

$$\therefore \frac{W}{P} = \frac{2\pi R}{\pi(R-r)} = \frac{2R}{R-r} \quad \ldots \quad (xi)$$

In making this calculation we have neglected the effect of friction and the weight of the movable pulley. If they are taken into account, the mechanical advantage will be less than $2R/(R-r)$.

Since the mechanical advantage depends on the difference between the two radii it is clear that by making it small we can make the mechanical advantage very large. For instance, if the bigger pulley has a radius of 6 inches and the smaller of 5 inches, the mechanical advantage will be 12. In order to have the same mechanical advantage with the pulley block we shall have to use six pulleys in the lower block, which is not very convenient.

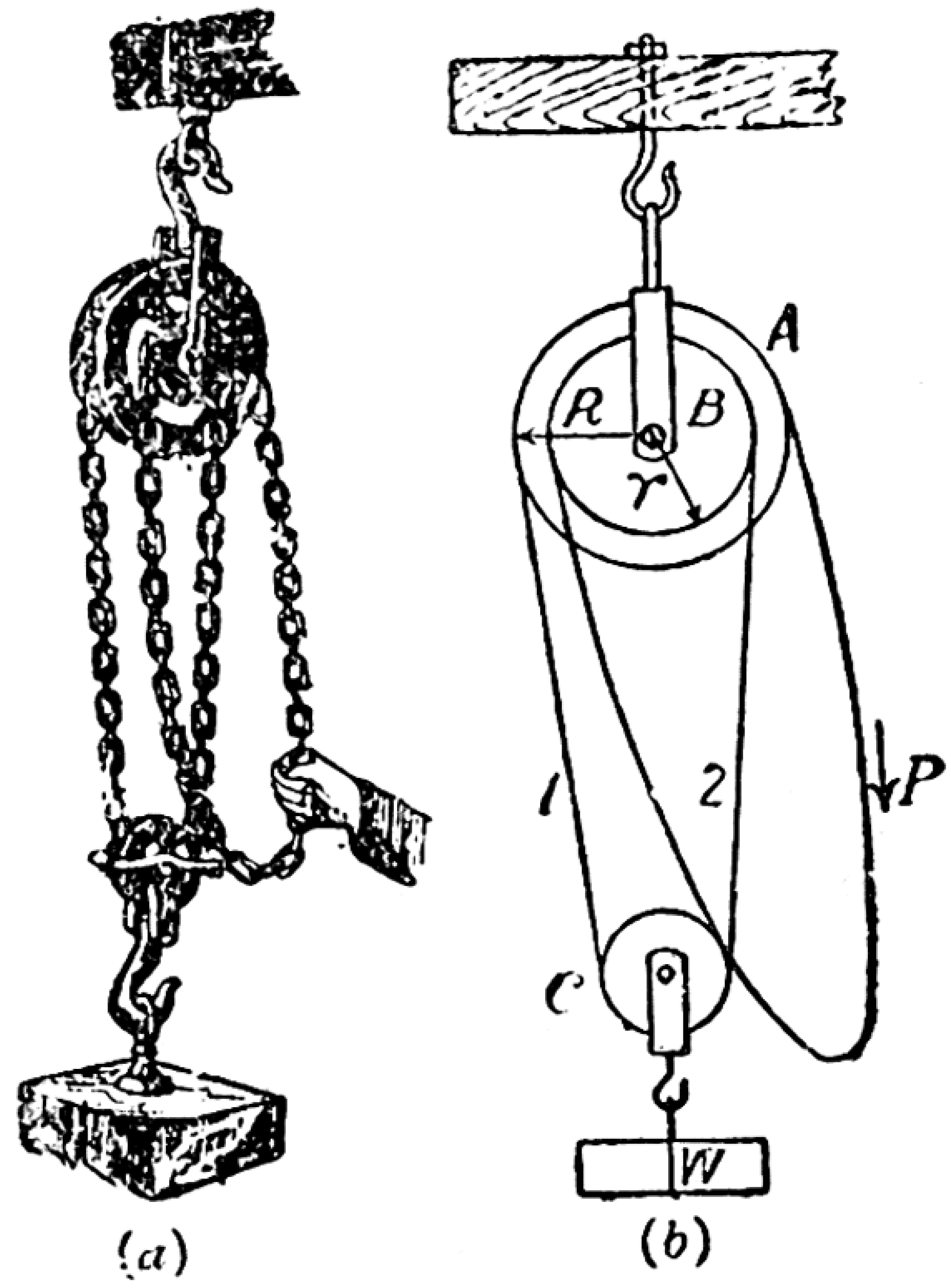


Fig. 65.

4. Inclined Plane.

82. Sometimes a weight, instead of being lifted vertically, is raised upward by means of a sloping surface. Who has not seen heavy barrels or packages being pushed up inclined planks in order to load motor trucks ? Such a slanting surface is called an inclined plane. If θ be the angle of inclination of the inclined plane with the horizontal, we know that $mg \sin\theta$ or $W \sin\theta$ is the component of the weight acting downwards along the plane. To keep the body in equilibrium we need a force $P = W \sin\theta$ acting in the upward direction.

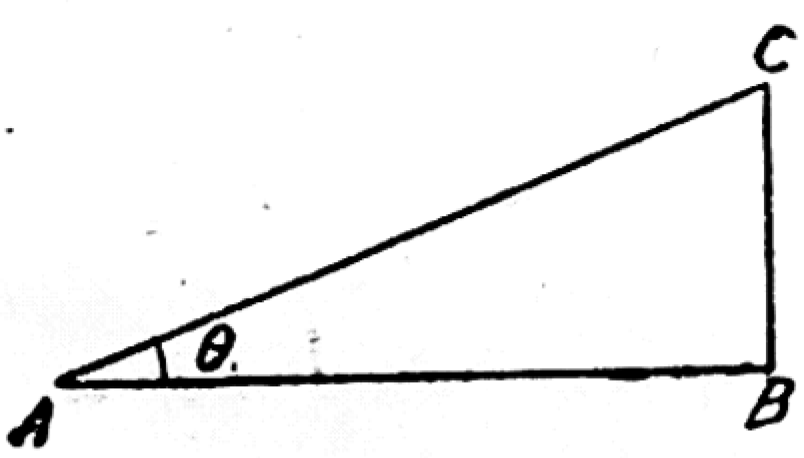


Fig. 66.

Thus the mechanical advantage is

$$\frac{W}{P} = \frac{1}{\sin\theta} \quad \ldots\ldots\ldots \quad (xii)$$

Since $\sin\theta$ is always less than one (unless θ is 90°), the mechanical advantage is greater than one.

It should be noted that the force acts along the whole length AC of the plane (Fig. 66) whereas the weight rises through its height BC only, which shows that we gain in power but lose in distance.

The input $= P \times AC$ and the output $= W \times BC$.

Applying the Principle of Work we get

$$P \times AC = W \times BC$$

or
$$\frac{W}{P}=\frac{AC}{BC}=\frac{1}{\sin\ \theta}.$$

5. Wedge.

83. It is simply a double inclined plane made of steel or hard wood. It is used by wood-cutters to split wood. When a carpenter uses an axe to split wood and a farmer uses a plough to break the soil they use a wedge. Nails, pins, needles, all act as wedges when they are driven through a resisting object. Cutting tools like a knife or a chisel are also examples of the wedge.

In order to find the mechanical advantage we shall suppose that the angle of the wedge is θ, and that it is driven into a piece of wood with a force P. Let the wedge exert on the wood a force W perpendicular to each face [AC and BC]. (The wedge is symmetrical about the vertical line DC.) The wood naturally will exert an equal and opposite force W on each face of the wedge. The vertical components of this force will balance P, *i.e.*

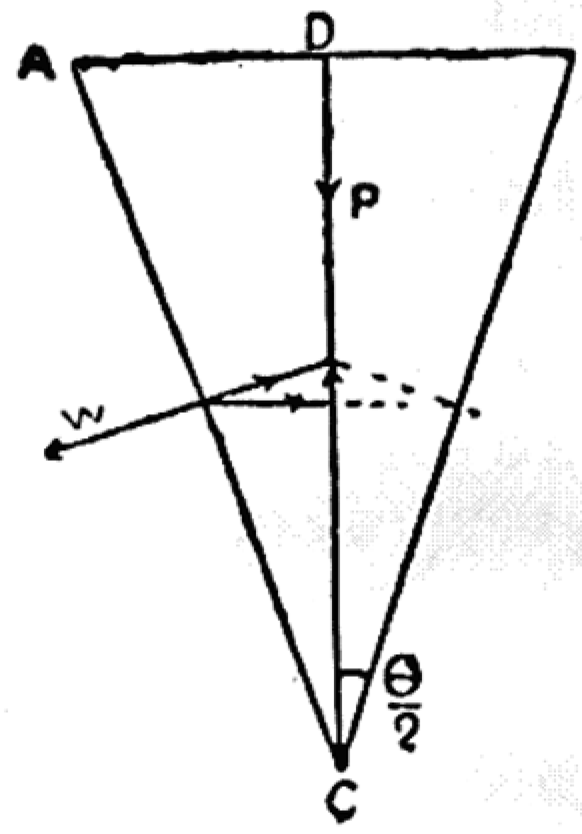


Fig. 67.

$$2W\ \sin\frac{\theta}{2}=P,$$

$$\therefore \text{ Mechanical Advantage } = \frac{W}{P}=\frac{1}{2\ \sin\frac{\theta}{2}} \quad \ldots\ldots \quad (xiii)$$

The above relation shows that the smaller the angle of the wedge *i.e.*, the smaller the thickness in proportion to its length, the greater the mechanical advantage and therefore the easier it is to drive the wedge against a resistance.

We can find the approximate value of the mechanical advantage of a wedge in a very much simpler manner. Suppose a wedge is 10 inches long and 2 inches thick at the top. When it is driven into a piece of wood it has to move 10 inches in order to move the two parts of the piece of wood two inches apart. In other words force moves five times as far as the parts of wood move apart. And so the mechanical advantage is 5.

The mechanical advantage according to relation (*xiii*) is 5·02.

It is supposed in the above calculations that there is no friction, but as a matter of fact without friction a wedge will not work; it will recoil as soon as it is driven inwards. The actual theory of the working of the wedge is difficult to explain, for the wedge moves not by pressure or continuous force but by impact, and it is almost impossible to state accurately the ratio of the force of the blow and the resistance overcome.

6. Screw.

84. It is another form of the inclined plane; the only difference is that it is *a movable inclined plane*, and, instead of going downwards directly, its motion is circular.

How the screw is formed from an inclined plane, will be easily understood from Fig. 68. Let $EFGB$ be a cylinder of diameter D. Cut a rightangled triangle of paper with base equal to AB in length and height$=BC$. Wrap it round the cylinder, keeping the base AB at rt. angles to the axis. Since AB is equal to the circumference the points B and A will coincide. The point C will come directly above the points A and B. The hypotenuse will mark out on the cylinder a path of the form of a spiral. This path is called the thread of the screw. The distance parallel to the axis of the cylinder between any two contiguous threads is called the *pitch*. It is evidently equal to the height of the triangle.

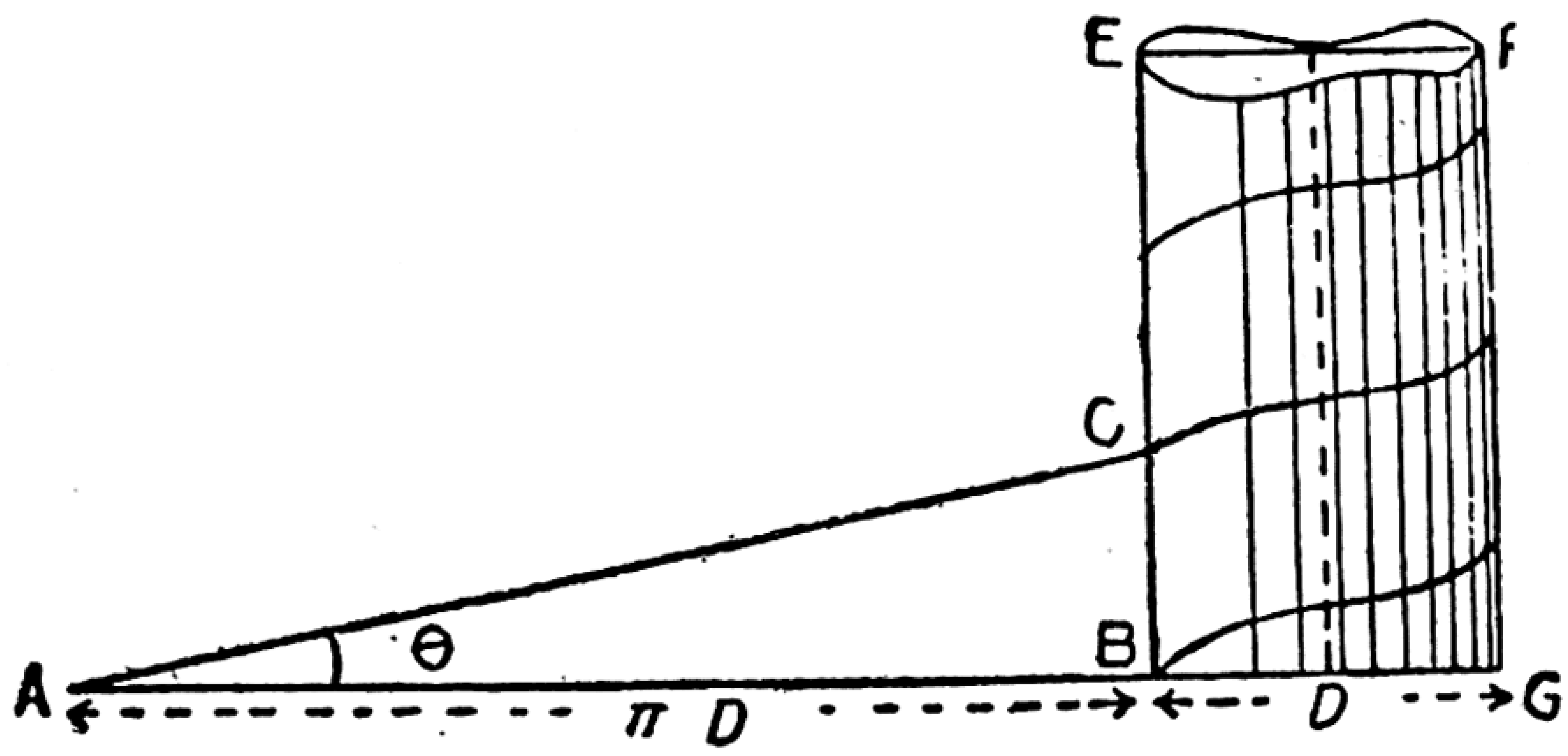


Fig. 68.

The thread is not a line in actual practice, as is supposed above, but has a definite thickness, on account of the fact that the screw is generally used to transmit a very great force.

If the thread is projected over the surface of the cylinder, the cylinder is called a *screw* or *bolt*; but if the thread runs like a groove on the inner surface of a hollow cylinder, it is called a *nut*. When the nut is fixed and the bolt is moved forward, after one complete revolution the screw or bolt is advanced by a distance equal to the pitch of the screw. The letter press, vice and screw jack are examples of the screw. To find the mechanical advantage of a screw let us consider the case of a screw jack (Fig. 69), which is frequently used for raising and holding up heavy pieces of machinery. The effort is applied at the end of a lever and the weight is lifted directly by the head of the screw. Let the power (P) be applied perpendicularly to the lever arm of length l. When the screw is rotated, in one complete revolution the force P acts through a distance $2\pi l$ and the head of the screw rises upward through a distance equal to the pitch of the screw (say h). If W be the force exerted at the end, the work done by the jack is $W \times h$, whereas the work done on it is $P \times 2\pi l$.

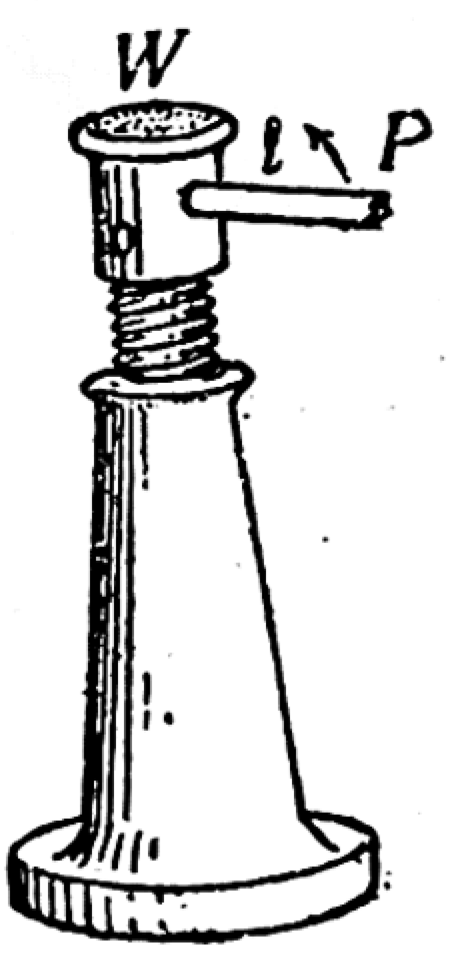


Fig. 69.
Screw Jack.

Applying the principle of work we get

$$P \times 2\pi l = W \times h,$$

or

$$\frac{W}{P} = \frac{2\pi l}{h} \qquad \ldots\ldots\ldots \qquad (xiv)$$

To form an idea as to how much mechanical advantage we can make with a jack, let us take an example.

Example.—A youth wants to raise a load of 5·5 tons with the help of a jack of $\frac{1}{4}$ inch pitch. The arm of the lever at the end of the jack is 14 inches. How much power should he apply ?

Use the relation $$\frac{W}{P}=\frac{2\pi l}{h}.$$

Substituting the values for W, l, and h we get

$$\frac{5{\cdot}5\times20\times112}{P}=\frac{2\times\frac{22\times14}{7\times12}}{\frac{1}{4}\times\frac{1}{12}}$$
$$=2\times\tfrac{22}{7}\times\tfrac{14}{12}\times4\times12$$
$$=2\times22\times2\times4.$$

Or $$P=\frac{5{\cdot}5\times20\times112}{2\times22\times2\times4}=35 \text{ lb. wt.}$$

Thus we see that *by exerting a force of 35 lb. wt. only, he can raise a load of 5·5 tons—a load which twenty people together will not be able to lift without the help of a machine.*

The mechanical advantage in the case of a real jack is hardly 30% of the calculated value ; the rest of the work done on it is spent in overcoming friction. It should be remembered that this loss due to friction is not wholly a disadvantage, for it is friction which keeps the jack from turning backwards of itself.

85. Balance.—Balance is simply a lever of the first order. We use it very often in our experiments for measurements of mass, hence we shall consider it somewhat in detail. A sensitive balance like the one which we use in our laboratories consists of a light metal beam AB, constructed like a girder so as to combine lightness with great rigidity. It is supported at the middle point C by a knife-edge pointing downwards which rests on an agate plate attached to the pillar of the balance. The pans are suspended with the help of stirrups and swing freely about the knife-edges fixed at the ends, A and B, of the beam with edges pointing upwards. A long metal pointer, rigidly fixed at right angles to the beam at its centre, swings in front of a scale attached near the foot of the pillar. The whole is enclosed in a glass case to protect it from air currents and rapid temperature changes.

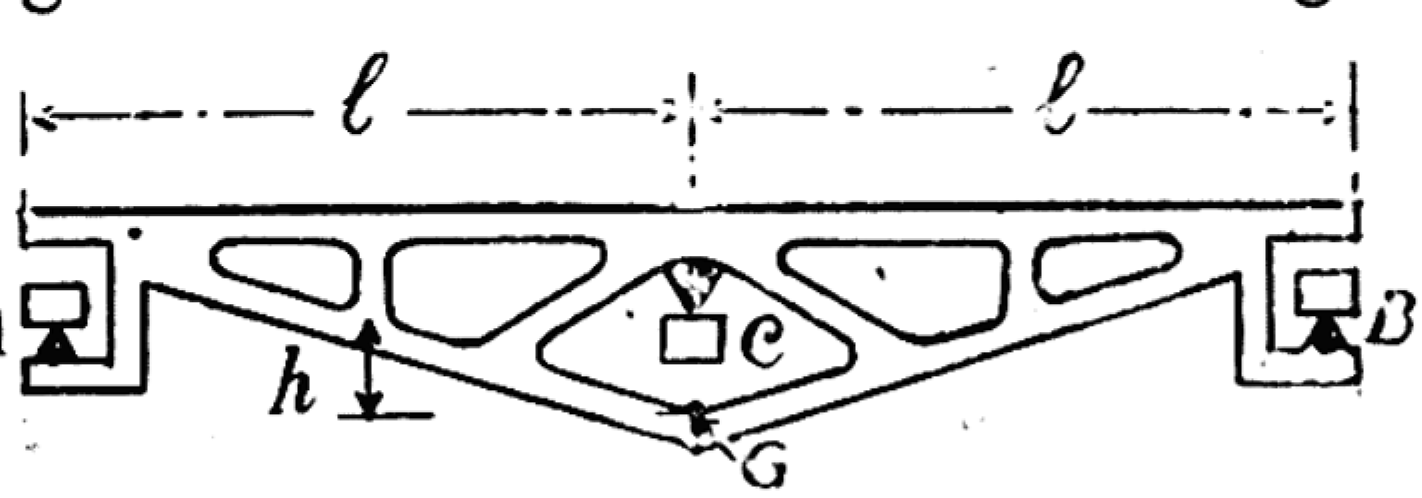


Fig. 70.

The knife edges act as frictionless pivots. Since the pans swing freely about knife edges A and B, the $C.\ G.$ of the pans and weights is *always* directly below the knife edges.

The requisites of a good balance are :—

(1) **Truth,** (2) **Sensitiveness** and (3) **Stability.**

86. Truth.—*A balance is said to be true if the beam becomes horizontal when equal masses are placed in the pans.*

Let us see what conditions must be fulfilled by a balance so that it may be true.

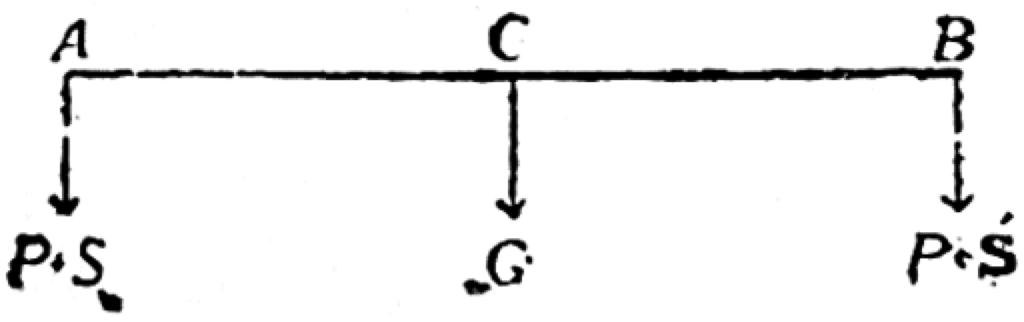


Fig. 71.

Let A B represent the line joining the knife-edges of the beam of a true balance and the arm AC be equal to a cm., the arm CB equal to b cm. in length and the pans weigh S and S' respectively. Let further two weights, P and Q be placed in the pans (Fig. 71).

By definition the beam must be horizontal when $P=Q$. Hence the moments about the point C must be equal, *i.e.*,

$$(P+S)a=(P+S')\,b.$$

In a true balance the beam will be horizontal also when the pans are empty.* This means that the following relation must also hold good :

$$Sa=S'b$$

Subtracting this from the former we get

$$Pa=Pb,$$

or
$$a=b,$$

and since $Sa=S'b$, we must have also

$$S=S'$$

i.e., the pans must have equal weights, which is our second condition.

Thus we see that in a true balance

(*i*) *The arms must be of equal length*, and
(*ii*) *The pans must have equal weights.*

87. Sensitiveness.—*A balance is said to be sensitive if a very small difference in weights causes a great deflection.*

Let us assume that the balance whose sensitiveness we wish to study is true. Denote CG, *i.e.*, the distance between the $C.G.$ of the beam and the central pivot by h. Place two weights P and Q in the pans, and let the beam be deflected through an angle θ. Since on account of deflection the centre of gravity G of the beam moves to G', the moment of the weight (W) of the beam will have to be considered along with the moments of other weights. As the beam is in equilibrium, the moments about C must be equal and opposite

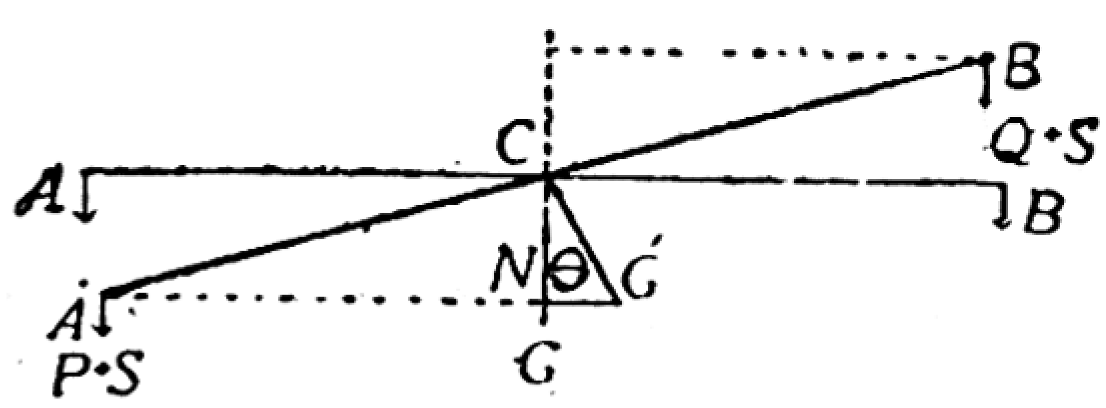


Fig. 72.

i.e. $$(P+S)a\cos\theta=(Q+S)\,a\cos\theta+Wh\sin\theta,$$

or $$a(P-Q)\cos\theta=Wh\sin\theta,$$

or $$\tan\theta=\frac{a(P-Q)}{W\times h}.$$

The relation shows that for deflection θ, or $\tan\theta$ to be large for a small difference between P and Q, a should be large, W (*i.e.*, the weight

*If both the pans are removed, even then the beam must be horizontal. In order that it may be so, the $C.G.$ of the beam AB must be vertically below C, for then the weight of the beam will have no moment about the central pivot C.

of the beam) should be small, and h, *i.e.*, the distance between the *C.G.* of the beam and the central pivot, should be small. In practice a cannot be made very large, for if it were done the balance becomes unmanageable, nor can the beam be made very light, lest it should bend by heavy weights. So the only method to increase the sensitiveness is to make the distance h, between C and G small.

88. **Stability.**—*A balance is said to be stable if it returns quickly to its position of equilibrium after being displaced.*

In order that a balance may be stable the moment tending to bring it back to the position of equilibrium should be as great as possible. When the pans are equally loaded, the only moment tending to bring the beam back is $Wh \sin \theta$. So that this expression may have a very big value for a fixed value of θ, h should be as large as possible. Note that by increasing W no advantage is secured, for although the moment tending to bring the beam back to horizontal position becomes large, the mass to be moved also becomes large. Hence the method to make the balance stable is to increase h, which reduces the sensitiveness. This shows that **stability is opposed to sensitiveness.**

89. To determine the True Weight of a Body with a False Balance.—(1) The simplest method to find the true weight of a body is that of substitution. The body to be weighed is balanced by a counterpoise, say lead-shot or sand. Then the body is removed and standard weights are put in its place in the pan till the beam is horizontal again. Since the standard weights are balancing the counterpoise of the given body, their mass must be same as that of the body.

(2) (*a*) If the arms of the balance be of unequal length, but the weights of the pans be such that the beam is horizontal when the pans are empty, proceed as below :

Let a, b be the lengths of the arms, S and S' the weights of the pans and w the true weight of the body which we want to find out. Place the body in the pan weighing S and let the weights placed in the other pan to balance it be x. Taking moments about the fulcrum C we get

$$(w+S)a=(x+S')b \quad \dots\dots\dots \quad (i)$$

But since the beam is horizontal when the pans are empty, we have

$$Sa=S'b \quad \dots\dots\dots \quad (ii)$$

Subtracting (*ii*) from (*i*) we get

$$wa=xb \quad \dots\dots\dots \quad (iii)$$

Now place the body in the other pan weighing S' and let the weights required to balance it be y. Taking moments we get

$$(y+S)a=(w+S')b \quad \dots\dots\dots \quad (iv)$$

Subtracting (*ii*) from (*iv*) we have

$$wb=ya \quad \dots\dots\dots \quad (v)$$

Multiplying the corresponding sides of the equations (*iii*) and (*v*) we get

$$w^2ab=xyab.$$

or $$w=\sqrt{xy} \quad \ldots\ldots \quad (vi)$$

(b) If the pans be unequal in weight (say the weights are S and S'), but the arms be equal in length, the following method should be used. Place the body first in one pan, say weighing S, and let the weights which balance it be x. The moments about the point C will be equal and opposite,

$$(w+S)a=(x+S')a$$

or $$w+S=x+S' \quad \ldots\ldots \quad (vii)$$

Now place it in the other pan and balance it with weights y. Take the moments as before.

$$(w+S')a=(y+S)a$$

therefore $$w+S'=y+S \quad \ldots\ldots \quad (viii)$$

Adding the corresponding sides of (vii) and (viii) and simplifying we get

$$2w=x+y,$$

$$w=\tfrac{1}{2}(x+y).$$

90. Steelyard.—It is a special form of a portable balance and is very useful for weighing bulky and heavy objects. It consists essen-

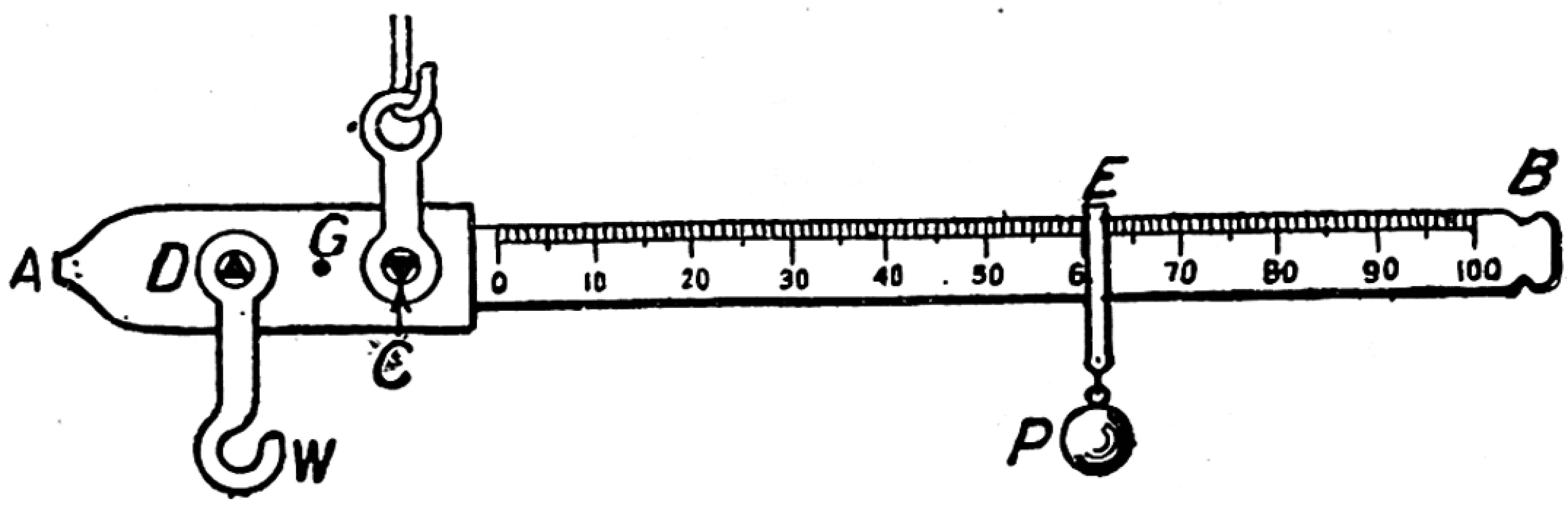


Fig. 73.

tially of a lever of the first order with a very short weight arm and a long power arm. Fig. 73 illustrates a common steelyard. The beam AB is so made that when there is no load on the hook and the sliding weight P is at zero graduation the beam rests in the horizontal position. When the body to be weighed is hung from the hook the beam tilts. The sliding weight is moved along the arm BC till the beam is once more horizontal. The arm CB has numbers engraved on it along its entire length in such a way that the position of the sliding weight where it balances the load hung from the hook gives the weight of the load. As to how it is done we shall explain in the following paragraph.

Let w be the weight of the steelyard and G the C.G. of the beam AB. When there is no load at the hook, the moment of the weight of the beam plus hook about the fulcrum balances the moment of the sliding weight P placed at 0 (zero) graduation. We can write this fact as

$$w\times CG=P\times 0C \quad \ldots\ldots \quad (ix)$$

or $$0C=\frac{w}{P}\times CG \quad \ldots\ldots \quad (ix\ a)$$

The relation (*ix a*) fixes the position of the graduation 0.

Now let a load W (not shown in the figure) be suspended from the hook and let E be the point at which P must be placed to make the beam horizontal. Taking moments as before we get

$$W \times CD + w \times CG = P \times CE \quad \ldots\ldots \quad (x)$$

Subtracting (*ix*) from (*x*) we get

$$W \times CD = P \times CE - P \times 0C$$
$$= P(CE - 0C) = P \times 0E$$

or

$$0E = \frac{W}{P} \times CD \quad \ldots\ldots \quad (xi)$$

This relation fixes the position of the point E.

It is obvious from relation (*xi*) that the greater the distance of the sliding weight from the zero graduation the heavier the load that is balanced. Suppose the movable weight is 10 lb. and CD is 2 inches and W is 60 lb. It is obvious that $0E = 12$ inches. The point E is marked 60. A point distant 1 inch from 0 on the arm CB will indicate a weight of 5 lb. and a point distant 2 inches will indicate a weight of 10 lb. etc.

In practice notches are cut along the entire length of the arm CB at a distance of every half inch and the corresponding weight at the hook which will be balanced is engraved on the arm.

It is very much easier to weigh a sack say weighing 85 lb. by running a small weight along a beam than to handle such weights as 60, 20 and 5 lb. By a suitable choice of the distance of the hook from the fulcrum and of the movable weight P, the steelyard may be graduated in whichever units we like.

Example.—A common steelyard weighs 10 lb., the distance of the hook from the fulcrum is 1·5 inches and the *C.G.* of the steelyard is 1 inch from the fulcrum and is on the same side as the hook. The movable weight is 10 lb. Where should the graduation corresponding to 1·5 cwt. be engraved ?

Here $CD = 1{\cdot}5$ inches, $CG = 1$ inch, $w = 10$ lb, $W = 168$ lb. and $P = 10$ lb.

Taking moments about the fulcrum we get

$$168 \times 1{\cdot}5 + 10 \times 1 = 10 \times EC$$

or

$$EC = \frac{262}{10} = 26{\cdot}2 \text{ inches.}$$

Hence the graduation corresponding to 1·5 cwt. should be at a distance of 26·2 inches from the fulcrum.

The weighing machines used for weighing lorries full of coal etc. are built on a similar principle. Levers are so arranged that by sliding a small weight along a bar huge weights can be balanced. The best machines are so sensitive that they will indicate a difference in weight if a lump of coal weighing no more than 5 lb. is taken away from a full lorry.

91. Bicycle.—We have explained above the working of simple machines. The real machines are complex in their construction.

They are mainly designed to increase speed or to multiply force. How the principles of Physics are made use of in such machines we shall explain by considering the example of a bicycle, the main object of which is to convert a slow speed into a fast one.

The essential parts of a modern bicycle are two road wheels fitted with pneumatic tyres and tubes, mounted in a straight line in a frame made of steel tubes, two pedals fixed to cranks, two toothed-wheels with a chain running over them, the brakes, a handle bar for steering and a saddle. There are some auxiliary parts like mudguards, bell, carrier and bicycle stand. The toothed wheel fixed to the hub of the rear wheel is called the "free-wheel." This is an arrangement so designed as to move the rear wheel only when the pedals are pushed forward. If the pedals are turned in the reverse direction the outer part of the free-wheel turns *freely* without turning the rear wheel. It, further, enables the pedals to remain at rest when the rider so desires, while the rear wheel and hence the bicycle continues to run. The toothed wheel in front is called the crank-wheel. The power is applied at the pedals and the motion produced is transmitted by means of a chain to the rear-wheel.

The frame consists of three triangles with a common base. Two of them are in the rear and the third is in the front. The latter is, however, truncated, to prevent shocks due to the roughness of the road

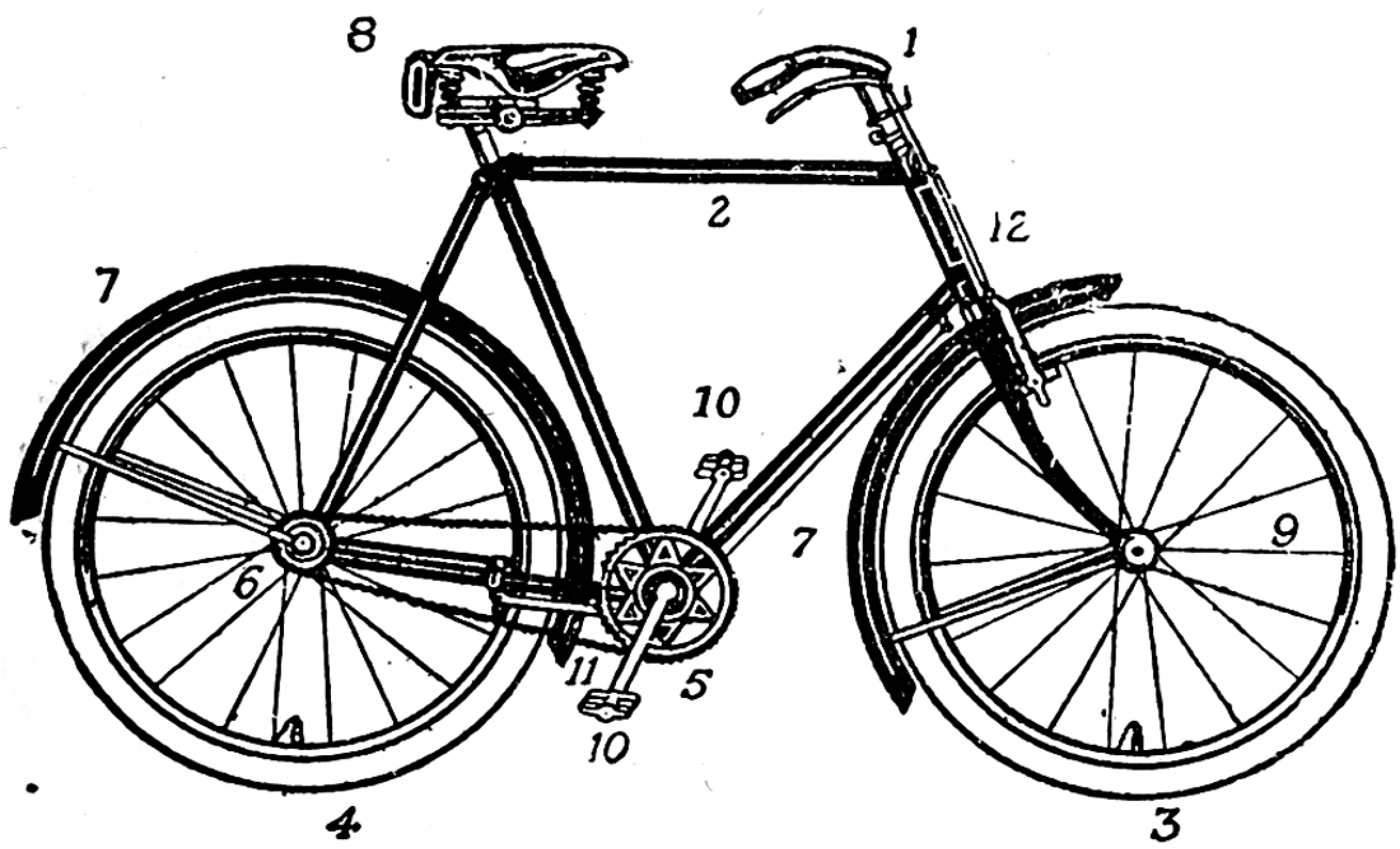


Fig. 74. Bicyle.

(1) Handle bar,
(2) frame,
(3) front wheel,
(4) rear wheel,
(5) crank wheel,
(6) free wheel,
(7) mud guards,
(8) saddle,
(9) spokes,
(10) pedals,
(11) chain,
(12) brakes.

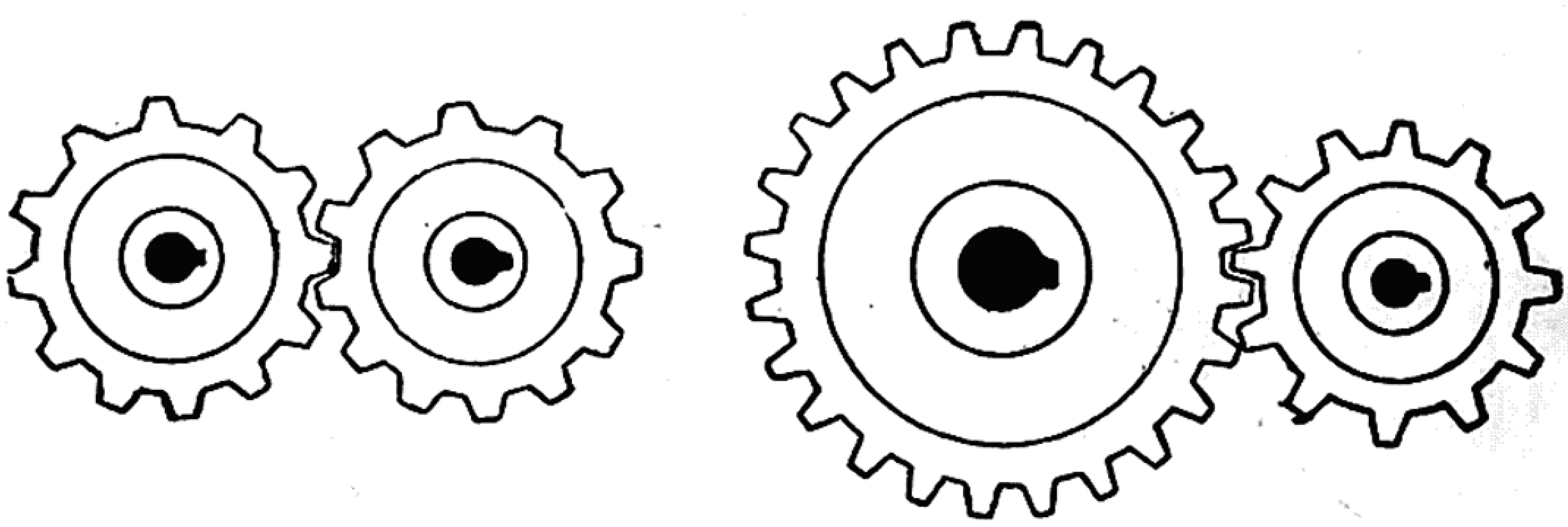
from being transmitted to the rider. Let us see why the frame is made of triangles. A triangle is a figure which cannot be distorted without altering the length or the form of its sides and since we do not want the length or form of a side of a bicycle frame to change on account of impacts we prefer the triangular frame. You might ask "Why do we use tubes in the frame-work and not solid rods ?" Some people think, it is because a tube is stronger than a solid rod of the same size. This is not so. A solid rod of a given diameter is certainly stronger than a

tube of the same size. But it is very much heavier. Since with a given weight of iron we can make a tube of very much bigger diameter than a solid rod of the same length it is herein (*i.e.*, the increase in diameter) that the secret of the strength of the tube lies. If rods of the same diameter as the tube were used the frame would be very heavy. We use tubes because thereby the frame remains light and is yet strong. It may be added that the front and back forks are also triangles of which the wheel-axles form the third side. Wheel rims become laterally stiff on account of a series of triangles of which the hub is the base and spokes the sides.

We have already said that to reduce shocks the front triangle is truncated. In addition to it the front fork is curved so that it springs slightly when the bicycle passes over rough roads and thereby reduces the severity of the shocks. The pneumatic tyres also help in reducing shocks. It may be pointed out that the rear wheel is the driving wheel and carries most of the weight of the rider. The front wheel bears only a part of the weight, its main object is to help the rider in steering bicycle by means of the handle bar.

A modern bicycle as described above is a remarkable piece of engineering. It is made of several hundred separate parts. It can easily carry a load ten times as heavy as its own weight (which is about 40 pounds*), over a road, rough or smooth.

The increase in speed is brought about by the use of gear-wheels. As to how it is done we shall explain by considering the following simple cases. Suppose we have two cogged wheels in contact with each other as shown in Fig. 75. When a cog of one wheel fits into a space of



(*i*) (*ii*)

Fig. 75.
Gear-wheels.

another wheel we say that the two wheels are inter-locked. So that there may be no slipping between two interlocking wheels the teeth (or cogs) of one wheel must exactly fit into the spaces of the other wheel. This will be so only if the cogs of both wheels are of the same size. The number of cogs in the case of such wheels is proportional to their circumference. A smaller wheel has a smaller number of cogs and a bigger one a larger number. Let us suppose a wheel of twelve cogs is interlocked with another wheel of 12 cogs. When 12 cogs of the first wheel turn, 12 cogs of the second wheel also turn— no more, no less.

*In the case of bicycles for racing purposes the weight may be as small as 20 pounds.

Now let us suppose that a wheel of 12 cogs is interlocked with a wheel of 24 cogs. When the first wheel makes a complete turn, its 12 cogs go round. The second wheel also turns 12 cogs but since 12 is only half of 24, the second wheel makes only half a turn when the first wheel makes a complete turn. This means that the second wheel turns at half the rate. In other words we have slowed down the speed to half. Now suppose that the first wheel has a larger number of cogs as compared with the second. For instance let the first wheel have 24 cogs and the second wheel 12 cogs. When the first wheel makes a complete turn, its 24 cogs move round. The second wheel advances 24 cogs, but since it completes one revolution when 12 cogs have moved

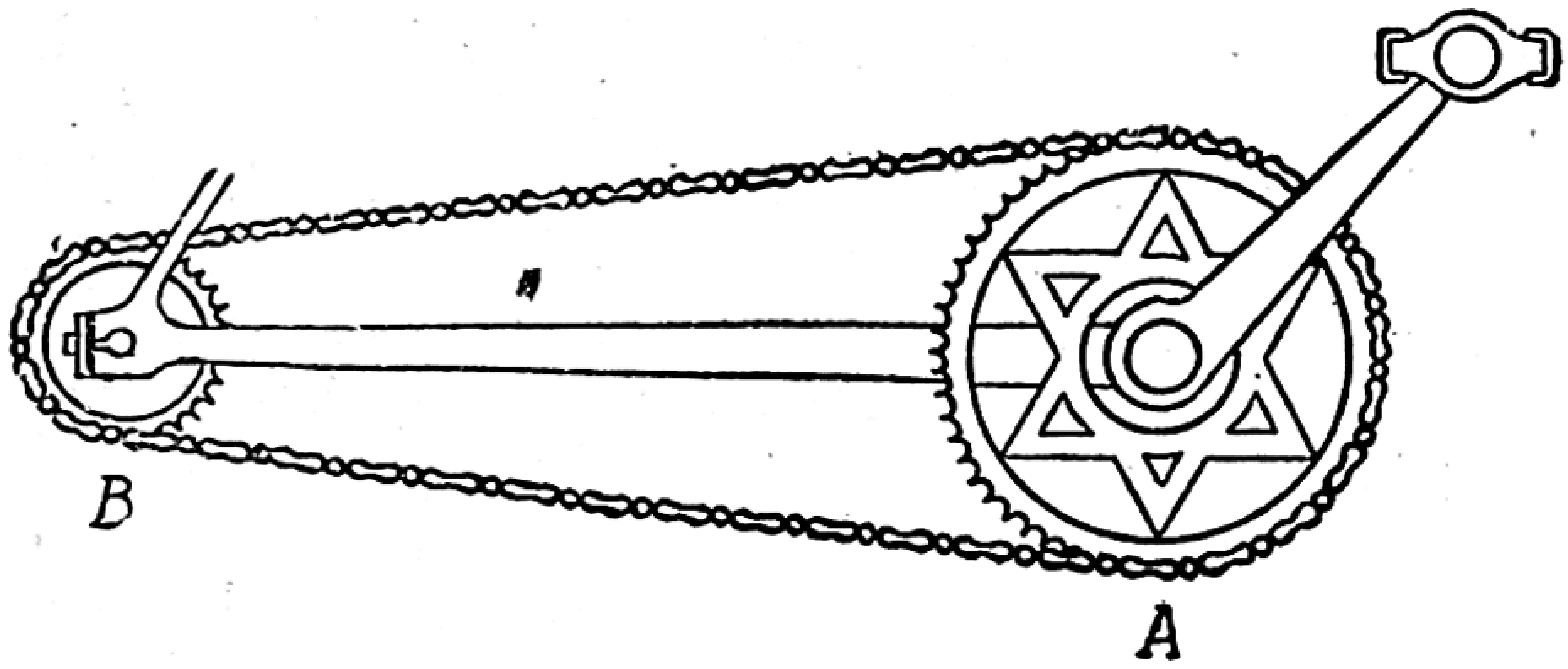


Fig. 76.

round, it must complete two turns for the movement of 24 cogs or in other words the second wheel has twice the speed of rotation as the first wheel.

By adjusting the number of teeth we can make the speed of the second wheel three or four times. Instead of having the wheels in contact they can be connected by a chain (as in a bicycle) whose links fit over the cogs. In Fig. 76 are shown the two toothed wheels of a bicycle. The wheel *B* is the free-wheel, and, as explained already, is fixed to the hub of the rear wheel, and the wheel *A* is the crank wheel.

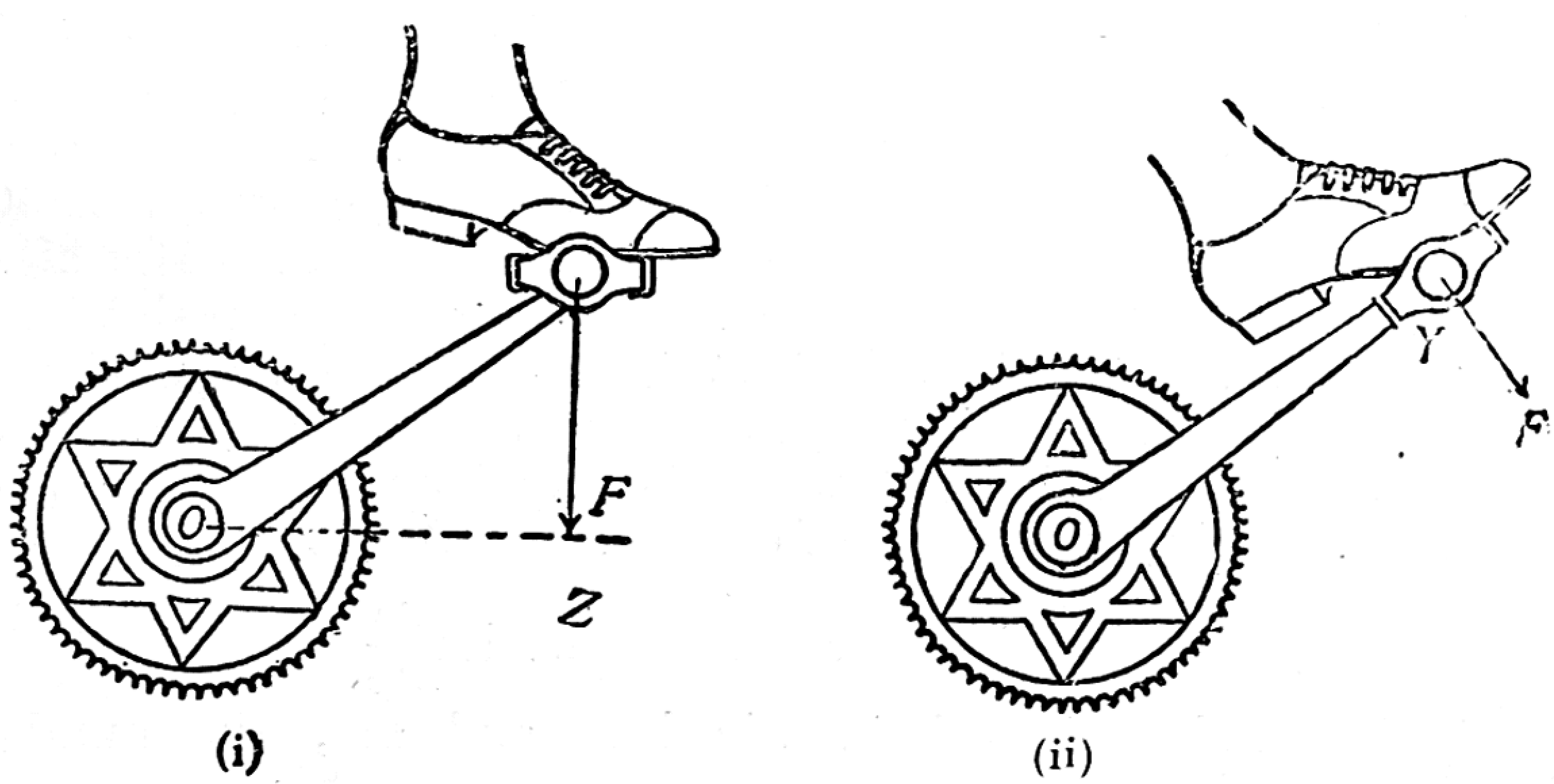


Fig. 77.

For these wheels we can write the above result as :—

$$\frac{\text{Revolutions of } B}{\text{Revolutions of } A} = \frac{\text{Number of cogs in } A}{\text{Number of cogs in } B}$$

Suppose the number of cogs on wheel A is 48 and on B 16. Then it means that for one revolution of A the rear wheel makes three revolutions. A word may be said at this stage as to the correct method of applying force. If force is applied in the correct manner, cycling becomes easier.

In Fig. 77 (*i*) the moment of the force applied by the foot $= F \times OZ$.

In case (*ii*) the moment $= F \times OY$.

It is obvious that in case (*ii*) the moment is greater than in case (*i*). This shows that the force should be applied perpendicular to the crank.

92. Work done in Cycling.—The work done in turning the pedals through one revolution $= F \times 2\pi r$, where r is the length of crank, and F the force exerted by the foot. This work is spent in overcoming the resistance of the air, the friction of the road, and the friction between the hub and the axle of each wheel. If the distance through which the bicycle moves during the time pedals make one revolution is n times the circumference of the rear wheel, the work done against friction $= F \times n \times 2\pi R$ where F is the total force of resistance and R the radius of the rear wheel. The smaller the total resistance to be overcome, the greater the distance through which the bicycle will move for a given amount of work done on it. It may be pointed out here that the energy used in pedalling is changed into heat. The air is made hotter, the road upon which the bicycle runs is made hotter, and so are made the tyres, and the bearings of the wheels.

It is easier to ride a certain distance on a bicycle on a level road than to walk the same distance. It is so because in cycling the work is done against friction only, whereas in walking a man has to raise the weight of his whole body each time he takes a step. The friction of the road, too, is small in this case because the wheels have to roll on the ground and not slide. The friction of the wheels upon their bearings is reduced by the use of ball bearings.

93. "Gear" of a Bicycle.—This term has come down to us from the days when bicycles were driven by applying force to the front wheel. Then a bicycle was known as a 60 in., 56 in. etc., according as the front wheel had a diameter of 60 in. or 56 in. respectively. A 56 in. wheel in one revolution travels approximately 176 inches. If a bicycle, whatever its make or type travels in one revolution of the pedals a distance of 176 inches, we say its gear is 56 inches. By gear we mean that diameter which the rear wheel should possess so that the bicycle may go in one revolution the distance which it actually travles while pedals make one turn. It is given by the following expression :—

$$\text{Gear} = \text{Diameter of rear wheel} \times \frac{n_A}{n_B}$$

where n_A is the number of teeth in wheel A and n_B, the number of teeth in wheel B, (*i.e.*, free wheel).

Usual diameter of the rear wheel is 24 inches.

$$\therefore \qquad \text{Its gear} = 24 \times \frac{n_A}{n_B}$$

In the example considered above $n_A = 48$, and $n_B = 16$,

$$\therefore \qquad \text{gear} = 24 \times 3 = 72.$$

Distance travelled (in inches) per one revolution of crank

$= \pi \times \text{gear}$ . . . . (*i*)

It is clear from this relation that with a low gear the cyclist travels a comparatively short distance and with a high gear a long distance per one revolution of the crank The force required to complete one revolution of the crank in the former case is much less than in the latter case. The user of a high gear, therefore, although he travels faster per one revolution of the crank than the user of a low gear, uses a greater force. He has an advantage only when the running conditions are easy, as for example, when he is going down a slope or when he is going along the wind. The contrary is the case when he is to go up a slope as in hill climbing or go against a strong wind. In such cases the user of a low gear is at an advantage. The low gear gives a higher mechanical advantage but a lower velocity-ratio and is useful when we have to start a machine or do hill climbing.

The Variable Gear.—Till some years back, the gear of a bicycle was fixed, and the cyclist had to choose a compromise between a high and a low gear. Under the most favourable conditions, he, for instance, would like to have a 90 in. gear and for hill climbing a 60 in. gear. But in one bicycle he could not have both these gears, so he had to choose a gear between these extremes say 72 in. and be obliged to tolerate it for all conditions of riding. Nowadays, however, he has the option of fitting a variable gear, so that he can use a high gear when the conditions are favourable, and a low gear when the conditions are unfavourable.

94. In using a bicycle, examples of the application of the wheel and axle (as for example in the pedals and the crank wheel, principle of lever (as in the use of handle bar), the laws of Friction (as in the use of brakes, ball-bearings etc.), will occur to the student. The flying off of mud from a bicycle wheel is an example of the centrifugal force. As said before the mud is carried by the bicycle wheel only if the adhesion between it and the tyre is great enough to pull it round ; otherwise it flies off at a tangent.

To make the tyres firm, a lot of air is compressed into the bicycle tube by means of a bicycle pump which we shall discuss in Sec. 145.

Why does a bicycle keep upright while running ? It is so because it tries to keep on in motion, in accordance with the First Law of Motion, in a straight line unless some force acts upon it.

EXERCISES

1. The arms of a false balance are in the ratio of 20 to 21. What

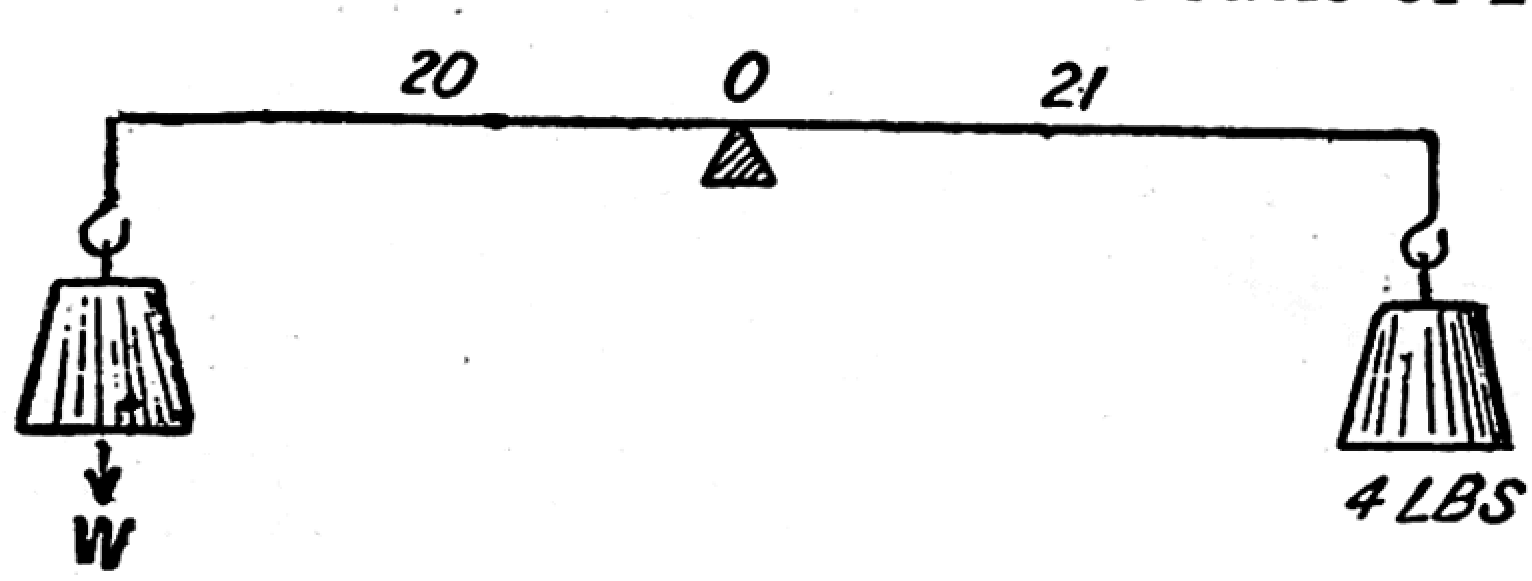


Fig. 78.

*In the case of cars we often use a low gear when starting them or going uphill

will be the loss to a tradesman who places articles to be weighed at the end of the shorter arm if he is asked for 4 lb. of goods priced at 3s. per lb ?

Let the weight of the article be W ; taking the moments about O we get

$$W \times 20 = 4 \times 21$$

therefore $$W = \frac{4 \times 21}{20} = \frac{21}{5} \text{lb.}$$

Now since he is charging for 4 lb. only but is giving away $4\frac{1}{5}$ lb., he loses the price for $\frac{1}{5}$ lb. or $7\frac{1}{5}d.$

2. There are three pulleys in the upper block and three in the lower block of a block and tackle ; neglecting the weights of the pulleys in the lower block find what pull must be exerted on the free end of the rope to raise a load of 114 lb.

In this case $$\frac{W}{P} = n.$$

and since $n = 6$, and $W = 114$ lb,

therefore $$\frac{114}{P} = 6,$$

or $$P = 19 \text{ lb.}$$

3. The rope round a movable pulley is carried over a fixed pulley and is held by a man weighing 180 lb. who rests on a board hung from the hook attached to the movable pulley. What pull must he exert on the rope to support himself ?

When the man pulls the rope downwards with a certain force, he diminishes the pressure on the board which is supporting him by the same amount. Let him pull the rope downwards with a force of P lb. to support himself.

His pressure on the board will be $(180 - P)$ lb. Therefore this is the weight to be lifted by the movabe pulley. Now the mechanical advantage of the movable pulley is 2, hence

$$\frac{\text{Weight supported}}{\text{Pull exerted}} = 2 = \frac{180 - P}{P}$$

or $$3P = 180 \text{ lb.}$$

or $$P = 60 \text{ lb.}$$

4. An object is placed in one scale-pan of an ordinary balance and is balanced by 20 lb. The same object is then placed in the other scale-pan and now it takes 21 lb. to balance it. When both scale-pans are empty the scale-pans balance. What is the matter with the balance, and what is the true weight of the object ? *Ans.* 20·493 lb.

5. A man weighing 10 stones is supported in a well by means of a windlass, the arm and axle of which are 30 inches and 9 inches in radius respectively. (1) What force must be applied to support him ? (2) to let him down with uniform velocity ? *Ans.* 42 lb. in each case.

6. With a pulley block, if a force of 6 lb. just supports a weight

of 28 lb. and a force of 8 lb. a weight of 42 lb. find the number of pulleys and the weight of the lower block. *Ans.* 7 and 14 lb.

7. A man resting on a board hanging from the axle of a wheel and axle supports himself by pulling at the rope passing round the wheel. If the weight of the man is 140 lb. and the radii of the wheel and axle are 2 ft. and 4 inches respectively, what force must he exert ?

Ans. 20 lb.

8. A screw jack has 4 threads to the inch and a lever 19 inches long. The efficiency of the jack is 50 per cent. What power must be applied to the lever to raise a load weighing 2 tons ? *Ans.* 18·75 lb. wt.

9. The crank of a bicycle has 28 teeth and the free-wheel 7. If the rear wheel has a diameter of 28 in., and the pedals turn round 60 times per minute find the speed of the bicycle. *Ans.* 20 miles/hour.

10. Explain the terms, mechanical advantage and velocity-ratio.

Prove that the efficiency of a machine is given by mechanical advantage divided by velocity ratio.

11. What is the purpose of different gears in a bicycle ?

Explain why in the case of a bicycle fitted up with a variable gear we change one from high to low gear while climbing a hill. What effects will this change have on the velocity ratio ?

12. Name the type of machine to which the following articles belong :—

(*i*)	Screw driver.	(*ii*)	Door knob.
(*iii*)	A chisel.	(*iv*)	Carving knife.
(*v*)	Carpenter's plane.	(*vi*)	Ice-cream freezer.

Ans. (*i*), (*ii*) and (*vi*) Wheel and axle and (*iii*), (*iv*) and (*v*) Wedge.

CHAPTER VII

Friction

95. In the preceding chapters we have several times used the term "friction" without explaining it. Let us now see what we mean by it. When a solid body moves over the surface of another solid body, as a cricket-ball rolls on ground, or when a body moves through a fluid, as a boat moves through water, or a bullet through air, its motion is *always* opposed by a force oppositely directed to the motion. The amount of the resisting force, of course, depends upon the nature of the pair of bodies concerned ; in some cases it is small, in other cases great, but never zero. This opposing force is called the **force of friction** or, simply, **friction**.

Up to a certain limit, *friction is a self-adjusting force* ; that is to say only as much of it is called into play as is just sufficient to prevent motion. Let, for instance, a wooden block be pulled gently over the surface of a table with the help of a string fixed to its side. A resistance will be felt which will prevent the motion of the block. If the pull is stopped the force of friction also stops.

The amount of the force of friction which can be called into play is not, however, unlimited. If we gradually increase the pull on the block we find that after a certain stage the force of friction can no more balance the pull, and the block begins to move. *This maximum force of resistance which can be called into play is called the* **limiting friction**.

Coulomb was the first physicist to investigate the motion of a body on the surface of another body. From his experiments he deduced the following laws for the *limiting friction* :—

(1) *The magnitude of the limiting friction is directly proportional to the normal reaction between the two surfaces.*

(2) *The magnitude of the limiting friction between two bodies is independent of the area and the shape of the surfaces in contact, so long as the normal reaction remains the same.*

These laws are experimental and are therefore only approximately true. They, however, represent with a fair degree of accuracy the actual conditions of mechanical problems when the surfaces are dry.

96. We can easily verify these laws in the laboratory. Take a well-planed wooden board B (Fig. 79) fitted with a ball-bearing pulley D at one end and clamp it on to a table. Take now a small wooden tray A whose lower surface is well planed and fix a hook in its side. Weigh it and place it on the board. Fasten one end of a thread to the hook and the other end to a scale-pan. Let the thread pass over the pulley. Add weights to the pan till the tray, on gently tapping just begins to slide along the board. Now place some weights in the

tray and again try the experiment. Repeat this process with three or four different weights. The normal reaction here is equal to the weight of the tray plus weights in the tray. The force of friction is equal to the weight of the scale-pan and the weights added to produce sliding. It will be seen from the results of the experiment that limiting friction is always proportional to the normal reaction, *viz.*, the ratio of the limiting friction to the normal reaction is constant. In order to make it clear we shall take the data of an actual experiment.

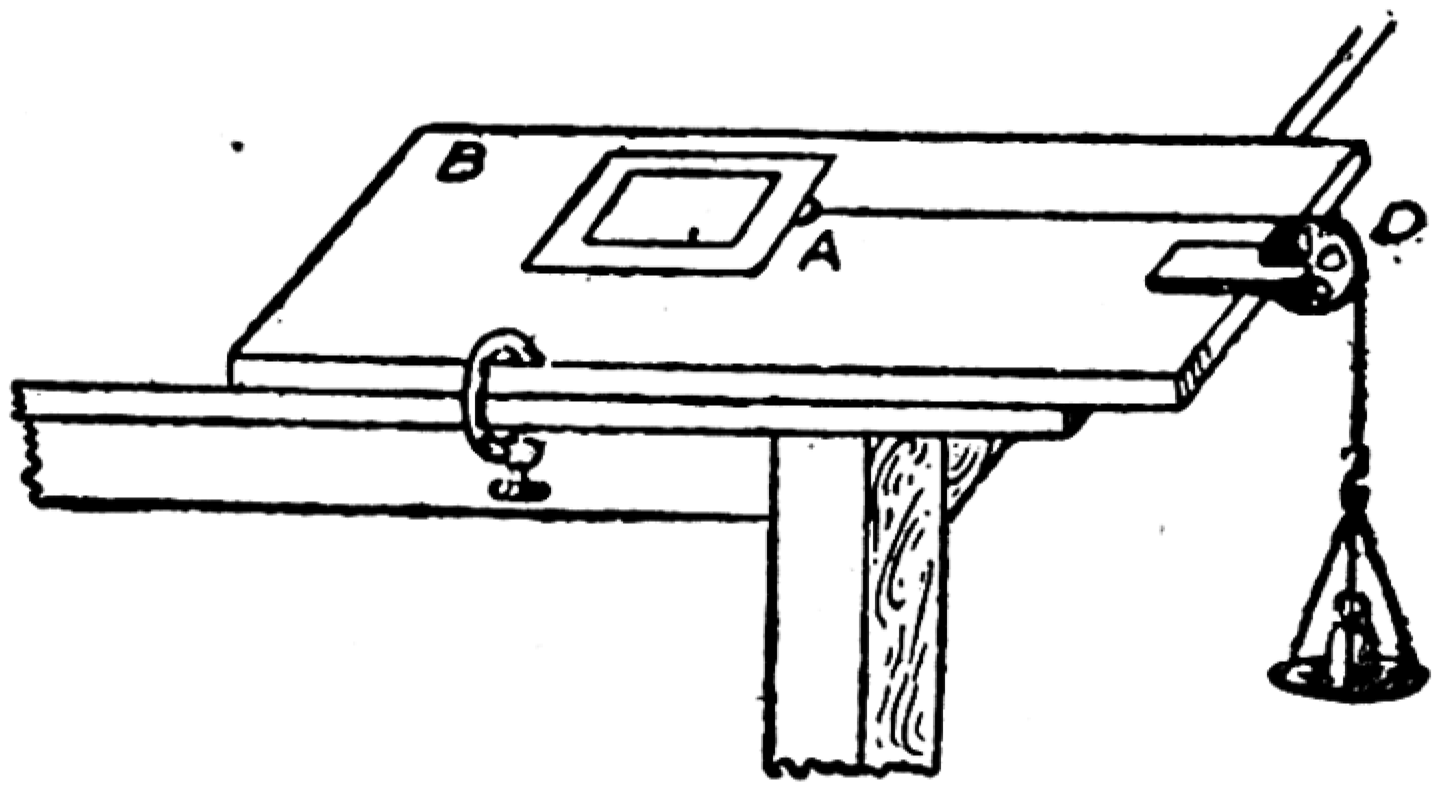


Fig. 79.

EXPERIMENT : *Limiting Friction between Wood and Wood.*
The weight of the empty tray = 450 gm.
The weight of the scale pan = 104 gm.

Additional Weights in Tray.	Weights added to the Pan just to produce Sliding.	Normal Reaction R	Limiting Friction F	Ratio $\frac{F}{R}$
0 gm.	46 gm.	450 gm.	150 gm.	0·33
400 ,,	168 ,,	850 ,,	272 ,,	0·32
800 ,,	313 ,,	1250 ,,	417 ,,	0·33

Mean ratio = 0·33 (approx.)

The above experiment shows that the ratio remains practically the same whatever the magnitude of the normal reaction.

By taking trays of different sizes and shapes, but of the same material, and repeating the experiment it can be shown that the ratio $\frac{F}{R}$ is just the same in every case.

If in place of wooden board we clamp a sheet of glass we can find the ratio $\frac{F}{R}$ for wood and glass. Similarly, any other pair of surfaces can be tried.

Experiments of the type given above show that the limiting friction depends upon the nature of the materials and not upon their size or shape.

97. The Law, $\frac{F}{R}$ = a constant, can be written as

$$\frac{F}{R} = \mu.$$

The constant (μ) is called the **co-efficient of friction** and is different for different pairs of surfaces. It must be clearly understood that *this equation holds good only when sliding is just to take place.*

The value of μ depends upon the state of polish of the surfaces and may be made very small by improving their polish and making them smooth. But it should be noted that however hard we may try *we can never get a pair of perfectly smooth bodies, i.e.* bodies for which μ is zero.

The following table gives the approximate value for the coefficient of friction for some important pairs of substances :

Wood upon wood (dry)		0·25	to 0·5
,, ,,	stone	0·6	
,, ,,	polished metal	0·5	to 0·6
Metals ,,	metals (dry)	0·15	to 0·20
,, ,,	stone	0·5	

98. We can study the laws in another way also. Let us suppose that the board instead of being clamped on to the table is hinged so that the angle between the board and the table can be gradually increased. Imagine the wooden tray to be placed on the board lying flat on the table to start with. Raise the free end of the board gradually and gently tap it from time to time, till the tray just begins to slide down. Note the angle at which the sliding *first* occurs. Repeat the experiment with trays of different sizes and weights but of the same material. It will be found that the angle at which the sliding just begins to take place is always the same. This shows *that the angle at which slipping first occurs depends upon the nature of the two substances in contact.* This angle is called the **angle of friction**.

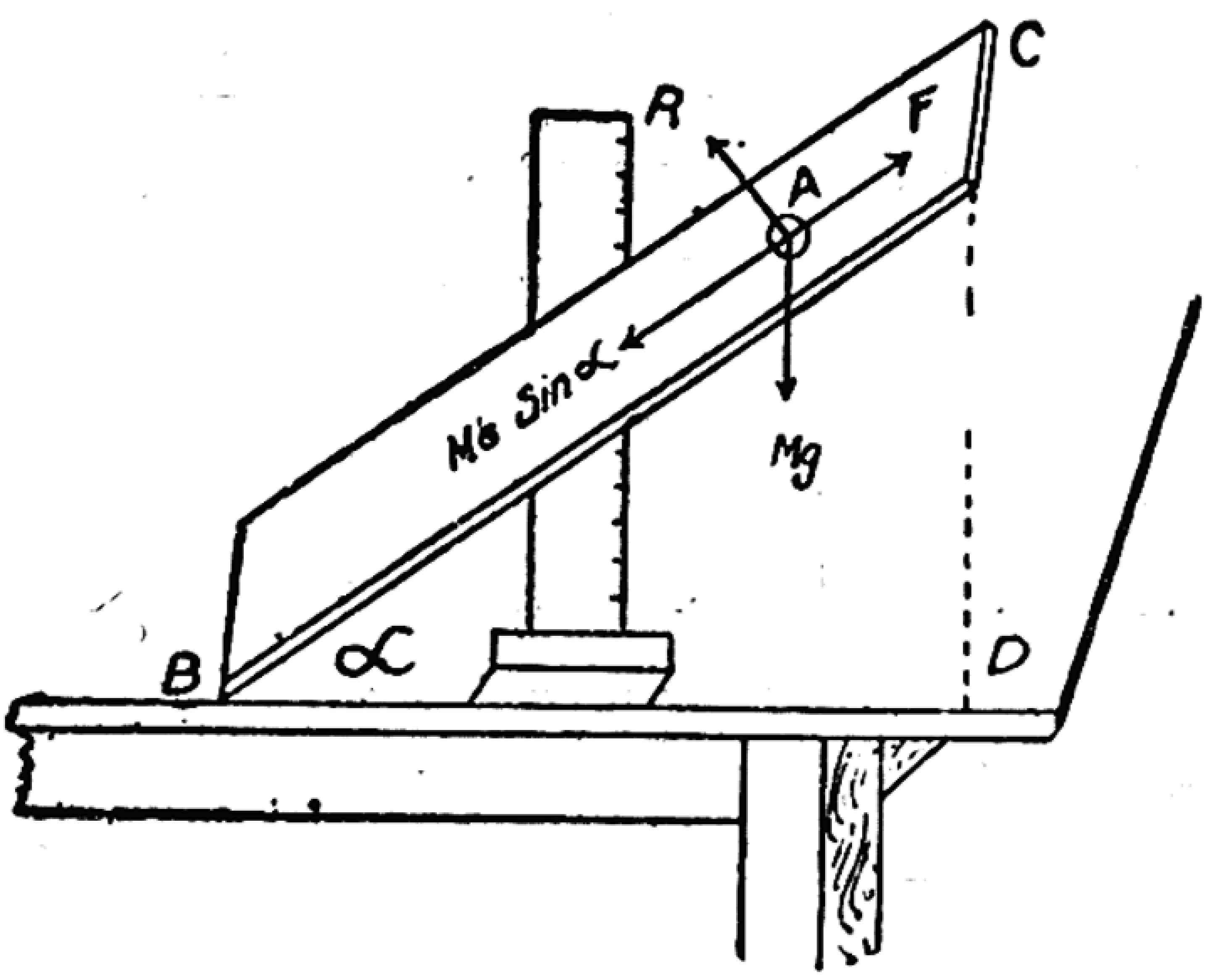


Fig. 80.

The angle of friction is different for different pairs of substances. For wood sliding on wood it varies from 12° to 25° ; for iron on wood from 10° to 30°, while for iron on iron it varies from 9° to 12°.

It is found that the coefficient of friction and the angle of friction are related to each other in a simple manner. Suppose Mg is the weight of the body A, placed on an inclined plane, and α is the angle of friction.

When the body just begins to slide down, the component due to weight ($Mg \sin \alpha$) is only slightly greater than frictional force. In other words, *just* before the slipping takes place, the forces acting upon the body are in equilibrium.

Let R be the normal reaction and F the limiting friction acting along the plane.

Resolving Mg along the plane we have

$$F = Mg \sin \alpha,$$

and resolving perpendicular to the plane we have

$$R = Mg \cos \alpha.$$

By dividing the corresponding sides we get

$$\frac{F}{R} = \tan \alpha.$$

But $$\frac{F}{R} = \mu$$

$\therefore$ $$\mu = \tan \alpha.$$

Thus we see that the *coefficient of friction is equal to the tangent of the angle of friction.*

99. Nature of Friction.—Friction is mostly due to the interlocking of the inequalities of one body into those of the other body. It is therefore diminished by polishing the parts that rub together and thereby making them smooth. If we oil these parts, the frcition is still further reduced. As said above, the force of friction tends to stop the motion and makes it more difficult to move the body. When the bodies are at rest, the inequalities of one surface fit into those of the other and the interlocking becomes greater. Owing to this fact it is much more difficult to start a body than to keep it in motion when once started.

100. Static and Dynamic Friction.—The force required to start a body to slide over the surface of another body is called the **starting friction** or the **static friction**. It is greater than the **sliding friction** or the **dynamic friction**, which is the force necessary to maintain a body in steady motion after the motion has once started.

It is experimentally found that the (dynamic) frictional force is independent of velocity, provided the latter is neither too great nor too small. It is clear that we shall have a second coefficient of friction corresponding to the sliding friction. It can be expressed mathematically as

$$\mu_d = \frac{F_d}{R}$$

where μ_d is the coefficient of dynamic friction, F_d is the frictional force called into play when one of the bodies is sliding and R is the normal reaction between the surfaces in contact.

Fig. 81 will give the student an idea as to how the force of friction varies under different circumstances. Along OY is represented the force of friction and along OX the force acting on a body. So long as no motion ensues the force of friction increases at the same rate as the pulling force; until the pulling force reaches a value corresponding to ON. The friction now has its maximum value and is equal to the limiting friction. When motion starts the force of friction is a little less than the limiting friction as shown by the part BC of the curve.

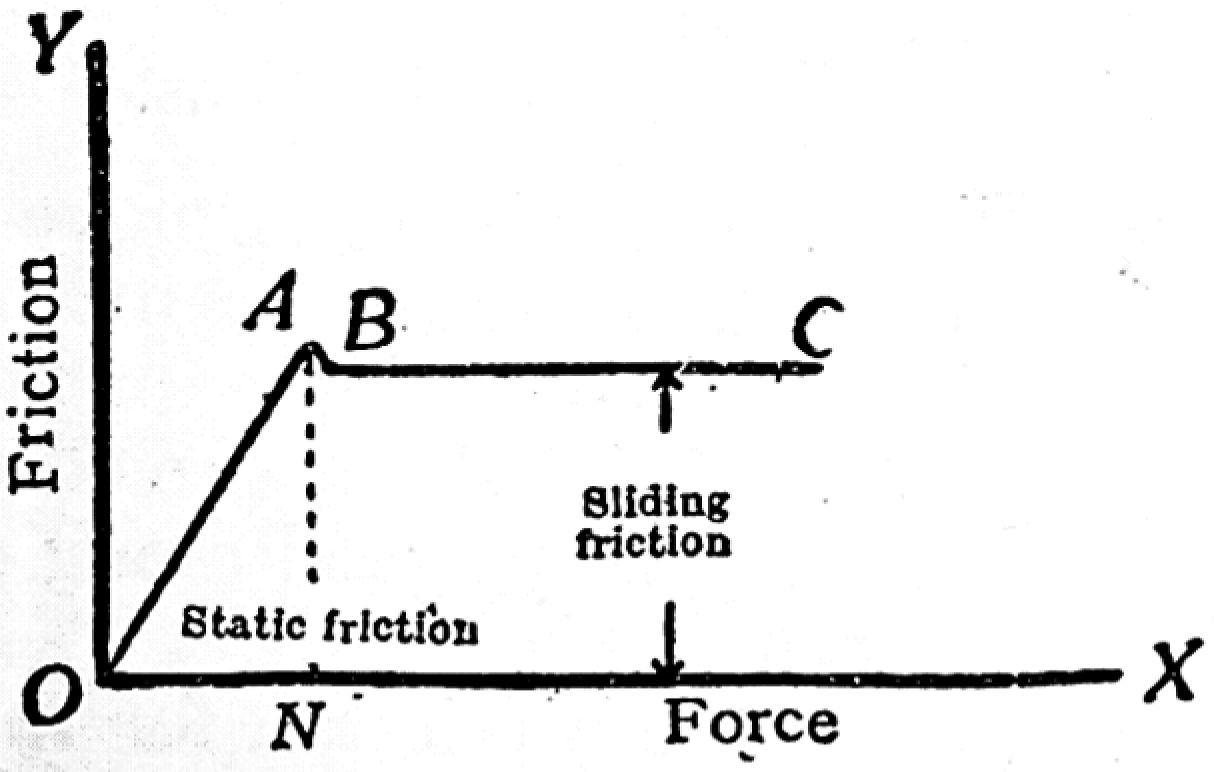


Fig. 81.

The following table gives the starting and sliding coefficients of friction for some of the substances :

Coefficient of Friction

Substances	*Static Coefficient*	*Dynamic Coefficient*
Oak on oak	0·62	0·48
Wrought iron on cast iron	0·19	0·18
Cast iron on cast iron	0·16	0·15

101. Rolling Friction.—If, instead of allowing a body to slide, we make it roll over a hard and smooth surface the friction is greatly reduced. In the case of rolling friction, both the body that rolls and the surface over which it rolls are slightly deformed. For example when a motor car runs along an asphalt road, both the tyre and the road are slightly deformed. But this deformation is extremely small, particularly when both surfaces are hard, and hence rolling friction is extremely low.

It is because the wheels of a carriage roll and not slide that the various methods of transport on land have become a practical proposition. If they were to slide the friction would be so enormous that rapid transport would be an impossibility. In the case of ordinary vehicles like tongas etc. there still remains the sliding friction at the axle of the wheel. By introducing ball bearings in the axle as in the case of a bicycle or a motor the sliding friction at the axle can be replaced by rolling friction.

It may be pointed out here that rolling friction of steel on steel is only about $\frac{1}{100}$ th part of sliding friction of steel on steel.

102. Fluid Friction.—The fluid friction, *viz.*, the resistance offered by a fluid* to the motion of a body is not independent of the velocity of the body as it is in the case of solid friction. For slow speeds it increases nearly as the velocity, but for rapidly moving bodies it increases nearly as the square of the velocity. For very high speeds it increases at a considerably greater rate. It is on account of this reason that the trains are not run very fast because the resistance of air, which is very small in comparison to the solid friction for small speeds, becomes many times more important at high speeds than the solid friction. The overcoming of this extra resistance due to air adds considerably to the cost of coal required to run the trains fast.

103. Work done against Friction.—Whenever a body is moved over the surface of another body, part of the work done is spent in overcoming friction. This part of the work is irrecoverable ; that is to say, it is lost to us for mechanical purposes. We shall consider the work done against the friction when a body is dragged along a rough horizontal plane.

Let us suppose μ is the coefficient of (dynamic) friction for the given pair of bodies and Mg is the weight of the body which is moved.

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*Ordinarily the fluid friction is largely due to the density of the fluid, the part due to viscosity being extremely small. Whatever is said above is true for the part due to density.

The normal reaction is equal to Mg. The forces of friction called into play will be equal to μMg. Since

$$Work = force \times distance$$
$$W = \mu Mg \times S,$$

where W stands for the work done against friction and S for the distance through which the body is moved.

Whenever a body is moved up an inclined plane work is done against friction in addition to the work done in lifting it. *The first part of the work is lost to us for mechanical purposes.*

Some work must be done against friction even in the case of machines, for we can never make a machine which has no frictional forces between its parts. Now you will understand why we have always to do more work on a machine than we get back from it or why *the efficiency of a machine is less than one* (see Art. 75).

103a. Methods of reducing Friction.—As said above to reduce friction the surfaces are highly polished and wherever possible rolling friction is made to take the place of sliding friction. It is to replace the rolling friction for the sliding friction at the axle of the wheel that ball or roller bearings are used.

To still further diminish the friction, lubricants like oil, grease and graphite are used. The efficiency of a lubricant is mainly due to the fact that it holds the surfaces somewhat apart so that their inequalities or projections do not interlock tightly and the rubbing takes place between layers of the lubricant instead of between the original surfaces. In order to keep the surfaces apart, the lubricant must not be squeezed out of the bearings by the weight of the machinery. It is, therefore, that in heavy machinery thick oil is used whereas in light machinery, like watches and sewing machines, thin oil is used.

When a body moves in air with small speeds it is the mechanical friction only that matters ; the resistance of air is extremely small. But at high speeds it is the other way round ; now it is the wind resistance which is the predominant factor. In the case of a motor car moving at a speed of ten miles/hour it was found that to overcome the air resistance only $\frac{1}{20}$ H.P. was needed but when it was moving at 100 miles/hour 5 H.P. were required for the same purpose. In the case of aeroplanes the wind resistance becomes still more important. Hence the necessity of "stream lining" the shape of the fast moving bodies. Have you ever noticed the shape of the new models of motor cars ? They are all *stream-lined.* In the United States even the fast trains are now stream-lined.

EXERCISES

1. Find the H. P. of an engine which is to move at the rate of 30 miles an hour, the weight of the engine and the load being 50 tons, and the resistance from friction 16 lb. wt. per ton.

We know that $W = F.\ S.$

where $F(=\mu Mg)$ is the force of friction and S the distance passed over.

Let us find the work done in one minute.

The friction is equal to 16×50 lb. wt.

The distance passed over in one minute $= 44 \times 60$ ft.

$\therefore$ Work done $=16\times50\times44\times60$ foot pounds per min.

or $$\text{H.P.}=\frac{16\times50\times44\times60}{33{,}000}=64.$$

2. Find the H.P. of an engine which is to move at the rate of 20 miles per hour up an incline rising 1 foot in 100, the weight of the engine and the load being 60 tons and the resistance from friction 12 lb. wt. per ton.

The force exerted by the engine should overcome resistance as well as drag the body up the plane.

The total force downward $=Mg\sin\alpha$ + Frictional force.

$$=\left\{\frac{(60\times2240\times1)}{100}+(12\times60)\right\}\text{ lb. wt.}$$

Since the engine moves through $\frac{88}{3}\times60$ ft. in one minute,

$$\text{the work done}=60\left\{\frac{224+120}{10}\right\}\times\frac{88}{3}\times60\text{ ft. lb. per min.}$$

$$=60\times\frac{344}{10}\times\frac{88}{3}\times60\text{ ft. lb. per min.}$$

$$=60\times344\times88\times2\text{ ft lb. per min.}$$

$$\text{The H.P. required}=\frac{60\times344\times88\times2}{33{,}000}=110{\cdot}08.$$

3. Find the work done in moving a weight of 1 cwt. along a plane 200 yards long, the coefficient of friction between the weight and the plane being $0{\cdot}43$.

$$W=\mu Mg\times S\text{ foot poundals}$$
$$=\mu\times M\times S\text{ foot. lb.}$$
$$=0{\cdot}43\times112\times200\times3\text{ ft. lb.}$$
$$=28{,}896\text{ ft. lb.}$$

4. Find the H.P. of an engine which is able in 4 minutes to generate in a train of mass 100 tons, a velocity of 30 miles per hour on a level line, the resistance due to friction being equal to 8 lb. wt. per ton. *Ans.* $166\frac{2}{3}$ H.P.

5. A train of mass 60 tons is kept moving at the rate of 30 miles an hour on its track by an engine whose H.P. is 160. Find the resistance due to friction per ton. *Ans.* 40 lb. wt. per ton.

6. A train of mass 100 tons, including the engine, is drawn up an incline of 3 in 500 at the rate of 40 miles per hour by an engine of 300 H.P. Find the resistance per ton due to friction.

Ans. $14{\cdot}69$ lb. wt. per ton.

7. A train of mass 50 tons is moving up an incline of 1 in 280 with a uniform velocity; the resistance due to friction is 16 lb. wt. per ton; if the H.P. of the engine be 100, find the velocity of the train.

Ans. $31\frac{1}{4}$ miles/hour.

8. A horse drawing a load along a level road at a speed of 3 miles per hour does 4500 ft.-lb. of work in 3 minutes; what pull does the horse exert continuously in drawing the load? *Ans.* $5{\cdot}7$ lb. wt.

9. Define the coefficient of sliding friction and the angle of friction for two rough bodies.

A mass of 30 lb. is resting on a rough horizontal plane and can be just moved by a force of 10 lb. wt. acting horizontally. Find the coefficient of sliding friction. *Ans.* $\mu=0{\cdot}33$.

10. Distinguish between starting and sliding friction. Which of the two coefficients is greater and why ?

11. "The wheel is one of the greatest inventions ever made ; without it the modern methods of travel on land would have been impossible." Discuss this statement.

12. Is a large brake on a bicycle wheel more effective than a small one ? Give reasons for your answer.

13. Why do you slip on a wet pavement but not on a dry one ?

14. Explain why a ladder is more apt to slip when you are high up on it than when you just begin to climb ?

15. Why is a man without a parachute killed when he jumps from an aeroplane while a man who falls with an open parachute remains uninjured ?

16. Explain as fully as you can the following statement : "It is cheaper in the long run to construct a tunnel through a hill than to pay for the extra coal that would be used by the engines if they had to pull all the trains up the hill and then come down to the same level."

CHAPTER VIII

Elasticity

104. We have already explained the characteristics of solids and liquids in Art. 2. We stated there that a solid has a shape of its own as well as a size which it does not change ordinarily whereas a liquid has only a volume (and hence size) of its own but no shape. Usually these characteristics are enough to enable us to classify a given substance as a solid or a liquid. But there are some substances which are hard to classify and of which it is difficult to say whether they are solids or liquids. For instace, shoemakers's wax when cold can be cast in the form of a bell which will ring, but if given sufficiently long time, flows and takes up the shape of the vessel containing it. Naturally the question arises—how are we to say in such a case whether the body is a solid or a liquid ? The answer which Physics gives is the following : "Subject the body to the action of a deforming force, and if it has even the slightest tendency to return to its original shape when the force is removed, *viz.*, if it has the *elasticity of form* (or what is called *rigidity*), it is solid, otherwise liquid." In simple words, this means that a *liquid yields to any force, howsoever small, tending to change its shape or to produce a movement of its particles.* This is sometimes expressed by saying that a liquid offers no resistance to a shearing stress.

Since it is elasticity on which our conception of the distinction between solids and liquids is mainly based, we shall try to explain what we mean by elasticity of a body.

105. Elasticity.—When a body is subjected to external forces, generally it undergoes a change either in shape or in volume, and as soon as these forces are removed it recovers more or less its original condition. Some bodies recover completely, while others have a permanent change left in them. The bodies which recover more completely are more highly elastic.

Let us see how this recovery is brought about. As soon as the external forces begin to act upon a body without causing it to move, a strain is set up in it, *viz.*, either a change of volume or a change of shape is produced, which can take place only if there is a relative displacement of the various particles with respect to one another. On account of this relative motion forces are called into play in the body which tend to bring it back to the original condition. The restoring force called into action owing to the strain is called *stress*. It is measured as force per unit area and is found by dividing the total force (called into play) by the area over which the force acts. By *strain* is meant the change in volume or shape which a body undergoes when it is subjected to external forces. If the volume changes, the strain is known as *volumetric strain* and if the length changes, it is called *longitudinal strain.* It is measured by dividing the change in volume (or length)

by the original volume (or length). If it is the shape that changes, the strain is called the *shearing strain.** When the stress called into play is very great, the body is said to be highly elastic ; for instance, when an attempt is made to change the shape or volume of an iron sphere, very great forces are called into play which resist the change. Iron consequently is highly elastic. In general the stress called into play is equal and opposite to the forces deforming the body, hence we say that the greater the force required to deform a body, the greater its elasticity. Note this statement. It is just opposite to what a layman understands by elasticity. According to him an elastic substance is one which can easily be distorted such as "rubber". In terms of Physics rubber has poor elasticity.

106. Hooke's Law.—Hooke observed that in the case of an elastic body the strain produced is proportional to the stress This is called **Hooke's Law**. This law applies to every kind of strain provided the stress is not too great. The following simple experiment will enable the student to verify this law for himself. Take a spring and fix it at one end. Attach to its other end a pan and let a pointer be attached near this end. Fix a scale behind the spring. Read the graduation opposite the pointer. Place first in the pan a weight of 10 grams, then of 20 grams, 40 grams, etc., and note the successive graduations opposite the pointer.

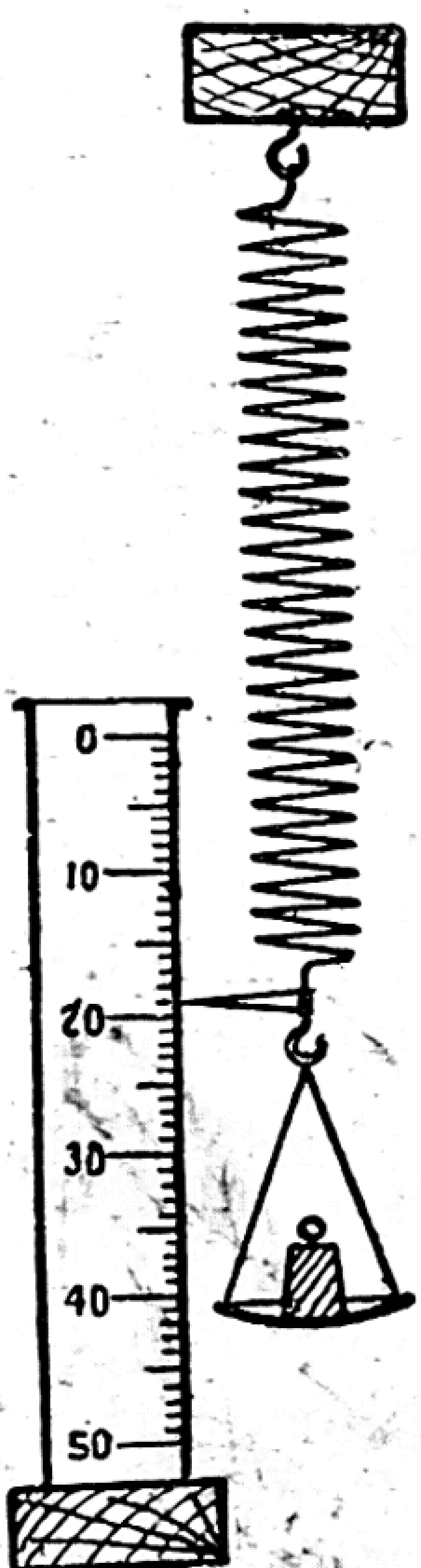


Fig. 82.

Now plot a graph of extension against the load (*i.e.*, weight of pan and weights added). It will be seen to be a straight line, showing that the change in length produced is proportional to the load producing it. Since change in length produced is proportional to strain and load is proportional to stress we can express the above result in the following form :

$$\frac{Stress}{Strain}=constant.$$

This relation is a mathematical expression for Hooke's Law. The constant is a measure of the elasticity of a substance, and is called the *modulus of elasticity.* When the strain is volumetric the constant is called *bulk* modulus. If the strain is shear strain the constant is called *shear modulus* or **rigidity**. If the strain is longitudinal the constant is called Young's Modulus.

Since the strain is a ratio of one length to another or of one volume to another it is a pure number and hence the modulus or coefficient of elasticity is expressed in the same units as those used in stating the stress.

$$Elasticity=\frac{Stress}{Strain}.$$

It should be noted that solids resist change of volume as well as change of shape, whereas liquids resist change of volume only but offer absolutely no resistance to change of shape. Gases resist even the

*It is measured in terms of the angle through a which vertical line in the body is rotated by the shearing stress.

change of volume to a much smaller extent than liquids or solids. From what has been said above it follows that solids possess the elasticity of form as well as of volume whereas liquids and gases (fluids) possess volume elasticity only.

107. Young's Modulus.—Let us consider the ratio, $\dfrac{stress}{strain}$, for solids when they are stretched or compressed. The easiest method of doing this is to consider the increase in length produced in a wire when stretched with a known force. Suppose a wire L cm. long of cross section a sq. cm. is fastened at the upper end, and carries a pan at the lower end. Place a weight in the pan and let the combined weight (*i.e.*, of pan and weight added) be m grams. Due to it a force equal to mg dynes acts on the lower end of the wire. Call this force F. Let the change in length produced be l cm.

$$\text{The strain} = \frac{\text{change in length}}{\text{original length}} = \frac{l}{L}, \text{ and the stress} = \frac{F}{a}.$$

$$\text{Therefore elasticity } E = \frac{Stress}{Strain} = \frac{F/a}{l/L} = \frac{FL}{al}.$$

The ratio $\dfrac{FL}{al}$ is the stretch modulus or *Young's Modulus of* elasticity of the material of the wire. It is generally denoted by Y.

A simple type of apparatus for determining Young's modulus is shown in Fig. 83. It consists of two vertical wires of the same material with their upper ends clamped close together on the same support M. One of them, say A, is kept taut throughout the experiment by suspending from its lower end a constant load which need not be known. It carries a small plate on which is engraved a scale S in half millimetres. The other wire B is the wire under test and carries a vernier V, which slides over the scale S, and a hanger as shown in Fig. 83. Since the load P of the wire A remains constant, its length does not change and hence the position of the scale S remains fixed during the experiment. Load and unload the wire B two or three times to remove bends or kinks in it and then put a weight of two kilograms on the hanger. Measure the length of the wire from the point of suspension to the zero of the vernier and note the reading on the scale and the vernier. Increase the load W by two kilograms at a time and read the vernier after each addition in load. Find the mean increase in length produced by a load of two kilograms. Measure the diameter of the wire with help of a screw gauge and calculate its cross-section. Using the relation $Y = \dfrac{FL}{al}$, determine Young's modulus for the substance of the wire.

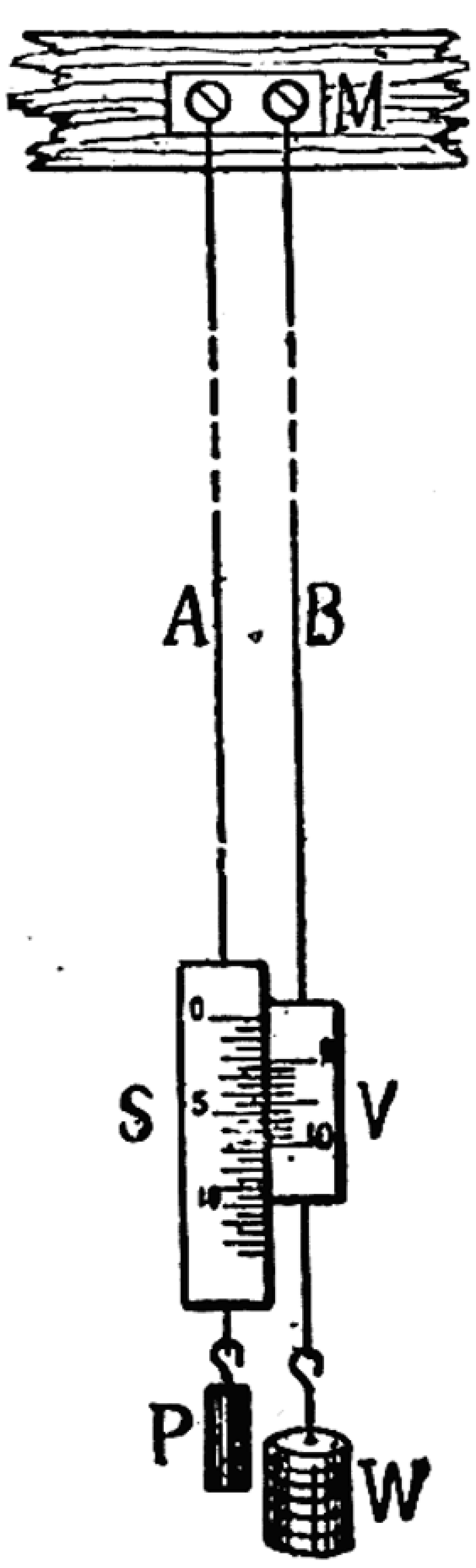


Fig. 83.

You might ask the use of the wire A. It enables us to automatically eliminate errors due to the change in temperature during the experiment and the yielding of the support M. For if the temperature of the room changes, the length of both wires is affected to the same

extent and if the support M yields when the load is increased, both scales V and S are lowered by an equal amount.

108 Volume Elasticity or Bulk Modulus.—Let us suppose a cube of steel of 2 inches side is lowered into sea to a depth of 3,000 ft. where the pressure is 1300 lb. per sq. inch. Since this pressure acts from all sides on the cube, it will decrease in size by $\frac{1}{2700}$ cubic inch.

$$\text{The strain produced} = \frac{1}{2700} \times \frac{1}{8}$$

The stress producing this strain $= 1300$

$$\therefore \text{ Elasticity of steel or Bulk modulus} = \frac{1300}{\frac{1}{2700} \times \frac{1}{8}}$$

$$= 1300 \times 2700 \times 8$$

$$= 28 \times 10^6 \text{ lb./in.}^2$$

The Bulk Modulus is usually denoted by K.

Reciprocal of Bulk modulus *i.e.*, $\frac{1}{K}$ is called the *Compressibility* of a substance.

Let us now consider V cubic centimetres of a fluid, (whether liquid or gaseous) and subject them to a pressure of P dynes. On increasing the pressure to $P+dp$ where dp represents the increase in pressure, let the volume be reduced to $V-dv$ where dv denotes the change in volume.

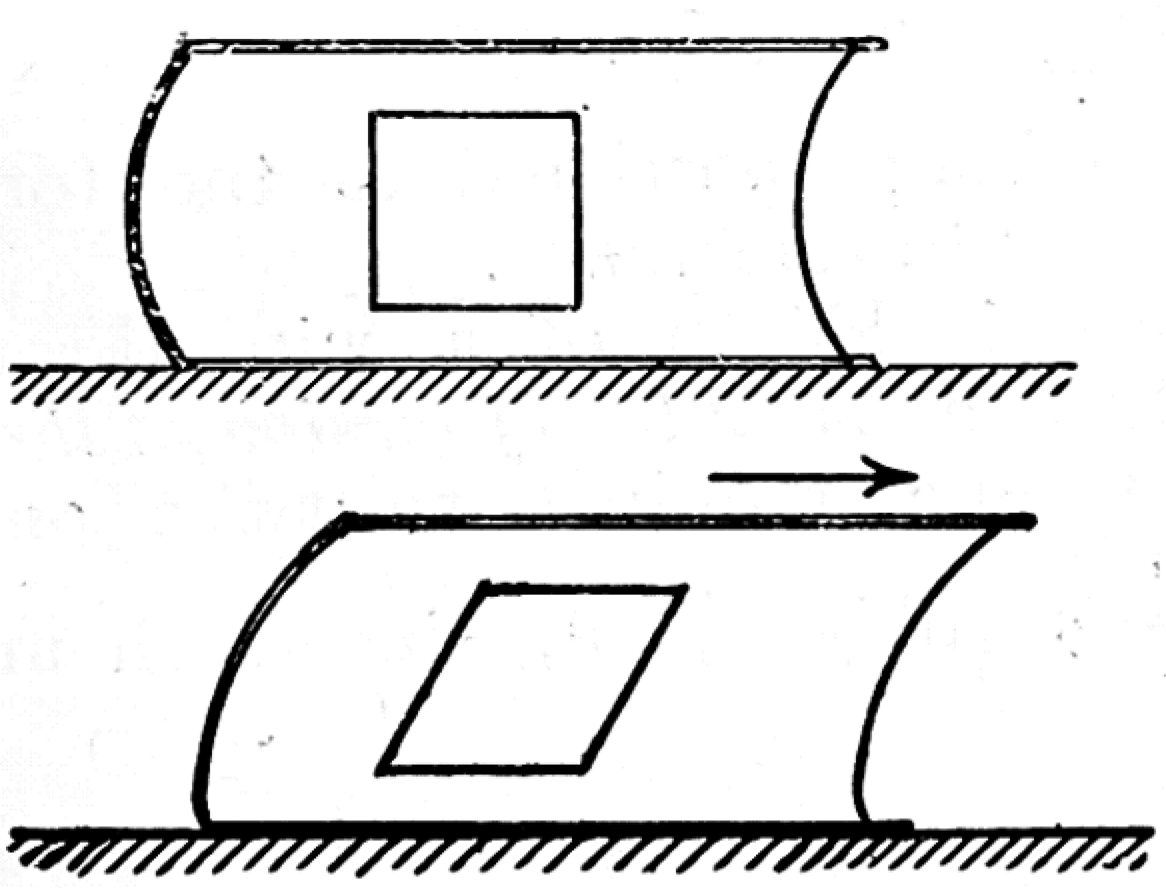
Since Strain $= \frac{\text{Change in volume}}{\text{Original volume}}$, it is equal to $\frac{dv}{V}$. The stress which is producing this change in volume is the change in pressure, *i.e.*, dp.

Therefore Bulk modulus or

$$K = \frac{dp}{dv/V} = \frac{V\,dp}{dv}.$$

$$\text{The compressibility} = \frac{dv}{V\,dp}$$

Shear Modulus or Rigidity.—Now let us consider the elasticity of form. Suppose a cube is deformed in such a way that its shape



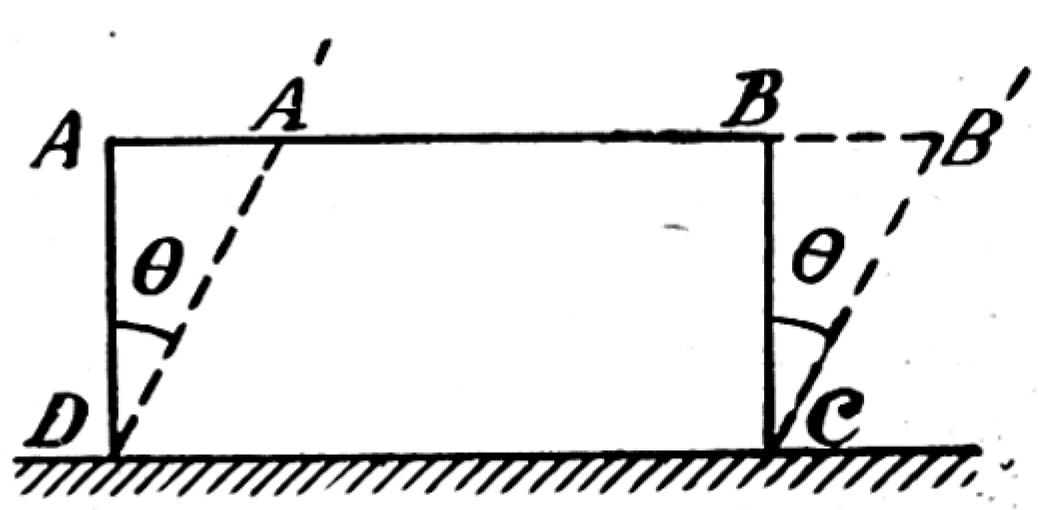


Fig. 84 (a).

changes but the volume remains unaltered. Imagine a thick book, like a dictionary, placed on a table with its upper cover being forced forwards parallel to the table and the lower end held fast. The leaves will all move over their neighbours from top to bottom. The change in form becomes evident from the change of the square marked in pencil on the end of the book into rhombus. A deformation of this type is called a shear and is measured by tan θ, θ being the angle of shear (Fig. 84).

Under similar conditions, solid bodies behave in the same way but to a very much lesser extent, and hence it is sufficiently accurate in their case to express shear in terms of θ instead of tangent θ. It is usual to write

Shearing strain $= \theta = \dfrac{AA'}{AD} = \dfrac{\Delta x}{L_0}$, where Δx is lateral displacement and L_0 is the original length.

If the force applied be F and area be A, shear $= F/A$.

$$\therefore \quad \text{Shear modulus or rigidity} = \frac{F/A}{\theta}$$

It should be clearly understood that to bring about a shear we require a couple and not a single force.

The idea of shear is involved in the twisting of a wire, whose one end is clamped and the other end which is free is turned by applying a couple. Its each successive circular section moves over the other and the angle of twist θ (usually called torsion) produced by the applied torque* depends upon the length L, the radius r of the wire and shear modulus or modulus of rigidity of the material. It can be proved that

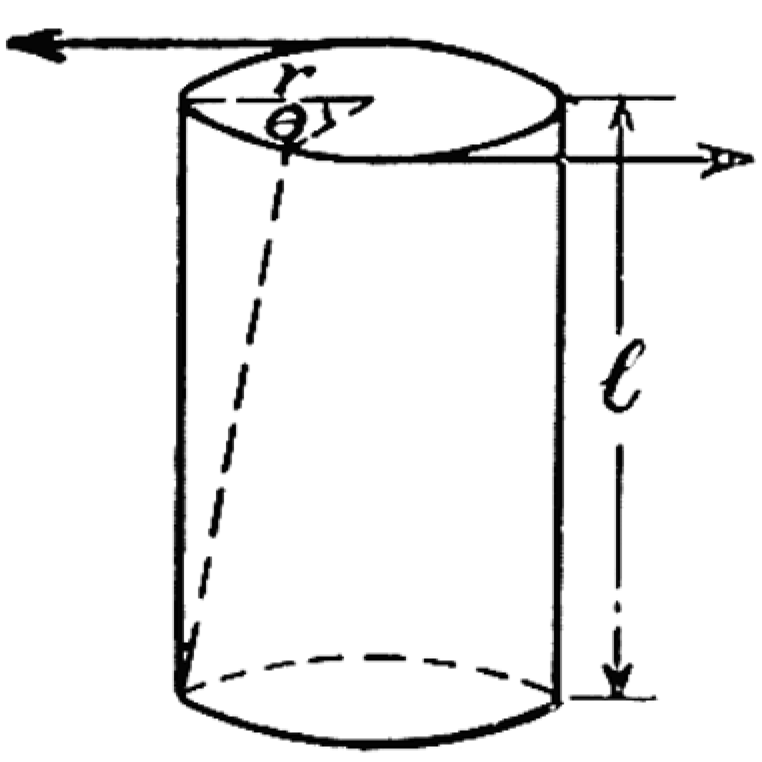


Fig. 85.

$$\theta = \frac{2GL}{\pi n r^4} \text{ radians,}$$

where G is the applied torque or moment of the couple used. The expression shows that the same torque which will produce a twist of 5° in a wire of 1 mm. diameter will twist a 0·5 mm. wire through 80°. We can write this result as

$$G = \frac{\pi n r^4}{2L} \theta.$$

It shows that the twist produced is proportional to the torque—a result which the student will do well to remember.

The torque required to twist a unit length of a wire through a unit angle (*i.e.*, one radian) is called the *modulus of torsion of the wire.* Let it be denoted by T. The torque required to twist one end of wire of length L through an angle θ will be $\dfrac{T}{L}\theta$, since each unit of length is twisted through $\dfrac{\theta}{L}$.

*Moment of the couple.

Equating the two expressions for the torque required to produce a twist of angle θ in a wire L cm. long we get

$$\frac{T\theta}{L}=G=\frac{\pi n r^4}{2L}\theta$$

or

$$n=\frac{2T}{\pi r^4}.$$

EXERCISES

1. Find the change in volume which 1 c. c. of water at the surface will undergo when it is taken to the bottom of a lake 100 metres deep, given that the volume elasticity is 22,000 atmospheres.

Let us find out the compression produced in one c. c. of water at atmospheric pressure when it is taken to the bottom.

$$E=\frac{dp}{dv/V}=\frac{100\times100}{76\times13\cdot6}\times\frac{1}{dv/V}=22{,}000.$$

$$\therefore\ \frac{dv}{V}=\frac{100\times100}{76\times13\cdot6}\times\frac{1}{22{,}000}=0\cdot00044.$$

This is the compression or the change in volume produced in 1 c.c.

2. Find the change in volume which 1 c.c. of water will undergo when taken from the surface to the bottom of an ocean 1 mile deep, given that volume elasticity of sea water is 20,000 atmospheres.

Ans. 0·0075.

3. State Hooke's Law.

A spring 60 cm. long is stretched by 2 cm. by the application of a force of 200 gm. wt. What would be its length when a force of 500 gm. wt. is applied? *Ans.* 65 cm.

4. If Young's Modulus for copper is $1\cdot2\times10^{12}$ dynes/cm.^{2}, calculate the increase in length of a copper wire 2·5 metres in length and 1 mm. in diameter when stretched by a weight of 5 kilograms. *Ans.* 1·3 mm.

5. Young's Modulus for steel is 2×10^{12} dynes/cm.^{2} What should be the length of a steel wire of 0·5 mm. radius so that it may increase in length by 1 mm. when stretched by a force of 4 kilograms.

Ans. 4 metres.

6. Explain the terms stress, strain and Young's Modulus of a substance. Calculate their values in the case of a rod 10 ft. long, $\frac{1}{80}$ square inch in cross-section when a load of 25 lb. stretches it by $\frac{1}{125}$ of an inch.

Ans. 2000 lb. wt. per sq. inch ; $\frac{1}{15000}$; $3\cdot0\times10^7$ lb. wt. per sq. inch.

7. What is elasticity?

Which is more highly elastic, rubber or steel? And why?

CHAPTER IX

Mechanics of Liquids

109. Viscosity.—We have said in Art. 104 that a fluid offers no resistance to a shearing stress, which, in other words, means that a fluid yields to a shearing stress, however small the stress may be. But in how long a time the effect becomes perceptible depends upon the nature of the fluid. For instance, water and alcohol yield at once to the deforming forces, whereas honey and glycerine take some time, though very small. Pitch offers so much resistance that unless the deforming force is applied for a long time, no effect is produced. For example, a lump of pitch when placed in a funnel would not appreciably flow through even in some days; but if left there for some months, it would slowly make its way down the funnel. This is expressed by saying that fluids are viscous to different extents. This property, which is due to the friction between the molecules of a fluid, is called **viscosity.** We define it *as a temporary resistance offered by fluids to the deforming forces or the shearing stress.*

A perfect fluid must be perfectly mobile; it must have no viscosity whatsoever. No substance of this type exists in nature; all fluids, even gases, have viscosity.

We can compare the viscosity of liquids by allowing them to flow through a narrow tube. The mobile liquids like water or alcohol flow through quite rapidly whereas viscous liquids like honey or glycerine take comparatively a much longer time. It is found that the velocity with which a liquid flows through a long, narrow tube depends upon

(1) The difference of pressure on the two ends of the tube;

(2) the radius of the tube;

(3) the length of the tube; and

(4) the nature of the liquid, *i.e.*, its coefficient of viscosity.

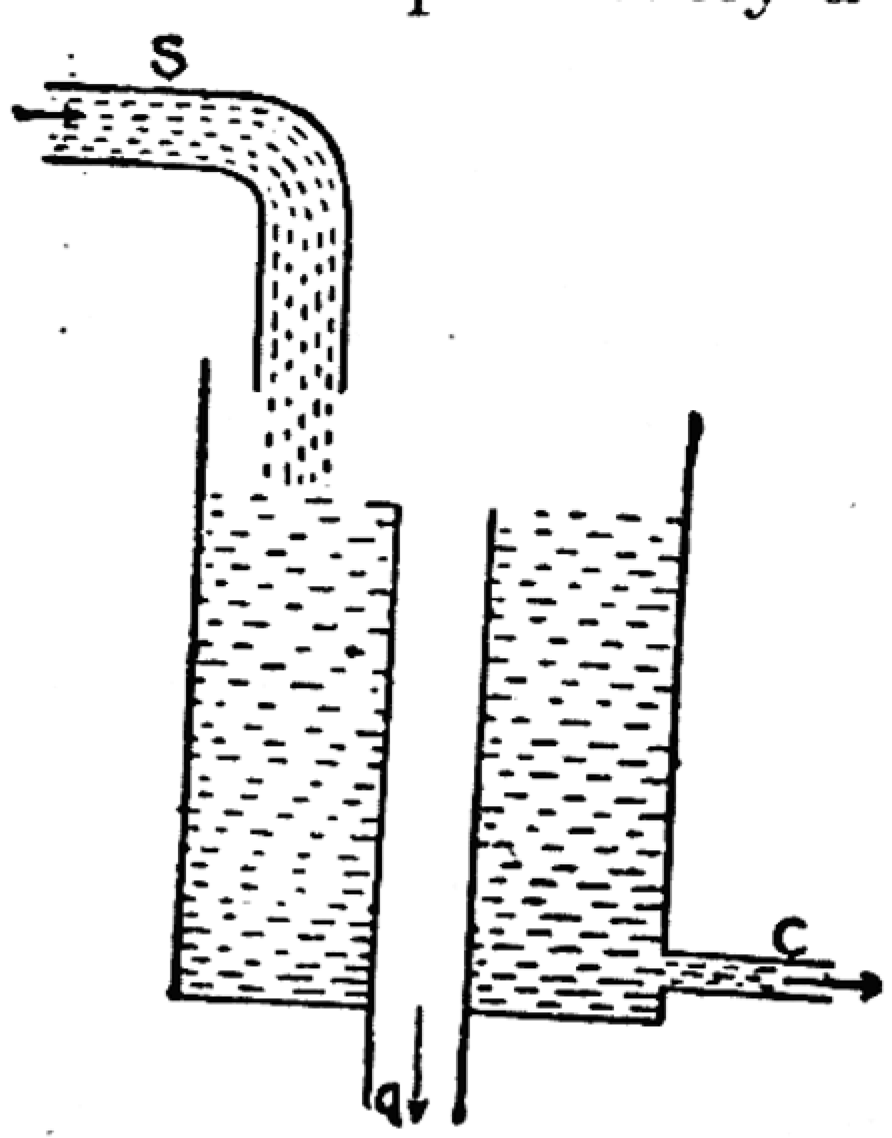


Fig. 86.

In order to compare the coefficients of viscosity of different liquids the apparatus shown in Fig. 86 is used. It is a cylinder having a glass tube q passing through the middle of it to serve as an overflow. It has a capillary tube C inserted into its side near the bottom. Through the tube S the liquid flows into the cylinder. This arrangement enables us to maintain a *constant difference of pressure* between the two ends of the tube C. Measure the volume of the liquid which flows through the tube C in t seconds. Let it be V c.c.

Next, take the second liquid. Calculate first what should be the height of the column in order to get the same difference of pressure as before and adjust the position of q to give the required height of the

liquid. Remember, the height will depend upon the density of the liquid : the greater the density the less the height. Note the volume that flows through the tube in the same time, *i.e.*, t seconds. Let it be V' c.c. The coefficients of viscosity would be inversely proportional to these volumes. If the coefficient of viscosity in the first case be η, and in the second case η' ; then

$$\frac{\eta}{\eta'}=\frac{V'}{V}.$$

Although gases seem to be perfectly mobile, yet it should be noted that they also possess viscosity. It is on account of the viscosity of air that a small drop of water such as in a fog or a cloud falls slowly through it. It is found that the coefficient of viscosity of a substance varies with temperature. In the case of a liquid it diminishes with the rise of temperature, whereas in the case of a gas it increases. The following table gives the coefficients of viscosity at 0° and 15°C.

Coefficients of Viscosity
(C.G.S. Units)

		0°C	15°C
Pitch	...	51×10^{10}	13×10^{10}
Water	...	0 0179	0·0114
Air	...	173×10^{-6}	181×10^{-6}
Hydrogen	...	86×10^{-6}	89×10^{-6}

110. Surface Tension.—We know that the molecules of a substance possess cohesion, *i.e.*, attract each other. If this force were destroyed, blocks of iron, rocks, houses, and in fact all solids would crumble to powder by their own weight.

In liquids the magnitude of this force is comparatively much less than in solids but it is by no means negligible. It is on account of cohesion only, that liquids possess a definite free surface, and behave as if they were covered with an elastic skin or membrane. If a needle be gently placed on the surface of water in a beaker, it floats on the surface, although it is more than seven times as dense as water, which shows that the surface must be under tension to be able to support this weight. The same thing is shown by the fact that insects run about on the surface of water without sinking.

This force in the surface is called the surface tension.

The magnitude of this force depends upon the nature of the liquid ; in the case of mercury it is very great, and in the case of alcohol, very small. It also depends on the nature of the medium in contact with the liquid. If a drop of alcohol be dropped over the surface of water near the floating needle in the above example, the needle will be seen to move off to one side. The drop of alcohol, where it strikes the water, weakens the film, which by elasticity pulls itself away from that point, taking the needle along with it.

The following table gives the surface tension of some liquids in contact with air :

Surface Tension

Alcohol	...	26 dynes/cm.	Olive oil	...	38 dynes/cm.
Benzol	...	29 ,,	Petroleum	...	32 ,,
Mercury	...	450 ,,	Water	...	73 ,,

The effects of the surface tension are generally studied with the help of soap films. A metal ring is dipped in a soap solution. On removing, it carries a thin soap film with it. A small cotton loop, which has been previously moistened with the soap solution, is placed carefully on the film. The loop can be made to have any form as shown in Fig. 87 (*a*).

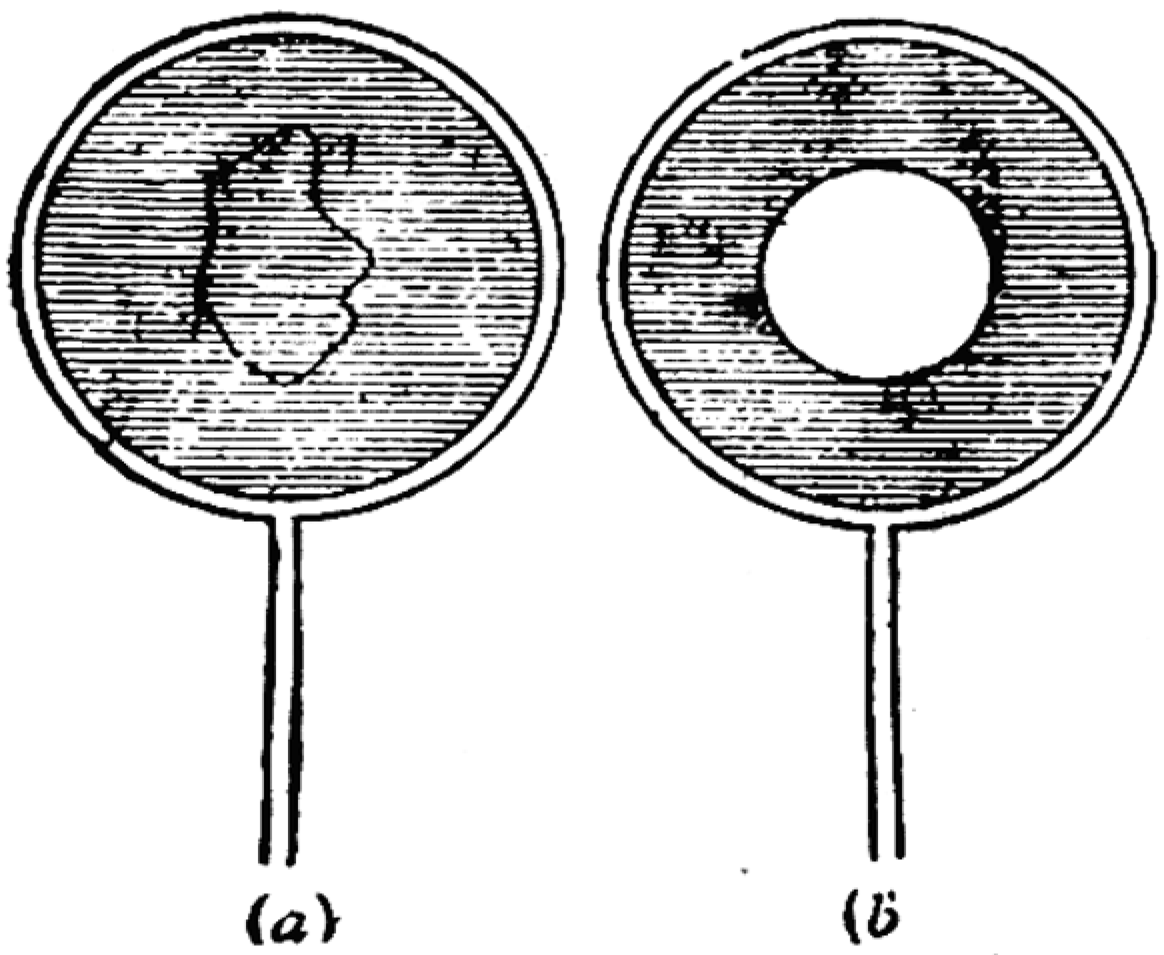


Fig. 87.

If, however, the film within the loop is pricked, it disappears from there, and the loop is pulled out in the form of a circle, as shown in Fig. 87 (*b*). If it is now deformed in any way, on being released it immediately takes up the circular form. The reason why the loop behaves in this way is that in the first case the tension acts equally on its both sides (inside and outside) ; but when the film inside is broken, the force is acting only on one side and at right angles to the thread at each point and hence the loop is pulled out into a circle.

The effect of the surface tension is to make the area of the free surface as small as possible It is on account of this fact that a sphere has the least surface for a given volume and that raindrops are spherical.

The reason why the free surface of a liquid is not so commonly observed to be spherical as surface tension leads us to expect, is that ordinarily the force of gravity and other external forces are much stronger than surface tension.

That it is actually so can be verified by eliminating the disturbing forces. By adding alcohol to water make a solution of the same density as olive oil, and half fill a cylinder with this solution. Now introduce some where in the middle of this solution a large drop of olive oil with the help of a pipette. The drop will be noticed to float there as a perfect sphere.

molecular range

To explain how the surface tension is brought about let us consider two molecules, *P* and *Q* (Fig. 88) ; one of them, *P* is wellbel ow the surface, while the other, *Q*, is just near the surface. *P* is pulled equally on all sides by the neighbouring molecules, whereas *Q* is mainly pulled downwards and laterally, for there are fewer molecules above it to pull it upwards. Similar is the case with all other molecules which are near the surface. The result is that the molecules on the surface are bound together to form something like a stretched membrane over the surface of the liquid.

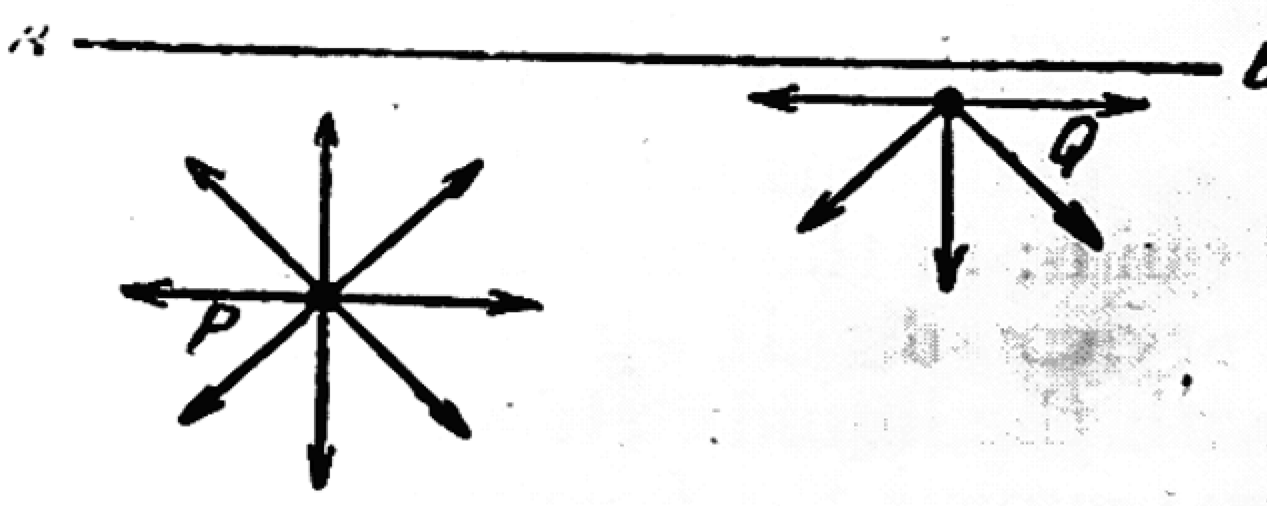


Fig. 88.

110a. Angle of Contact.—Consider a molecule *P* in water near the walls of the containing vessel. This molecule is acted upon in addition to its weight by two sets of forces.

(*i*) the pull of the other water molecules, *i.e.*, the resultant force due to *cohesion*, and

(*ii*) the pull of the molecules of the solid wall *i.e.*, the resultant force due to *adhesion*.

If the resultant R of the forces acting on P, acts vertically downwards, the surface will be horizontal, since the free surface of a liquid at rest is perpendicular to the resultant force acting on its surface. The angle at which the liquid surface meets a solid wall is called the *angle of contact*. In the case of water and silver the angle of contact is 90° and the surface of water is horizontal.

Fig. 89. (*a*) represents the case where the resultant adhesive force A is greater than the resultant cohesive force B. It is clear from the diagram that the surface of the liquid to be perpendicular to R must run upwards near the wall and form a *concave meniscus*. The angle of contact for clean glass and water is 8°. A liquid which runs upward is said to wet the surface.

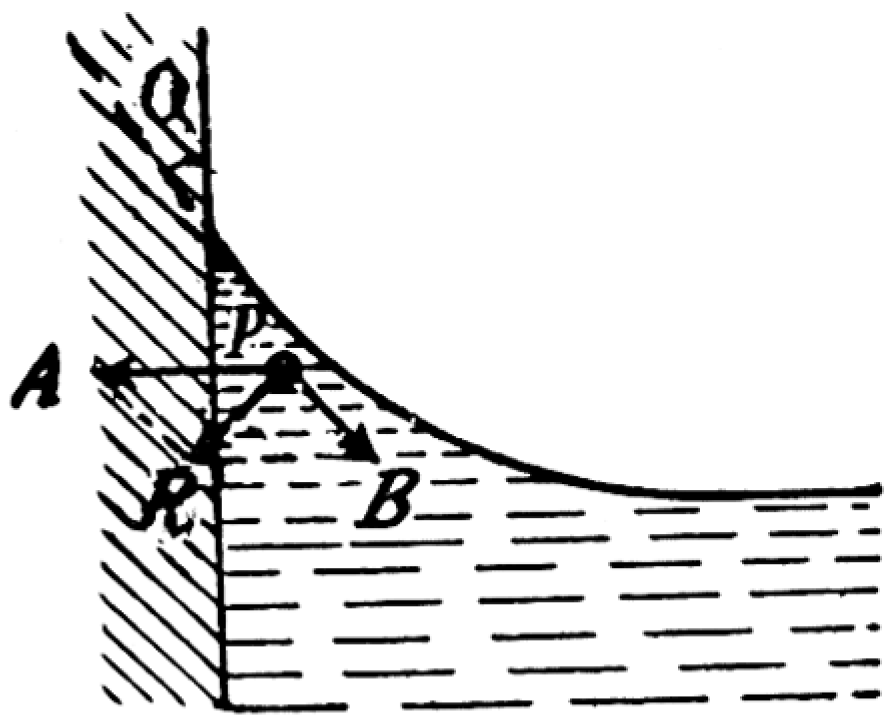


Fig. 89 (*a*).

Fig. 89 (*b*) represents the case where the resultant cohesive force B is very much greater than the resultant adhesive force A. In order that the free surface near the wall may be perpendicular to R, the surface curves downwards. This is the case with mercury against glass. The angle of contact between clean glass and mercury is 140°. When the liquid curves downwards it doos *not* wet the surface.

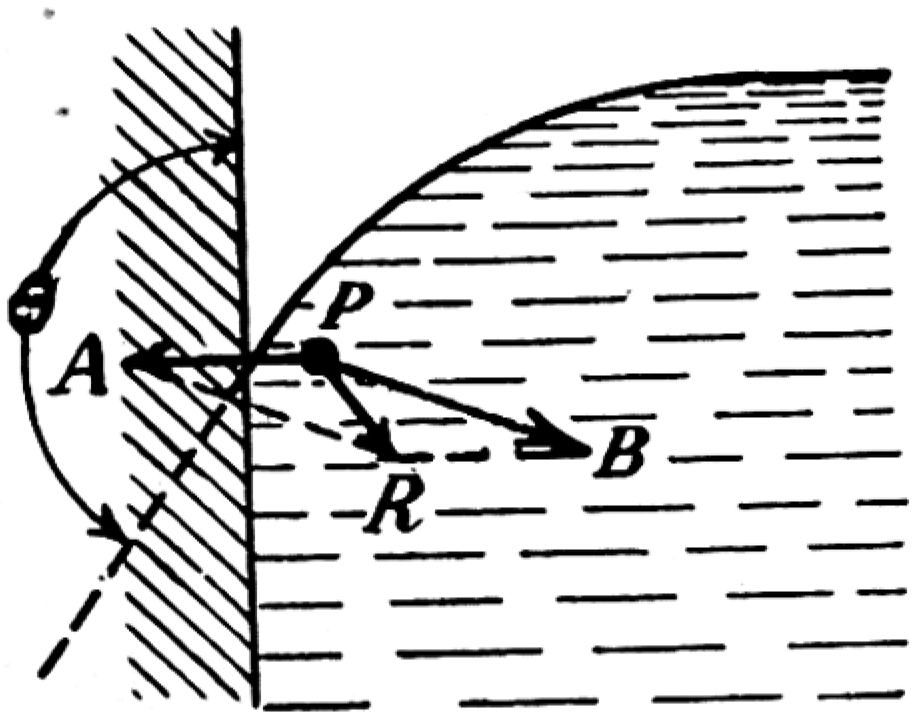


Fig. 89 (*b*)

If the angle of contact is less than 90° the liquid wets the surface, if it is more it is non-wetting.

110b. Capillarity.—If we dip a narrow tube in a liquid which wets the tube it rises in the tube above its level in the outer vessel. The height to which the liquid rises depends upon the bore of the tube as well as upon the nature of the liquid. Liquids which do not wet the tube in place of rising are depressed below the level outside.

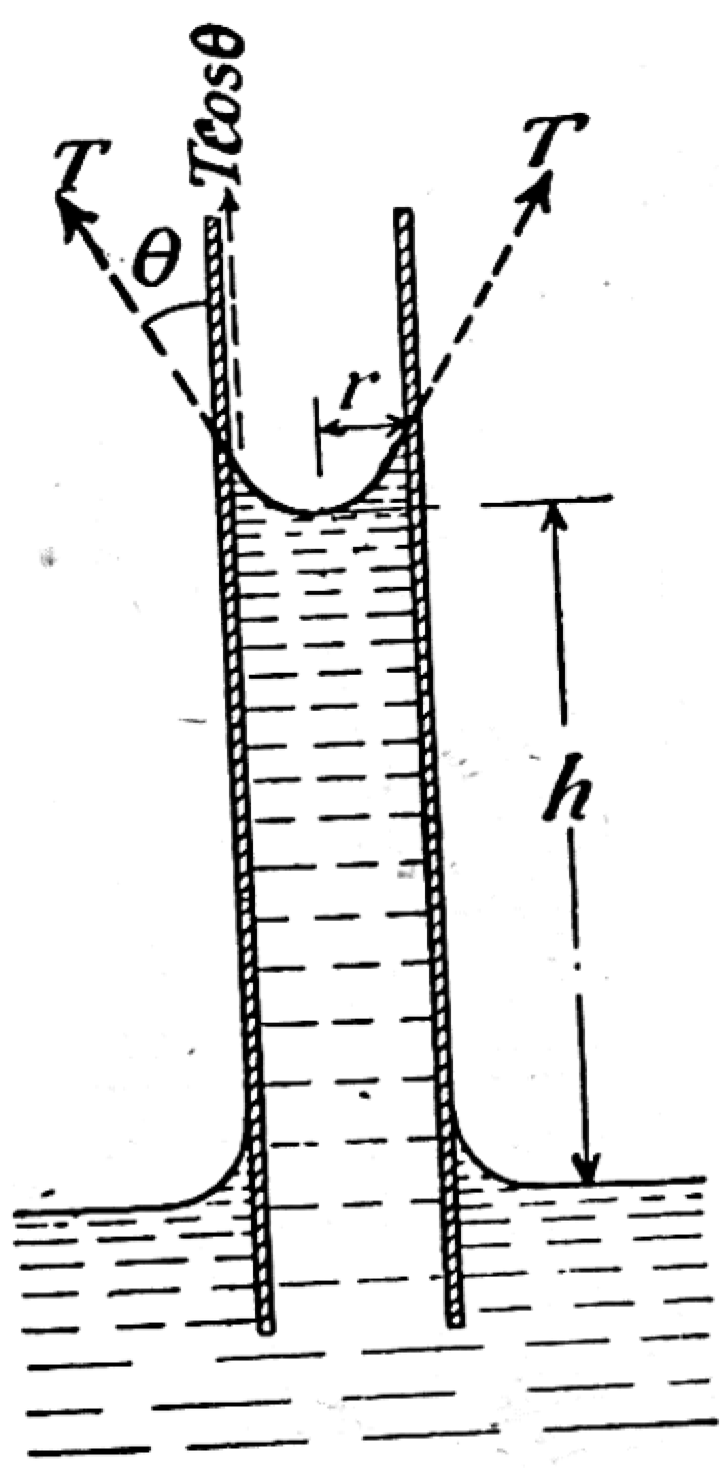


Fig. 90.

To find the height to which a liquid will rise or get depressed, dip an end of a capillary tube in the liquid and suppose the liquid rises to a height h cm. in the tube above the level of the liquid outside. Let its angle of contact with the walls of the tube be θ. Surface tension will act along the junction of the tube and liquid *i.e.*, along a line of length $2\pi r$, where r is the inner radius of the tube, and will be directed upward at an angle θ with the wall. The vertical component of this force

$=2\pi rT\cos\theta$. As a result of this upward force the liquid rises in the tube until the downward pull of gravity on the column balances the upward force due to tension. The weight of the column is equal to $\pi r^2 h\rho g$ dynes, where ρ is the density of the liquid. Thus

$$2\pi rT\cos\theta=\pi r^2 h\rho g$$

or

$$h=\frac{2T\cos\theta}{r\rho g}$$

Since in case of water θ is very small, we can suppose $\cos\theta=1$,

$$\therefore\qquad h=\frac{2T}{r\rho g}$$

For a liquid like mercury for which angle of contact $\theta>90°$, $\cos\theta$ is negative and so is h, which shows that the liquid will be depressed below the level outside.

The rise of kerosene oil in a lamp wick or the absorption of ink by a blotting paper or the rising of melted wax in the wick of a candle are familiar examples of capillary action.

Bricks and mortar are porous so that the water of the soil rises through them by capillarity and keeps them constantly damp. To prevent this a *damp-proof* course consisting of some non-porous material, (*e.g.*, stone-slab or cement concrete layer) is inserted between two horizontal rows of bricks just above the ground level.

Sandy soil, made of comparatively large particles, has large air spaces whereas clay, made of finer particles, has narrow air spaces, with the result that water does not rise so readily through sand as it does through clay. Hence sand is a drier soil than clay.

The elevation or depression of liquids in capillary tubes is called *capillarity.*

110c. Measurement of Surface Tension.—The capillary tube method is the most commonly used method in the school and college laboratories. An essential condition for reliable results is that the tube should be absolutely clean. Clamp the capillary in a vertical position and let its lower end dip in the liquid whose surface tension is to be measured. To find the height of the liquid in the capillary tube a vernier microscope is used. The microscope is focussed on the lowest portion of the meniscus and a reading is taken. To read the level of the liquid in the beaker fasten a piece of a knitting needle in a suitable clamp and adjust its position so that its lower end *just* touches the liquid. Focus the microscope on the end of the knitting needle and take the reading on the scale. The difference of these two readings gives the height.

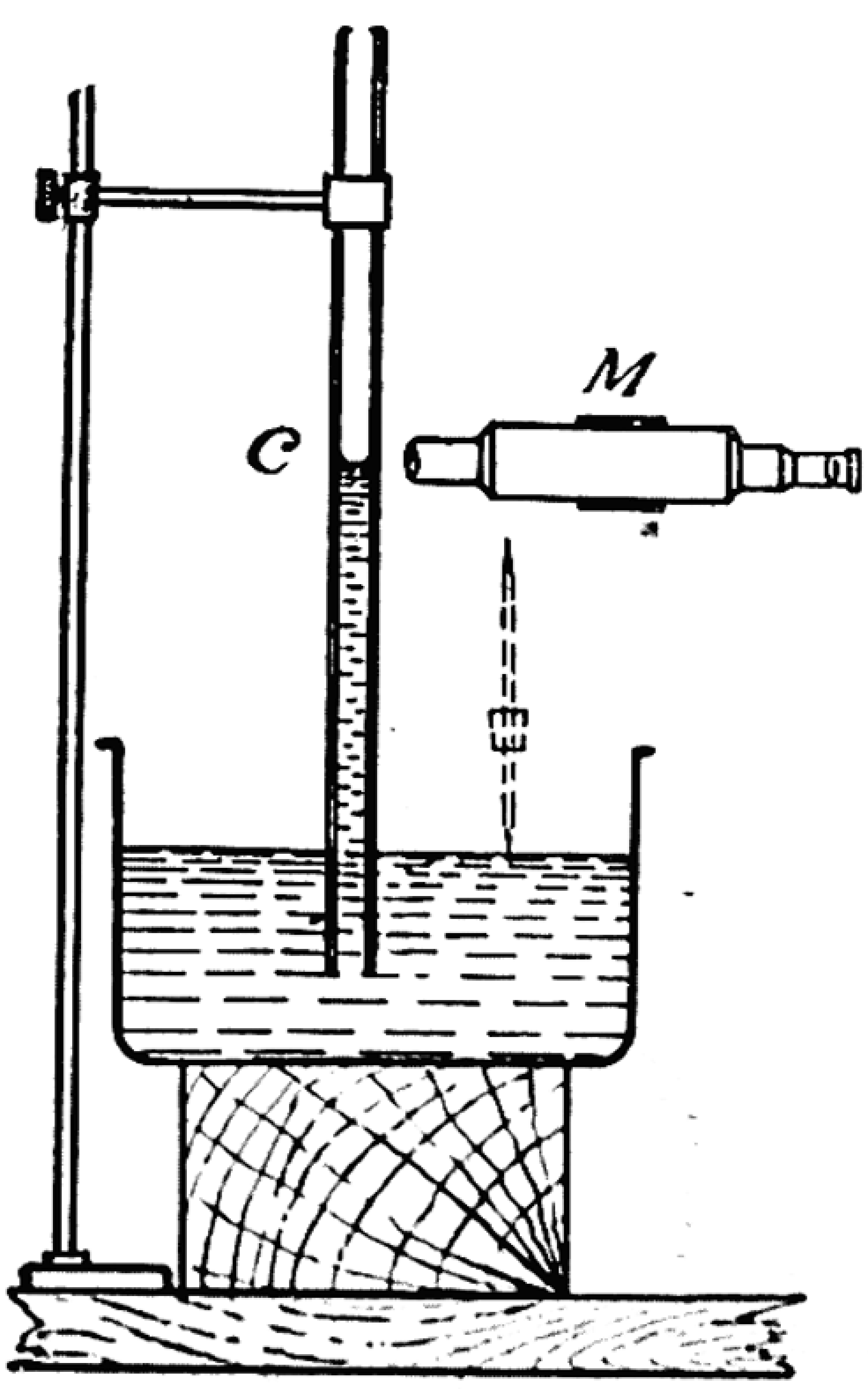


Fig. 91.

The height so found should be corrected by adding one-third of the radius of the tube in order to take into account the liquid above the lowest portion of the meniscus.

To find the diameter of the tube it is

first cut in two parts near the point C and its diameter read with a vernier microscope along two perpendicular directions. Half the mean gives the radius at the point.

111. Free Surface of a Liquid at rest is Horizontal.—Let us suppose that the free surface instead of being horizontal is represented by $ABCDEF$ (Fig. 92) where the part BCD is higher than the part DEF. Draw an inclined plane through BD. The weight (mg) of the liquid lying above the plane BD tends to pull it down, and since liquids yield to a shearing stress, howsoever small, the liquid lying above BD will move down to places which are lower in level. This sliding down will continue so long as there is a difference in level. It will stop only when the surface is horizontal*.

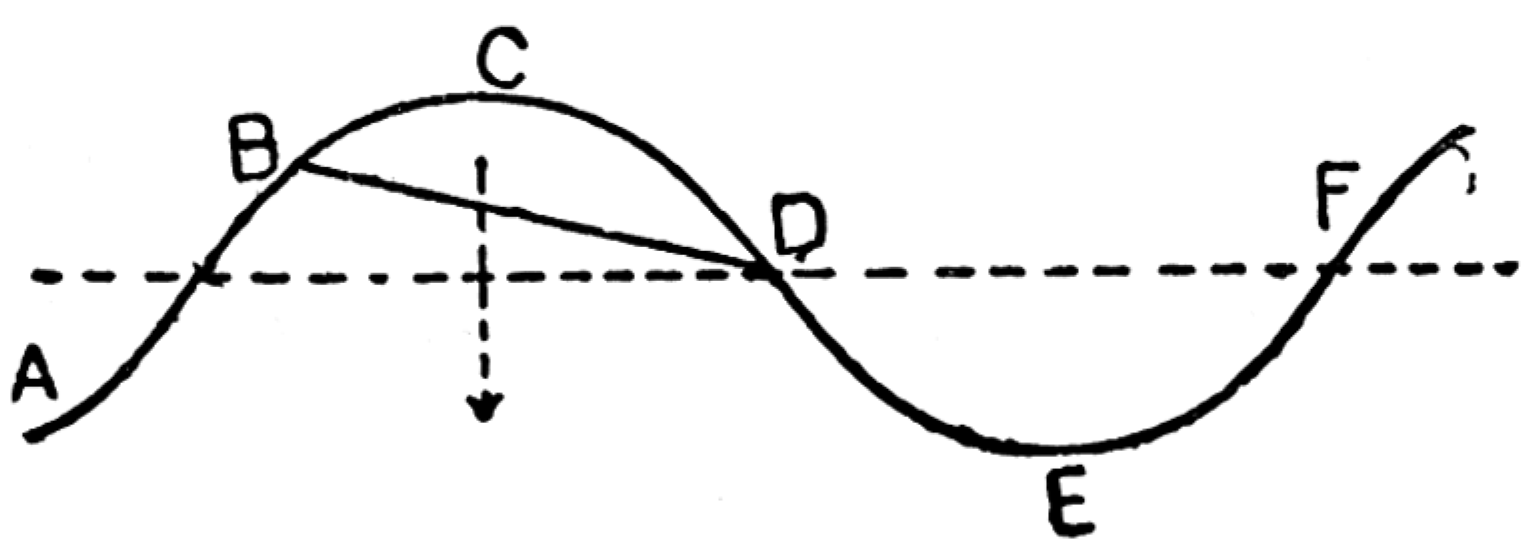


Fig. 92.

The above fact that the free surface of a liquid at rest is horizontal is sometimes expressed by saying that *a liquid finds its own level.* It can be proved experimentally by taking tubes of various shapes connected together by a horizontal tube (Fig. 93). When water is poured into any one tube, it stands at the same level in all of them.

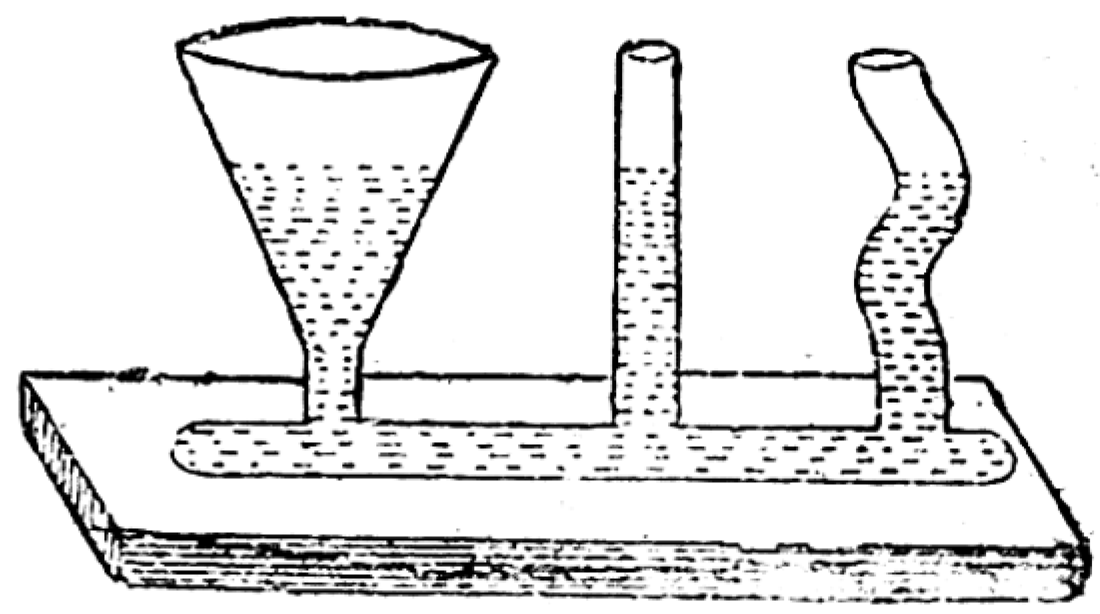


Fig. 93.

112. Liquid Pressure.—We have already seen that when two solid bodies are placed one above the other, the upper body presses the lower one and the lower body pushes in the upward direction the upper one. We called these forces action and reaction. Each of these forces is called a **thrust.** Every body whenever it is in contact with another body exerts a thrust no matter whether it is a solid, liquid, or gas. We have already dealt with the thrust between two solid bodies, now let us study the thrust when one of the bodies is a liquid. We all know, for instance, that if a man wants to dive into water, he cannot do so by falling flat on it for the surface of water offers resistance (upward thrust). In order to dive easily a diver clears the water with his hands and his body follows.

But there is a difference between the thrust exerted by a solid body on another solid body and the thrust exerted by a liquid on a solid body. In the case of solid bodies, the direction of reaction is determined by the fact whether the surface is smooth or rough, while in the case of a liquid at rest the thrust is always normal to the surface in contact with it. This is in fact one of the fundamental properties of a liquid.

*It should be noted that a horizontal surface is not necessarily flat (*i.e.*, plane). It is so only when we deal with comparatively small surfaces. A large expanse of water, as for instance a sea, possesses a curved surface which is a part of the spherical surface of the earth. In such a case when we say that the surface is level and horizontal, we mean that the different parts of the surface are equidistant from the centre of the earth and hence possess the same curvature as the earth.

113. To prove that the thrust of a liquid at rest is always perpendicular to a surface in contact with it.—Let us suppose ABC (Fig. 94) is a surface in contact with a liquid (say water). Consider the forces acting at the point B. The thrust of the liquid will be equal and opposite to the thrust of the surface at the point B. If the thrust of the liquid is not normal, let it be along MB; according to Newton's 3rd law the thrust of the surface on the liquid will be along BM. This force can be resolved into two components one along BN perpendicular to the surface and the other along BL parallel to it. Since a liquid cannot withstand even the slightest tangential force (*i.e.*, shearing stress) the liquid will move under the influence of the component along BL, and therefore will no longer be at rest. Thus we see that, if a liquid is at rest, the thrust must be normal to the surface in contact with it.

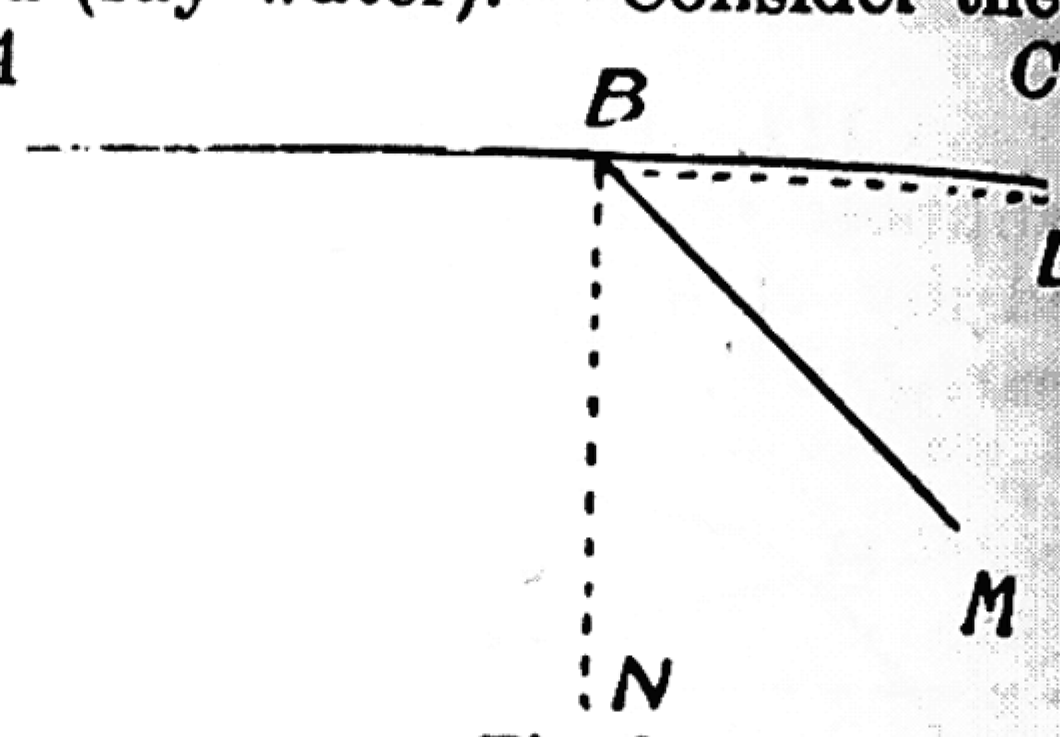


Fig. 94.

114. Pressure.—The thrust that a surface experiences per unit area when in contact with a liquid is called the *pressure*. It is uniform if the thrust on any two equal areas ,however small, is equal. When it is so, the thrust on a surface is equal to *pressure* $\times$ *area*. Mathematically we can express this fact as

$$P=\frac{F}{A}, \text{ or } F=PA,$$

where P is the pressure, F the thrust, and A the area of the surface.

For quantitative measurement of pressure it is necessary to fix upon a unit of pressure.

Since pressure is force per unit area, the unit of pressure depends upon the units of force and area. In the C.G.S. system pressure is measured in dynes or grams-weight per square centimetre whereas in the F.P.S. system it is measured in poundals per square foot, or pounds-weight per square inch. Engineers use the last unit. According to them the atmosphere, for instance, exerts a pressure of 14·7 pounds-weight per square inch.

115. Transmissibility of Liquid Pressure.—We have already studied one fundamental property of the liquids, *i e.*, the pressure of a liquid at rest is perpendicular to the surface in contact with it ; now we shall study the second fundamental property, *viz.*,

The liquids transmit pressure equally in all directions.

This law was first stated by Pascal and is hence, sometimes called **Pascal's law.** It can be verified in a simple manner by taking a closed vessel (Fig. 95) fitted with four water-tight-pistons, which, for simplicity, we shall assume to possess equal cross sections. Let us suppose that the pistons are at rest to start with. Push A inwards with a force of 1 lb. wt. It will be seen that to keep the other pistons at rest the same force must be applied to each one of them. This

shows that the increase of pressure applied at one point is transmitted equally to all other points. It should be noted that *it is the pressure which is equally transmitted and not the thrust.* If, for instance, the cross section of the piston B had been twice that of A, on pushing the piston A inwards with a force of 1 lb. wt. the piston B must be pushed (inwards) with a force of 2 lb. wt. to keep it in equilibrium. On a piston with three times the cross section of A, the force required would be 3 lb. wt., and so on. A practical application of this increase of force is met with in the **Bramah Press** which consists essentially of two cylinders fitted with pistons of different sizes and connected with a pipe.

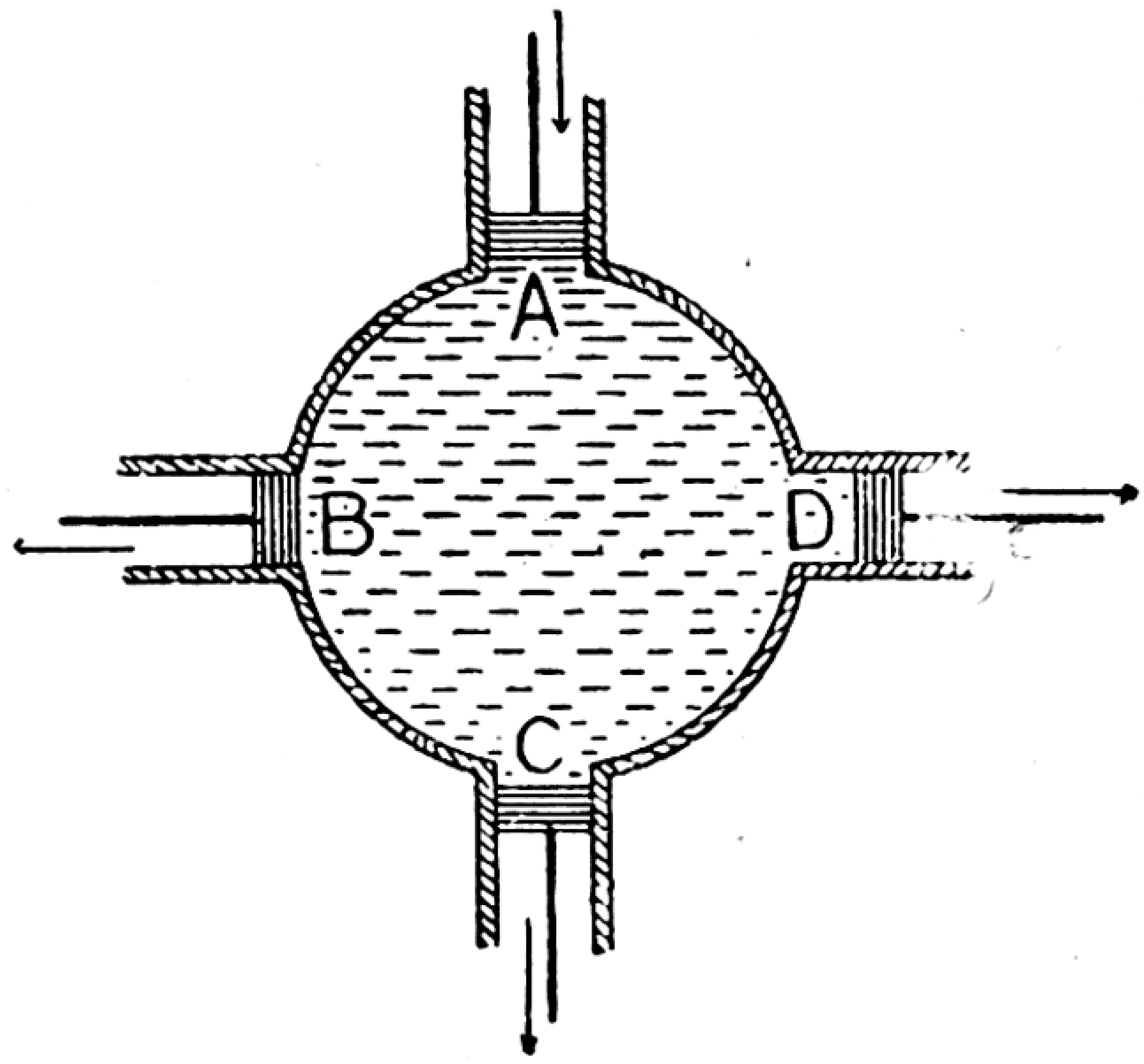


Fig. 95.

A general sketch of the Bramah Press is shown in Fig. 96. P is the pump-plunger, Q the press-plunger, T a pipe connecting the two cylinders, and S a reservoir of water.

Let us suppose that the cross sections of the plungers P and Q are a and b sq. inches respectively, and that P is pressed down with a force R. The pressure on this side is $\frac{R}{a}$ lb per sq. inch ; it is transmitted to the bigger cylinder. The thrust, W, on the piston Q, will be $\frac{R}{a} \times b$. Since b is greater than a, W is greater than R. By suitably increasing b in comparison with a we can make W as great as we please. This shows how a small downward force on P can be transmitted as a great upward force on Q. By placing goods between Q and the top of the framework, they can be pressed, and hence the name *Bramah Press*. It should be remembered that whatever is gained in power is lost in distance for P must move through a greater distance than Q in order that work done on the two sides may be equal.

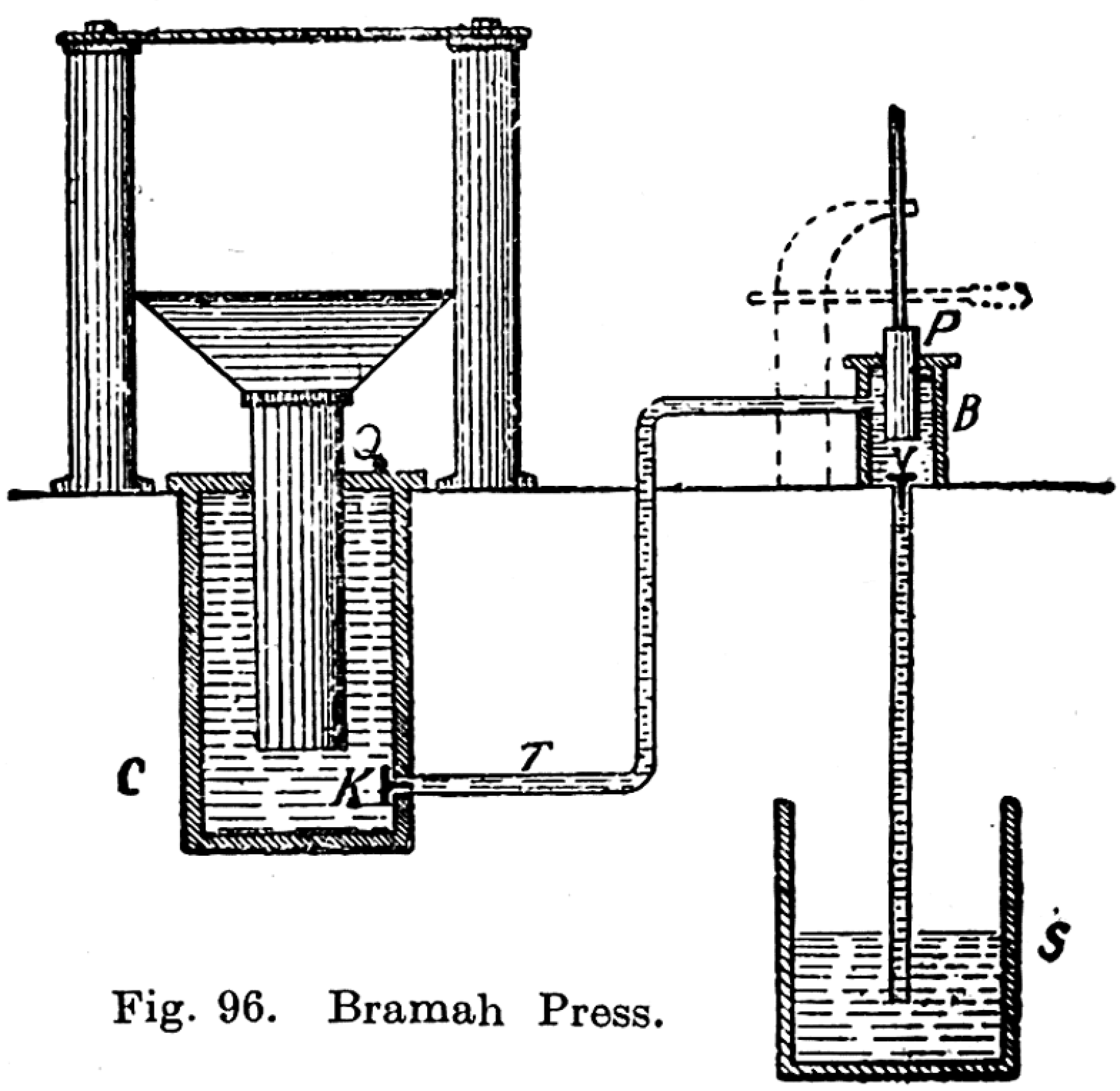


Fig. 96. Bramah Press.

$$\text{Mechanical Advantage} = \frac{W}{R} = \frac{b}{a} \times R \times \frac{1}{R} = \frac{b}{a}.$$

If r' and r be the radii of pistons Q and P respectively, then

$$\frac{b}{a}=\frac{\pi r'^2}{\pi r^2}=\frac{r'^2}{r^2}$$

Further, if a lever arrangement be used to press down the pump-plunger the mechanical advantage will become still greater. If the power-arm is L and the resistance arm l, the mechanical advantage of the lever is $\frac{L}{l}$ and of the Bramah Press $\frac{b}{a}\times\frac{L}{l}$.

To understand the working of the press let us start with the piston P at the bottom of the cylinder B. When it is raised by means of the lever, the pressure in the cylinder B is diminished and the valve V opens upward allowing water from the reservoir S to come into B. When P is lowered the valve V closes, but the valve K opens allowing thereby the water in the cylinder B to be forced into the cylinder C. The valve K prevents the return of water from cylinder C to B when the piston P is raised again. At each upstroke, water is drawn from the reservoir S into the cylinder B and at the downstroke it is forced into the cylinder C, where it pushes up the piston Q. Note, the piston P moves in every downstroke through the whole length of the cylinder B, whereas the piston Q rises in each stroke through only a small distance. It is only after a large number of strokes that the piston Q moves through a measurable distance. After the goods have been compressed, the water is allowed to come back from the cylinder C to the reservoir by turning on a tap (not shown in the figure) and the piston Q descends by its own weight and becomes ready for use again.

The motor car hoist or lift is another common example of the application of Pascal's law. The students must have seen motor cars being raised up on a platform for cleaning etc.

The giant machines used in testing strength of materials like girders or in pressing metal sheets into desired shape like mudguards or presses used to extract oil from seeds are all examples of the application of Pascal's law.

116. Pressure of a Liquid varies directly with the Depth.—Let us find the pressure at a point h cm. below the free surface AB of a liquid in a vessel. Take a unit horizontal circular surface S round the point and consider a vertical column R of the liquid h cm. high and 1 sq. cm. in cross section. The vertically downward pressure on S is due to the weight of the column R of the liquid, which is proportional to the volume $(=h\times 1$ c.c.) of the column. If ρ is the weight of one cubic centimetre of the liquid, the weight of the column will be $h\times\rho$ grams. This shows that the pressure at a point is proportional to its depth.

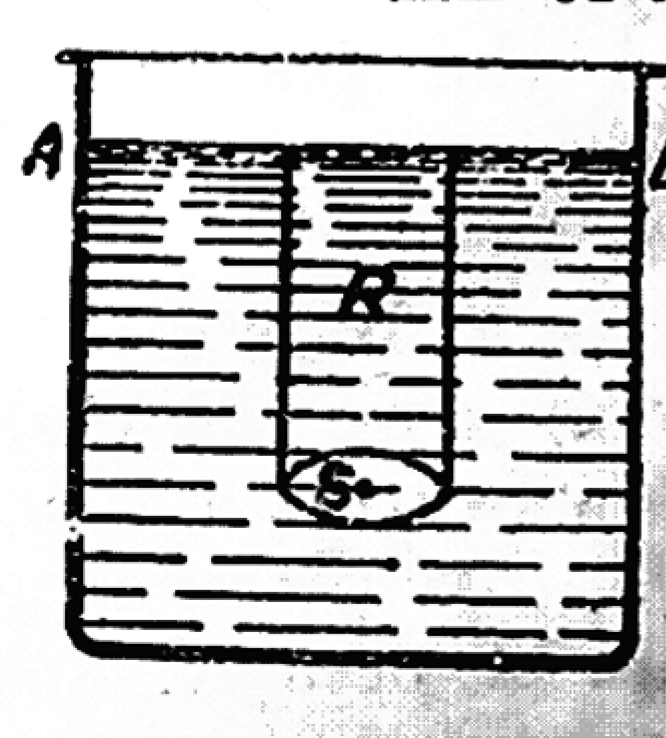
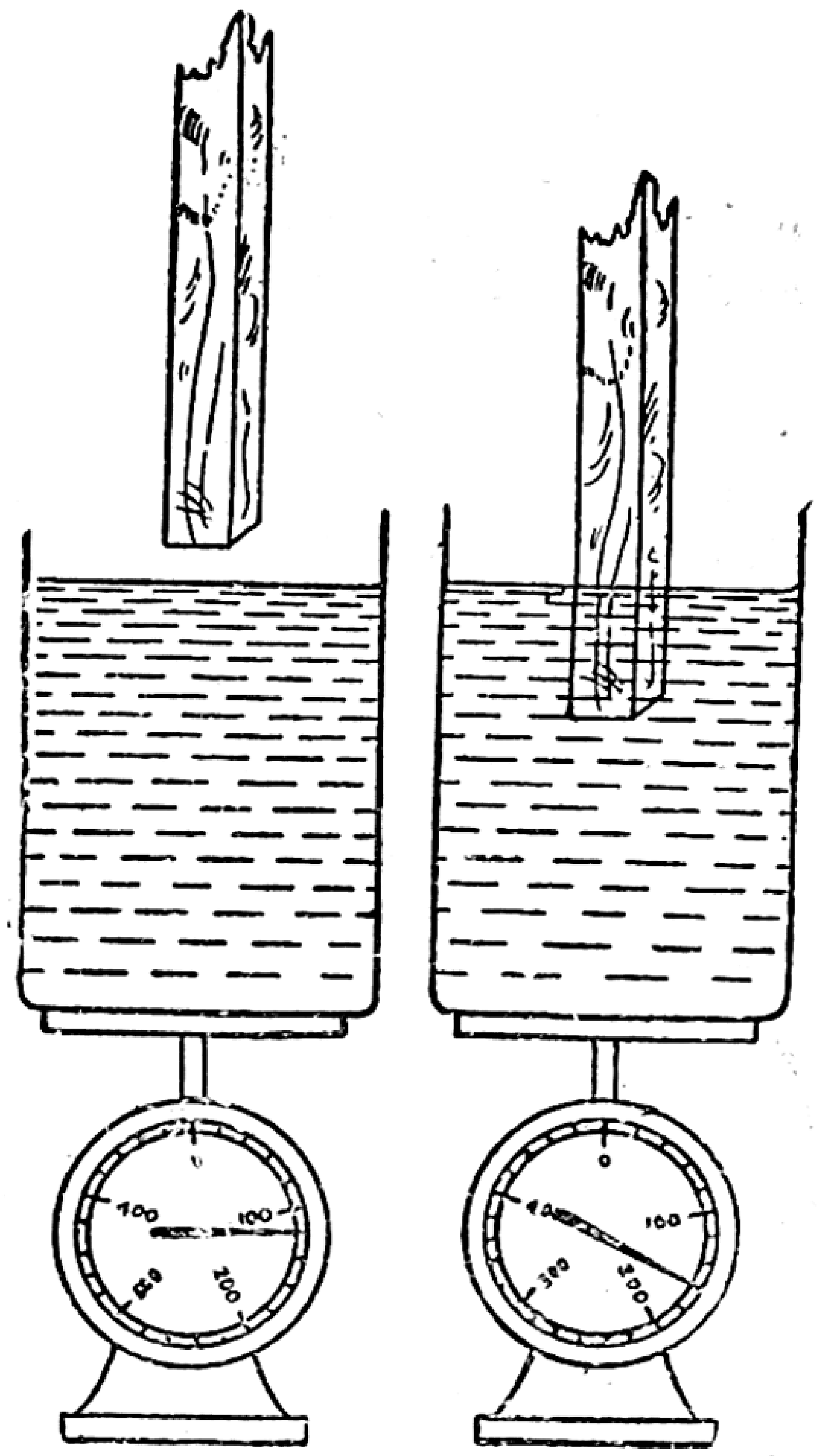


Fig. 97.

A very important corollary follows directly from this result. Since the free surface of a liquid at rest is horizontal, all points at an equal distance below the free surface have the same pressure ; in other words, the pressure in a horizontal plane is the same everywhere in the case of a liquid at rest.

117. Pressure at the base depends only on the depth of water and not on its quantity.—It was Pascal who first experimentally showed this statement to be true.

We can verify this statement by means of the following simple experiment. Take a beaker two-thirds full of water and place it on the pan of a weighing machine [Fig 98. (*i*)]. Read the weight of water and beaker. Dip about 1 inch of a wooden rod about 1 inch wide and 1 inch thick in the water without letting it touch the beaker. You will observe that the weight of the water and beaker indicated by the weighing machine is greater than before [Fig. 98 (*ii*)]. The rod is not touching the beaker, hence it cannot directly increase the weight. But since due to dipping of rod, the level of water rises, *i.e.*, the depth becomes greater, the increase in weight may be due to the increase in depth. To see that it is really so, dip now about 2 inches of the rod and notice that the increase in weight is greater. This shows that the *pressure depends upon the depth and not on the quantity of water in a vessel.*


(i) (ii)
Fig. 98.

A very interesting application of this principle is met with in the **Hydrostatic Paradox**. The apparatus commonly used for the demonstration of this paradox consists of two wooden discs of about 1 foot diameter connected together by a collapsible bellows of leather or water-proof canvas joined to a tube as shown in Fig. 99. The tube rises to a height of 4·5 feet or thereabout and ends in a funnel. A man may raise his own weight by pouring water down the tube. It seems as if a mere thread of water is supporting the weight of a man.

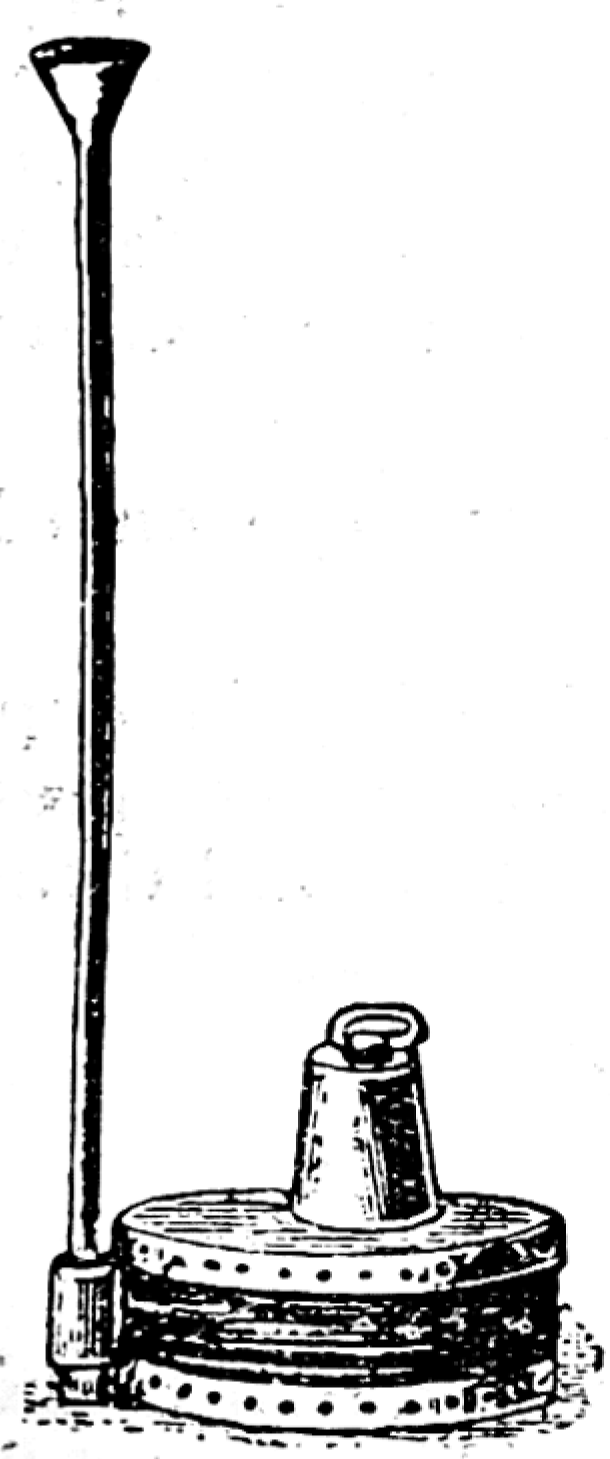


Fig. 99.

In order to see that in reality there is no paradox in this case, let us calculate the height of water column in the tube above the level of water in the bellows which will support a man who weighs 150 lb.

Let the height be h ft., ρ the density of water, and s the area of the top of the bellows. The thrust of water in the wooden disc of the bellows $= h \times \rho \times s$.

This must be equal to the weight of the man. In other words

$$h \times \rho \times s = 150 \text{ lb.}$$

Since weight of 1 cubic foot of water or $\rho=62{\cdot}5$ lb. and $s=0{\cdot}75$ sq. ft. (1 foot being the diameter)

$$h=\frac{150}{62{\cdot}5\times0{\cdot}75}=3{\cdot}2 \text{ feet.}$$

118. Upward Pressure of Liquids.—Now we shall show that there exists an upward and lateral pressure at a point inside water equal and opposite to the downward pressure.

To prove experimentally the existence of upward pressure, take a glass tube 15 to 20 cm. long about 4 cm. wide, with one end ground flat so that it can be closed water-tight by a metallic disc *A*. To the centre of the disc fasten a thread and hold the disc firmly against the tube by means of the thread. Push the tube down into the water in a jar. Let the thread go. It will be found that the disc does not fall off. It is held in position by the upward thrust of water.

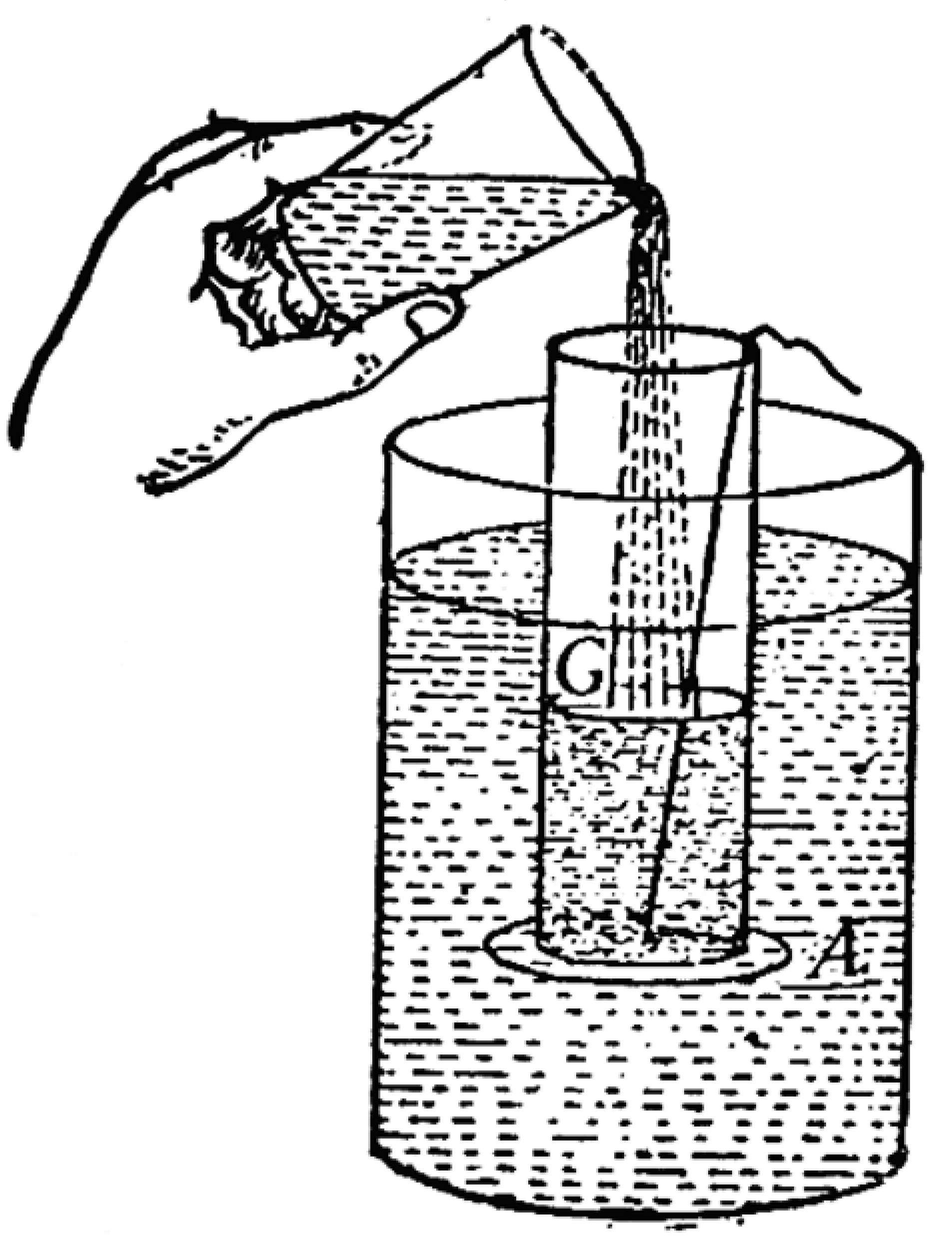


Fig. 100.

Now gently pour coloured water into the tube (Fig. 100). It will be seen that the disc falls off as soon as the level of water inside is the same as outside. This shows that at any point the upward pressure exerted by a liquid is equal to the downward pressure.

It may be remarked here that liquids also exert **Lateral Pressure.** We all know, for instance, that if a hole be made in the side of a vessel full of water, the water rushes out. This shows that water must have been exerting a pressure at the point before the hole was made.

The lateral pressure at a point inside a liquid like the downward pressure depends upon the depth of the liquid at that point. To calculate the thrust at the side of a vessel the student should remember that the lateral pressure increases gradually from zero at the surface to its full value at the bottom. In order to explain how the thrust is calculated in such cases we shall take an example.

Example.—Calculate the thrust of water 10 ft. deep on a lock gate whose width is 25 ft.

Since the pressure goes on increasing uniformly with the depth it is clear that the average value of the pressure is equal to the pressure half-way down. Multiplying this by the area we get the total thrust.

$$\text{Lateral thrust}=\text{A}\times\frac{h}{2}\times d=250\times5\times62{\cdot}4$$

$$=78{,}000 \text{ lb. wt.}$$

119. Upward Thrust exerted on a Body immersed in a Liquid.—Take a metallic rectangular block 10 cm. long, 5 cm. broad and 2 cm.

thick and hence of volume 100 c.c. When immersed in water it will displace 100 c.c. of water. Since 1 c.c. of water weighs 1 gram, it is obvious that the weight of water displaced will be 100 gm. Weigh the block with a spring balance first in air and then in water. Its weight in water will be seen to be less by 100 grams, *i.e.*, to be less by the weight of water displaced. Every substance when weighed in water is found to lose in weight, the loss being equal to the weight of the water displaced. It was Archimedes who, while he was taking his bath in a tub of water, first observed that bodies lose in weight. Hence this result is called after him Archimedes' Principle. It is stated as below :—

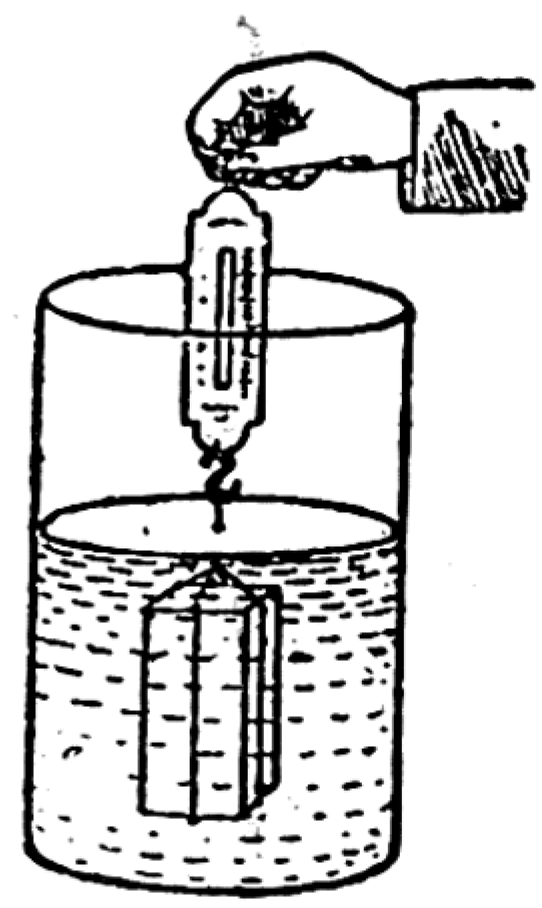


Fig. 101.

A body immersed in a liquid, loses weight by an amount equal to the weight of the liquid displaced.

A question arises, "Why does a body lose weight when weighed in water ?" The answer is that it does so, because it is *on the whole* pushed upwards by the liquid and the amount of push is equal to the weight of the liquid displaced. In view of this fact the Archimedes Principle is often stated as follows :

When a body is immersed in a liquid, it experiences an upward thrust equal to the weight of the liquid that it displaces.

This upward thrust is called *Buoyancy*. It acts vertically upward through the centre of gravity of the liquid displaced.

We can easily deduce the principle of Archimedes from the consideration of the forces acting on a body immersed in a liquid. Let us suppose a metallic block is suspended in a liquid as shown in Fig. 102, with its upper face d cm. below the surface of water. There are four vertical forces acting on the block ; two of them tend to push it up and the other two tend to push it down. They are given below :

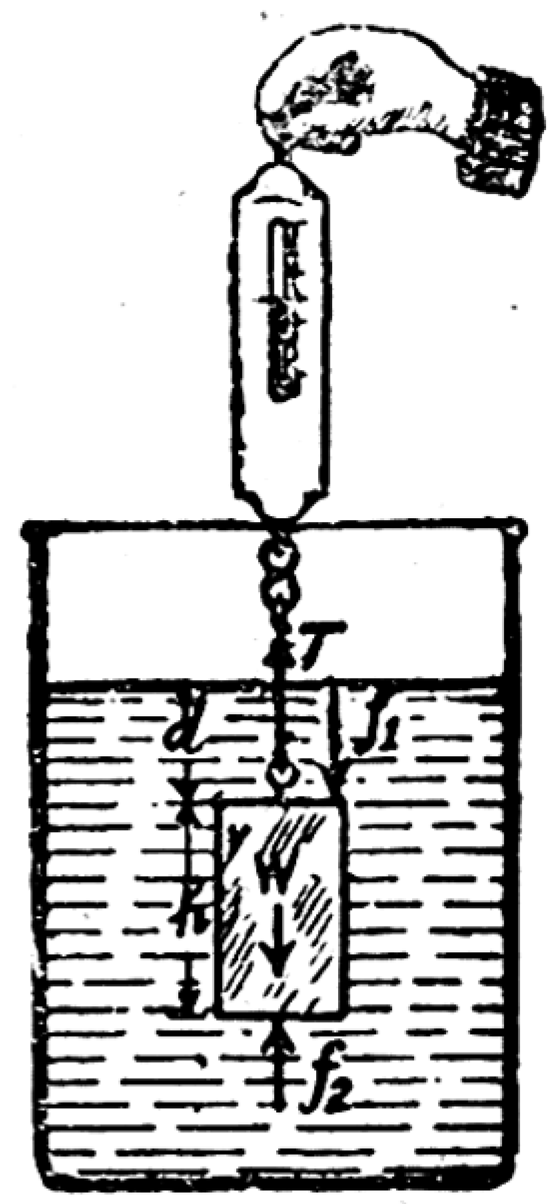


Fig. 102.

Downward forces
(1) The real weight of the block, say W grams.
(2) The downward thrust due to pressure of the liquid on the top face, say f_1.

Upward forces
(3) The tension T of the spring, which measures the apparent weight.
(4) The upward thrust due to liquid on the lower face, say f_2.

Since the body is in equilibrium, these forces must balance, *i.e.*,

$$W+f_1=T+f_2 \quad \dots\dots\dots\dots \quad (i)$$

or

$$T=W-(f_2-f_1) \quad \dots\dots\dots\dots \quad (ii)$$

But f_2-f_1 is the resultant upward thrust due to the liquid acting on the block, hence the above equation can be written as

Apparent weight=Weight in air−Resultant upward thrust of the liquid.

Now let us see what is the resultant upward thrust of the liquid equal to.

The downward thrust acting on the top face of the block is equal to the pressure $d \times \rho$ at that depth multiplied by the area, a of the face, *i.e.*

$$f_1 = d\rho a$$

The upward thrust acting on the lower face of the block is likewise equal to the pressure at that depth, $(d+h)\rho$ multiplied by a the area of the face, *i.e.*,

$$f_2 = (d+h)\rho a.$$

$$\therefore \quad f_2 - f_1 = h\rho a \quad \ldots\ldots\ldots \quad (iii)$$

But ha is the volume of the block and hence the volume of the liquid displaced ; therefore $h\rho a$ is the weight of liquid displaced. Substituting this value in equation (*ii*) we find

The apparent weight = Weight in air—The weight of the liquid displaced.

This result is precisely what we have called above the Principle of Archimedes.

At this stage arises a very interesting question. Does the water exert an upward thrust on a body immersed in it without itself experiencing a downward thrust of the same amount ? The answer is, no. For water does experience a downward thrust of the same amount. We have already learnt on page 125 that as soon as a rod is immersed in a beaker placed on a weighing machine, weight increases. Let us perform that experiment again. Place a vessel two-thirds full of water on the pan of a weighing machine. Let the machine indicate a weight of 300 gm. Suppose we immerse in water in the beaker $\frac{1}{4}$th part of the metallic block referred to in § 119, p. 127, without letting the block touch the beaker and observe the reading on the machine. It will be seen to be 325 gm. *i.e.*, greater by 25 gm. which is just the weight of water displaced by the block. This example shows that the upward thrust on the block and the downward thrust of the block on the water are equal and opposite. This is a very interesting example of action and reaction.

It may be remarked here that the Archimedes Principle affords *an accurate method for finding the volume of a solid body of irregular shape.* The volume found by this method is accurate to $\frac{1}{100}$th part of a cubic centimetre.

120. Floating Bodies.—The weight of the liquid displaced by a body may be greater than, equal to, or less than its own weight. If the weight of the solid body be W and that of an equal volume of the liquid be w, then by the Principle of Archimedes, *the weight of the solid body in the liquid will be* $W-w$ grams. The value of this expression will be negative, zero, or positive, according as W is smaller than, equal to, or greater than w.

(1) Let us first consider the case when the *apparent* weight is zero. Obviously in this case $W=w$ *i.e.*, the weight of the body = the weight of the liquid displaced. When such a body is immersed in a liquid, its weight is balanced by the upward thrust of the liquid and hence the body just floats.

(2) If the apparent weight of the body, *i.e.*, $W-w$*, has a negative value,* the body when immersed will experience an upward thrust greater than its weight and hence will rise to the surface and begin to

float when the weight of the liquid displaced is just equal to its own weight.

(3) If the value of the expression $W-w$ is positive, it simply means that the downward pull of gravity on the body is greater than the upward thrust acting on it when it is immersed in the liquid. Under the influence of this resultant downward force the body moves downwards. *The weight of the body in the liquid will be $W-w$ grams.*

It is clear from what is said in paragraph (2) that a body floats in water with a part of it above the surface if its weight is less than the weight of water that it displaces when completely immersed. This is what happens in the case of a ship. Its hull is so shaped that the weight of steel and other parts together with the weight of the cargo and air inside is equal to the weight of water displaced.

A fish by means of distension of the air bags in its body can change its volume without increasing its weight appreciably and thereby increasing the upward thrust on it. Hence by distending the air bags it moves upwards from the lower depths of water to the surface. A man can float on the surface of a river more easily by filling his lungs with air and expanding his chest thereby than when he has emptied them by breathing out.

A whale weighing about 200 tons on land weighs only about 5 tons in water. It is on account of Archimedes' Principle that it can move about, sink or rise without much effort. An animal of this weight will have difficulty to move about on land.*

120a. Equilibrium of Floating Bodies.—We have said above that in the case of a floating body the weight of the body is equal to the weight of the liquid displaced, and hence the resultant force acting on it is zero. This is a condition for zero motion of translation. But it may so happen that the two forces, the upward thrust and the downward pull of gravity, are equal and opposite and act along different lines, and thus give rise to a couple. For equilibrium, therefore, it is necessary that not only the resultant should be zero, but also that the forces should act along the same straight line so as to have no moment.

We thus arrive at the following conditions for the equilibrium of a floating body :—

(1) The weight of the body should be equal to the weight of the liquid displaced.

(2) The centre of gravity of the body and that of the fluid displaced (or centre of buoyancy) must be in the same vertical line.

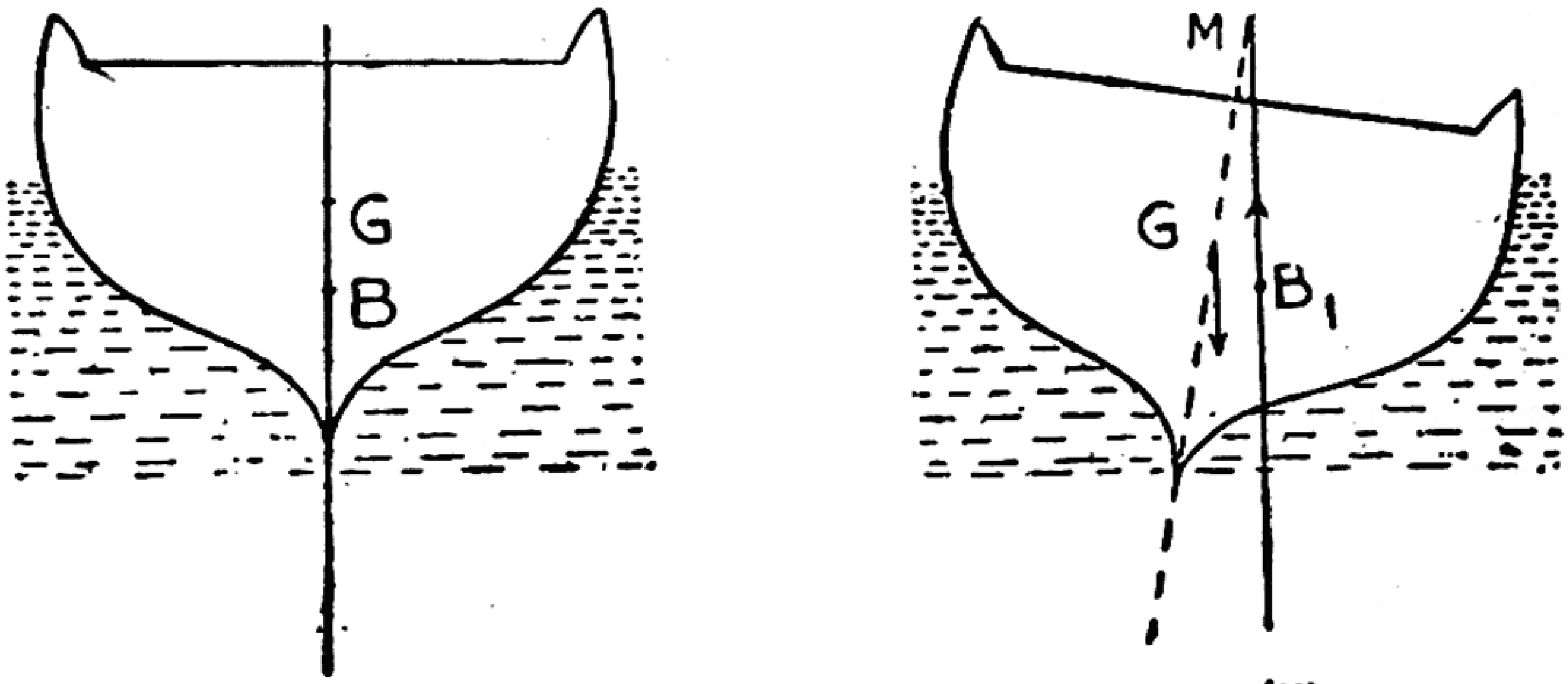
These two conditions are necessary for equilibrium, but they are not sufficient for stability.

Since the stability of equilibrium of floating bodies is very important on account of its application to ships, we shall briefly explain the condition for stable equilibrium of a floating body. Let us consider a ship floating in water. Fig. 103 represents the vertical section passing through G, the centre of gravity of the ship. Let the centre of buoyancy be at B [Fig. 103 (i)]. As long as the ship floats upright, the points B and G, are in the same vertical line. But when it is inclined, the

*The weight of an elephant rarely reaches 8 tons. Hence such an animal will weigh on land as much as 25 to 30 elephants together would do.

centre of buoyancy shifts to some point, say B_1 [Fig. 103 (*ii*)]. The weight acts at G but the thrust of water acts at B_1. This gives rise to a couple tending to bring back the ship to its original position. Hence the equilibrium is *stable*.

The point M, where the vertical line through B_1 meets the line passing through G and the original position of buoyancy, is called the **meta-centre.** It is a very important point. When it lies above G, the ship is in stable equilibrium. When it lies below G, the couple tends to move the ship further away from the position of equilibrium.



(i) (ii)

Fig. 103.

The ship is then in an unstable equilibrium. Therefore, the condition of stable equilibrium is *that the meta-centre must lie above the C.G.* of the floating body.

121. Submarine.—It is a ship so designed that ordinarily it floats on the surface of the sea but can be submerged in water when desired. You will ask how is it possible ? The answer is, by varying the specific gravity of the submarine. Let us see how it is done. In the lower part of the submarine are placed compressed-air cylinders as shown in Fig. 104. On either side are placed ballast tanks.

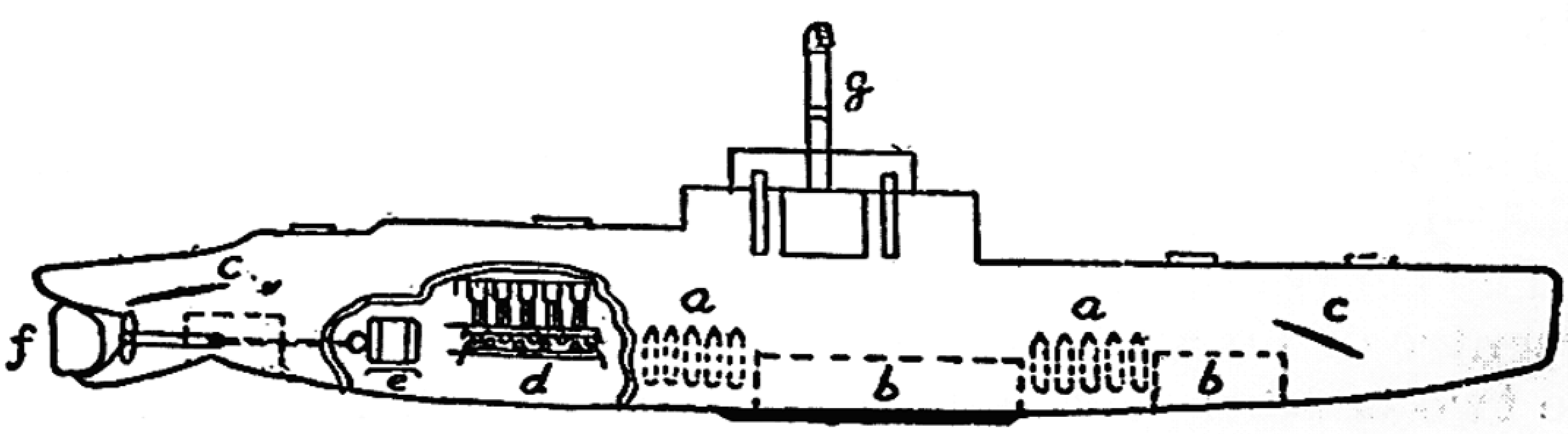


Fig. 104. Submarine.

(*a, a*) Compressed-air cylinders
(*b, b*) Ballast tanks.
(*c, c*) Horizontal rudders (called hydroplanes).
(*d*) Diesel engines.
(*e*) Electric motor for driving propeller.
(*f*) Vertical rudder.
(*g*) Periscope.

Ordinarily these ballast tanks are full of air and the specific gravity of the submarine is less than water with the result that it floats on the surface like other ships. But as soon as the ballast tanks are filled with water the submarine becomes slightly heavier than the water

displaced and hence it begins to sink below the surface. To dive, a screw propeller worked by an electric motor is used. The angle of descent is controlled by tilting the horizontal rudders which are fixed on either side of the submarine. To rise to the surface the water in the ballast tanks is forced out by compressed-air which is allowed to enter the tanks through a pipe from the compressed-air cylinders. When the submarine comes on the surface the cylinders are filled with compressed-air again. On the surface the submarine is propelled by Diesel engines and attains a very great speed. Under water it is driven by electric motors using power from large storage batteries and can make only 8 knots/hour. There is a pressure gauge on the side of a submarine. By its help the depth below the surface can be read directly. By the use of periscope the captain is able to scan the surface of the sea when the boat is under water.

Example.—A submarine weighs 2400 tons and has a volume of 112,000 c. ft. Find the capacity of the ballast tanks which must be filled with water so that the submarine may sink in water when desired. Take the weight of a cubic foot of water as 62·5 lb.

The volume of water displaced when the submarine is submerged

$$=112{,}000 \text{ c. ft.}$$

$$\text{The weight of this water} = \frac{112{,}000 \times 62{\cdot}5}{2240} \text{ tons.}$$

$$=3125 \text{ tons.}$$

The weight of the water to be pumped into tanks so that the submarine may sink

$$=3125-2400 \text{ tons.}$$

$$=725 \text{ tons.}$$

$$\therefore \text{ The capacity of the tanks} = \frac{725 \times 2240}{62{\cdot}5} \text{ c. ft.}$$

$$=25{,}984 \text{ c. ft.}$$

121a. Water Wheels and Turbines.—In §55 we said that water flowing downstream is capable of doing work. This natural source of power was used to save human labour even in prehistoric times. But on account of the crude construction of the wheels used to harness water power in the ancient days, the power developed by them was very limited, with the result that as soon as the steam engine was discovered they fell into disuse. On account of considerable improvement in their design their use is becoming popular again.

The machines commonly used for obtaining power from the falling or flowing water can be divided into two types (*i*) the water wheels and (*ii*) the water turbines. We shall first discuss water wheels.

In the ordinary water wheel the water is conducted to the wheel by means of a channel and if it is taken to the top of the wheel, the wheel is of the overshot type and if it is taken to the bottom the wheel is of the undershot type. We shall briefly describe the working of the undershot type.

In this case the flowing water is made to impinge with a high velocity on the paddles of the wheel when they are in the lowest position (Fig. 105). The water imparts a part of its kinetic energy to the paddles and flows on with a lower velocity. The greater the work done per second by the wheel the lower the velocity of the water leaving the paddles. As the quantity of water passing different cross-sections of the channel must be the same the depth beyond the wheel must become greater. The part beyond the wheel is called the "Tail end". The first part where the depth is smaller but velocity great is called the "Penstock".

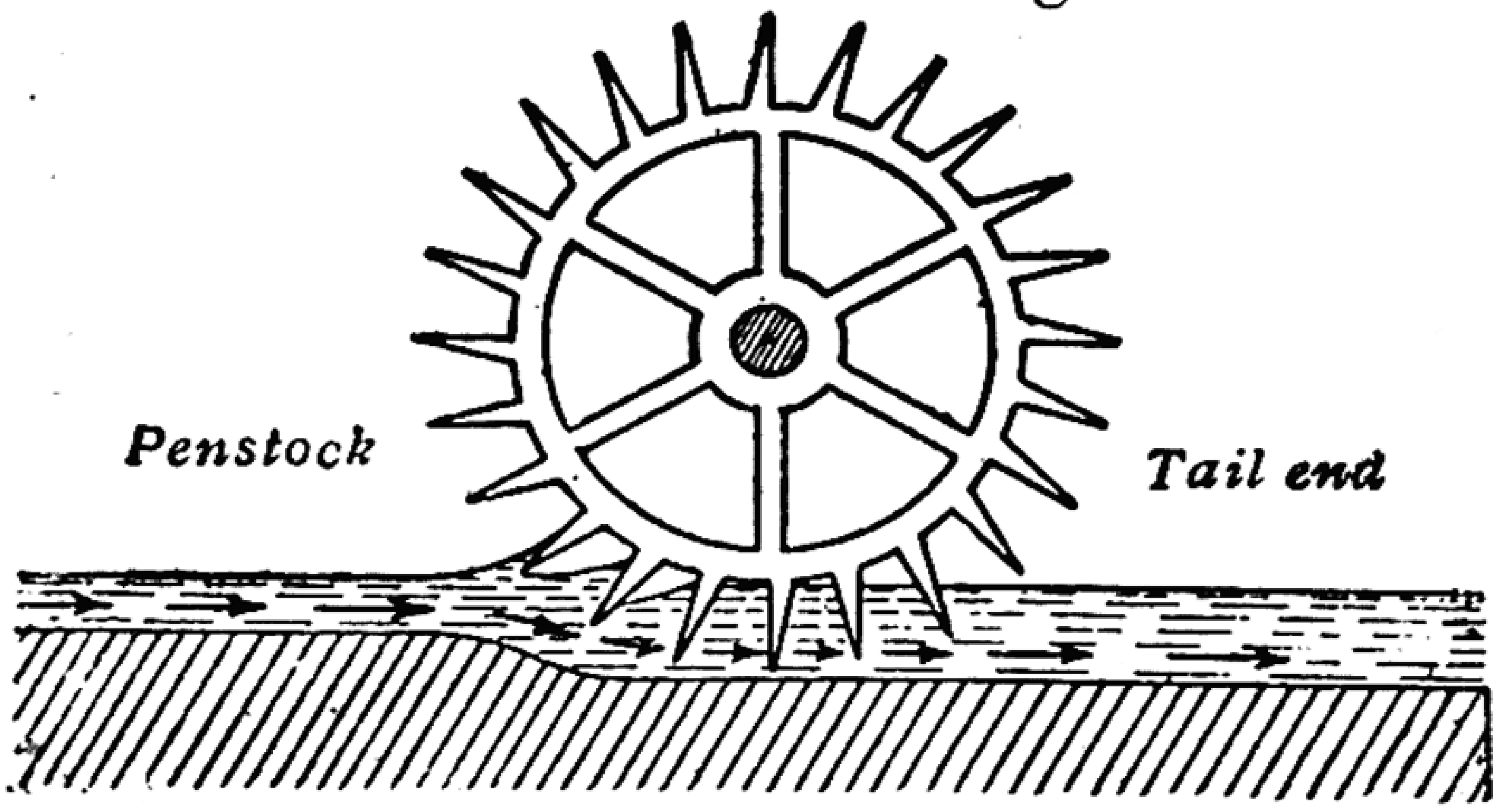


Fig. 105. Undershot Water-Wheel.

It may be pointed out here that the theoretical efficiency of an undershot wheel is 50% though in practice it seldom exceeds 25%.

The old type of an overshot wheel is also inefficient because the pressure is applied to but a few cups at a time.

Pelton Wheel.—It is a modern type of machine which is capable of developing considerable power and has an efficiency of 90%. It has buckets in place of flat paddles and each bucket has a ridge across its middle [See Fig. 106 (*a*)]. Water from a considerable height is conducted through a pipe to a nozzle which ejects it in the form of a jet along the ridge of each bucket as shown in Fig. 106 (*b*).

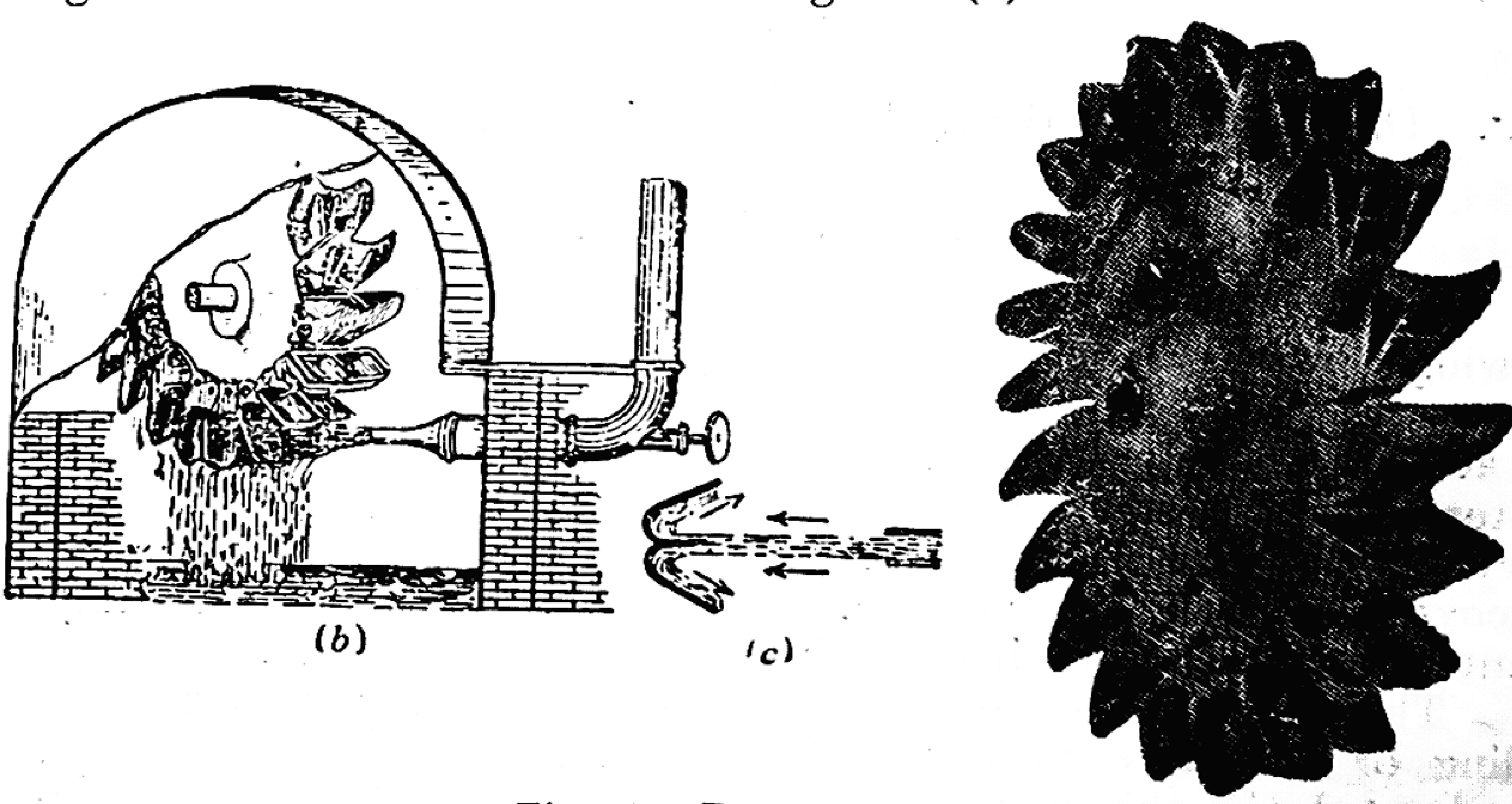


Fig. 106. Pelton Wheel.

The jet divides and circles round the curved buckets, gives up its momentum to them and is then discharged [See Fig. 106 (*c*)]. The shape of the buckets is so designed as to abstract the maximum amount of the momentum from the water. In the case of the best design the water leaves the buckets with almost zero velocity, thus transferring all its kinetic energy to the buckets.

The power developed by a Pelton wheel depends upon the head of water, the quantity of water and the size of the wheel. If the head of water is 800 ft. a Pelton wheel 8 ft. in diameter can easily develop 3,000 H.P. A Pelton wheel can be made so small as to give $\frac{1}{5}$th H.P. or so large as to give 16,000H.P. It may be enclosed in a casing or may be opened. Pelton wheels are used in many power houses in the western part of the United States.

Water Turbines.—If the head of the water is considerable Pelton wheels are used. But if the head is low, water turbines are used. A water turbine consists of curved blades mounted on a shaft completely enclosed in a casing. A set of guide blades fixed to the inside of the casing direct the water at right angles or nearly so against the blades mounted on the shaft. The water pushes against these blades and makes them rotate and along with them the shaft.

A water turbine has efficiency of 90% and can work even when the head of the water is as low as 10 ft. For heads of water less than 16 ft. the vertical shaft type is used, in all other cases the horizontal type is preferred.

EXERCISES

1. The press-plunger of a Bramah Press is 20 inches in diameter, and is required to lift 100 tons. What should be the size of the pump-plunger if the mechanical advantage of the handle is 10 and the power used is 10 lb. wt.?

It is evident that the force on the pump-plunger is 100 lb. wt. on account of the mechanical advantage of the lever. If the radius of the pump-plunger be r, using the relation $\frac{W}{R}=\frac{r'^2}{r^2}$ we get,

$$\frac{100\times 2240}{100}=\frac{10\times 10}{r^2}$$

$$r^2=\frac{10}{224},$$

or $$r=0{\cdot}211,$$

or $$\text{diameter}=0{\cdot}422 \text{ inch.}$$

2. Define the pressure at a point in a fluid. On what factors does it depend?

Oil to a depth of 15 cm. is floating on water in a jar. If the pressure at a point 5 cm. below the surface of the water due to oil and water is 14·75 grams per sq. cm., what is the density of the oil?

Ans. 0·65.

3. The pump and press-plunger of a Bramah Press have diameters of 1 inch and 12 inches respectively. The downward force on the pump-plunger is 56 lb. What is the force exerted by the press-plunger? If the length of the stroke of the pump-plunger is 1·5 inches, through what distance will the press-plunger rise after 12 strokes?

Ans. 3·6 tons ; 0·125 inch.

4. Calculate the difference of pressure between two points one mile apart lying on the same vertical line in sea water. Express the

result in lb. weight per sq. inch. (Given that a cubic foot of sea water weighs 66·1 pounds.) *Ans.* $2\,423\times10^3$.

5. How would you prove experimentally that a liquid exerts pressure in all directions? A tall vessel provided with a tap at the side near the bottom is filled with water and is made to float upright on a thick plate of cork. What will happen when the tap is opened? (*Cal. Univ.*)

6. A canal lock gate is 20 ft. wide and 9 feet deep. Find the magnitude of the thrust on it, given that the water in the canal is in level with the top of the gate and that the density of water is 62·4 lb. per cubic foot. *Ans.* 13·5 tons approx.

7. Find the total pressure on the bottom of a tank 4 ft. long, 2 ft. broad and 2 ft. deep when filled with water. Density of water may be taken as 62·5 pounds per cubic ft. *Ans.* 1,000 lb. wt.

8. If a resistance of one ton is overcome by a force of 5 lb. wt. applied to the pump-plunger of a Bramah Press, and the diameters of the plungers are in the ratio of 8 to 1, find the ratio of the arms of the lever employed to work the pump-plunger. *Ans.* 7 to 1.

9. Find the ratio of the areas of the pistons of a Bramah Press, if a force of 12 lb. wt. produces a thrust of 3 tons, the mechanical advantage of the lever being 7. *Ans.* 80 to 1.

10. Water is supplied to a house from a reservoir 810 ft. above sea-level. Find in lb. wt. per square inch the pressure in a pipe at the top of a house 60 ft. high above sea-level. *Ans.* 325·52 lb. wt.

11. In the water supply arrangement of a town a pipe 2 ft. in diameter and 50 ft. high is used as a reservoir. Will it make the pressure double if the pipe be 4 ft. in diameter?

12. What is Buoyancy? Will it be the same for a brick and a wooden block of the same size pushed under the surface of water? Will it vary with the depth?

13. Explain the principle of Archimedes. Does it apply to the movement of fishes? Explain as clearly as you can how a fish can move towards the surface or the bottom of the sea?

14. It is said that it is easier to swim in an ocean than in a river. Is it a fact? Give reasons for your answer.

15. "A man is found to be lighter by 3 or 4 ounces when weighed in air than in vacuum." Is it correct? If so, explain why?

16. A whale on land weighs 400 tons. The average density of its tissues is 1·05 gram per c.c. while sea water is 1·025. Find its weight in water. *Ans.* 9·5 tons.

17. A body of density D is let free in a liquid of density d where D is greater than d. Prove that the downward acceleration of the body while sinking in the liquid is given by the relation

$$a=\left(1-\frac{d}{D}\right)g.$$

18. What fraction of the volume of a floating iceberg will be outside the surface of water if the density of ice were ·918 gm./c.c. and that of sea water 1·025 gm./c.c. *Ans.* 107/1025.

CHAPTER X

Density

122. Take cubes of different materials, but of the same size say of silver, brass, iron, glass and wood, and weigh them. You will find that the silver cube is heaviest and the wooden eube lightest. The brass cube will be found to be lighter than the silver cube but heavier than others whereas the iron cube will be lighter than the silver and brass cubes but heavier than glass and wooden cubes. Their weights are different, although their volumes are equal. This shows that *volume for volume* silver is heavier than brass, brass heavier than iron, iron heavier than glass, and glass heavier than wood. Mark the words "volume for volume". Unless volume is specified it is meaningless to say that silver is heavy and wood is light. For a cubic foot of wood will be very much heavier than a cubic inch of silver. To find which substance is heavy and which is light we compare the weights of their unit volume and a substance which has a greater weight is said to be denser. For instance in the example given above silver is said to be denser than brass, brass denser than iron, iron denser than glass, and so on.

Mass per unit volume of a body is called **density.** If the mass of a body be M and volume V, the density D is given by the relation

$$D=\frac{M}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

Evidently the density of a substance depends upon the unit of the mass and volume. In the C.G.S. system, density of a substance is equal to the mass in grams of one c.c. of it. Since the mass of 1 c.c. of water is one gram,* the density of water is 1 gram per c.c. In the F.P.S. system the density of water is 62·4 (or, more accurately, 62·43) lb. per cu. ft.

123. Specific Gravity.—It is obvious from what is said above that the density of a substance can have many numerical values. To remember them all would be difficult. To avoid this difficulty the idea of relative density or specific gravity is introduced. By specific gravity or relative density we mean the ratio of the density of a substance to the density of a standard substance in the same units. The specific gravity is a number and has only one value for a substance. In the case of solids water is taken as the standard substance whereas in the case of gases hydrogen or air is used as the standard.

Take a cube of brass 100 c.c. in volume, and weigh it. Its weight will be found to be 850 grams. Density of brass, therefore is 8·5 gm./c.c. Density of water in the same units, as said above, is 1 gm./c.c. The

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*To be more accurate, the mass of 1 c.c. of distilled water at 4°C, is equal to 1 gram.

relative density or specific gravity of brass, is hence, 8·5. No matter what units of volume and mass are used the *relative density* of brass will always come out to be 8·5.

We have said that

$$\text{Specific gravity} = \frac{\text{Density of the substance}}{\text{Density of water}} \quad . \; . \; . \; (ii)$$

Since density of water is 1 in the C.G.S. system, it follows from relation (*ii*) that in the C.G.S. units the specific gravity is numerically equal to the density of the substance. It is different when other units are used.

The relation No.(*ii*) enables us to find the density of a substance in any units we like. To do this we have to re-write it in the following form :—

$$\textit{Density of a substance} = \textit{sp. gravity} \times \textit{density of water} \quad . \; . \; . \; (iia)$$

Let us find the density of iron in the F.P.S. units given that sp. gravity of iron is 7·4 and that the density of water in the F.P.S. units is 62·43 lb. per cu. ft.

$$\left.\begin{array}{l}\text{The density of iron}\\ \text{in F.P.S. units}\end{array}\right\} = 7{\cdot}4 \times 62{\cdot}43$$

$$= 462 \text{ lb. per cu. ft. (approx.)}$$

Equation(*ii*) can be written as

$$\text{Specific gravity} = \frac{\text{mass of unit volume of substance}}{\text{mass of unit volume of water}}$$

$$= \frac{\text{mass of a given volume of substance}}{\text{mass of the same volume of water}} \quad \ldots (iii)$$

A glance at relation (*iii*) tells us that if we know the specific gravity and the volume of a body we can find its mass. For

Mass of a body = specific gravity × mass of an equal volume of water
= specific gravity × V × mass of unit volume of water,

where V is the volume of the body.

124. Determination of Density.—We shall briefly discuss some of the methods without explaining the practical details, for which the student must refer to books on Practical Physics. In order to know the density of a substance we must find both its mass and volume ; the ratio of the two gives the density.

(*a*) *Density of a Solid Body.*—To find the mass of the body weigh it in air.

To find its volume measure its linear dimensions and calculate the volume, if the body is regular in shape. If it is irregular, slide it into a graduated cylinder containing water (or some other liquid if it be soluble in water) up to a certain mark. Note the rise in the level of the water. The change in the level gives the volume.

If the irregular solid be lighter than water, tie it to a heavy body of known volume in order to make it sink. Determine the combined volume, and subtract from this the known volume of the heavy body. The difference is the volume of the irregular body.

Knowing the mass and the volume of the body find the density by using the relation

$$D=\frac{M}{V}.$$

The method of finding volume by displacement is not an accurate one. If the volume is to be determined accurately use Archimedes' Principle.

(*b*) *Density of a Liquid.*—Take a vessel of known capacity and weigh it. Fill it with the liquid and weigh it again. The difference gives the weight of the liquid. Dividing the weight by the volume, find the density.

We shall now pass on to the various methods of finding the specific gravity of different bodies. In these methods we do not directly measure the volume.

125. Determination of Specific Gravity.—In order to determine the specific gravity of a body we find its weight and the weight of an equal volume of water; the ratio of the two weights gives the specific gravity.

To find the specific gravity of a solid, weigh it first in air and then in water; the loss in weight is, by the Principle of Archimedes, equal to the weight of an equal volume of water.

$$\therefore \text{ Sp. Gr.}=\frac{\text{Weight of body in air}}{\text{Loss of weight in water}}.$$

Example.—A solid weighs 58·56 gm. in air and 51·24 gm. in water. Find its specific gravity.

$$\text{Loss of weight in water}=58{\cdot}56-51{\cdot}24=7{\cdot}32$$

$$\therefore \text{ Specific Gravity}=\frac{58{\cdot}56}{7{\cdot}32}=8{\cdot}0$$

If the given solid be lighter than water, a metal piece heavy enough to sink the solid is tied to it before immersing it in water. This metal piece is called a *sinker*. Knowing the weight of sinker in water and knowing the weight of sinker and solid in water, we can find the loss in weight of the solid in water and hence find the specific gravity.

Let the weight of solid in air $=w$
" " " " sinker in water $=s$
" " " " solid and sinker in water $=w'$
The combined weight of *solid in air* and the sinker in water (*i.e.*, $s+w$) $=w''$
The loss of weight of solid in water $=w''-w'$

Therefore $$\text{Sp Gr.}=\frac{w}{w''-w'}.$$

Example.—The weight of a piece of cork in air was found to be 0·67 gm. and the weight of sinker and cork in water 52·84 gm. The weight of sinker alone in water was 55·54 gm. Find the specific gravity of cork.

The weight of cork in air $=0{\cdot}67$ gm.

The combined weight of cork in air and sinker in water

$$=55{\cdot}54+0{\cdot}67=56{\cdot}21.$$

Hence the loss of weight in water

$$=56{\cdot}21-52{\cdot}84=3{\cdot}37.$$

$$\therefore \text{ Specific Gravity}=\frac{0{\cdot}67}{3{\cdot}37}=0{\cdot}2$$

The Specific Gravity of a Liquid.—(*i*) *Loss of Weight Method.* In this method we make use of the Archimedes' Principle as explained below : Weigh a given piece of solid, (say a brass cylinder) in air, and let the weight be a. Weigh it in water, and let the weight be w. Finally weigh it in the liquid, and let the weight be l. The loss of weight in water is $a-w$. This is the weight of an equal volume of water. Similarly the weight of the same volume of the liquid is $a-l$, hence the

$$\text{Sp. gravity}=\frac{a-l}{a-w}.$$

(*ii*) *Specific Gravity Bottle Method.*—To determine the specific gravity of a liquid accurately we use a *specific gravity bottle*, which consists of a small bottle with a tightly fitting glass stopper (Fig. 107) having a fine hole bored through it.

To find the specific gravity of a liquid with its help proceed as below :—

Weigh the empty sp. gravity bottle. Fill it up completely with the liquid and push in the stopper. Wipe off the liquid that overflows through the hole. Weigh it again ; the difference between the two weights gives the weight of the liquid filling the bottle ; let it be l.

Fig. 107.

Pour off the liquid and dry the bottle with hot air. Fill it next with water in the same manner as in the case of the liquid. Let the weight of water filling the bottle be w.

$$\text{Sp. gravity}=\frac{l}{w}.$$

It should be noted that in this method we take equal volumes of the two liquids.

The specific gravity bottle method is also used for determining the specific gravity of powdered solids.

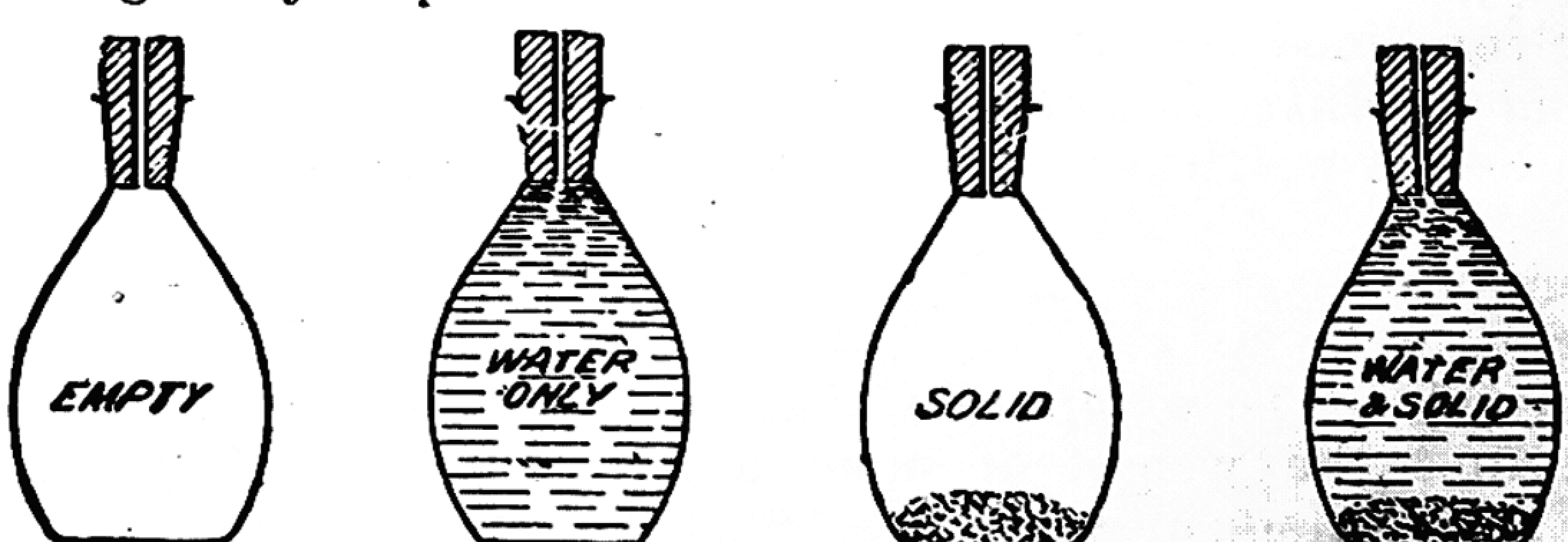

Fig. 108.

The Specific Gravity of a Powder.—First weigh the empty bottle,

then fill it with water and weigh it. The difference in the two weights gives the weight of water filling the bottle. Let it be w. Empty the bottle and dry it. Put in the solid and weigh. Subtract the weight of the bottle and get the weight of the solid ; let it be a. Fill it up with water now, the solid also being in the bottle, and weight it ; the difference in this weight and the last weight (of solid along with the bottle) gives the weight of water filling the space *unoccupied* by the solid. Let us call the difference in weights. e. It is evident that the weight of water having the same volume as the solid is $w-e$.

Hence $$\text{Sp. gravity} = \frac{a}{w-e}.$$

If the solid be soluble in water (as, for instance, powdered common salt) use in place of water a liquid like kerosene oil or turpentine oil in which the solid does not dissolve. Find the relative density of the solid with respect to the liquid in the manner given above and multiply this with specific gravity of the liquid*. The result will give the specific gravity of the solid with respect to water.

126. Hydrometers.—No doubt we can determine the specific gravity of a liquid with the help of a sp. gravity bottle accurately, but it is not a quick method. For commercial and other purposes we need a method, which should be simple and quick although it may give only approximate value of specific gravity. For such purposes we use **hydrometers.** They all depend upon the *principle of floatation i.e.*, when a body floats in a liquid its weight is equal to the weight of the liquid displaced. Evidently either the hydrometer may have constant weight, in which case it will sink to different depths in different liquids, or its weight may be changed in order to sink it to the same level. Corresponding to these two alternatives there are two types of hydrometers.

1st Class : Hydrometers of constant weight but of variable immersion : and

2nd Class : Hydrometers of variable weight but of constant immersion.

We shall consider both these types one by one.

127. Hydrometers of Variable Immersion but of Constant Weight.—In order to understand the principle of hydrometers of this type, let us consider a tube closed and loaded at the lower end so that it floats vertically in a liquid. Place a millimetre scale in it with the zero of the scale at the bottom. Float it in different liquids ; as the weight of the hydrometer remains unchanged, *in each case the weight of the fluid displaced is the same*, but since

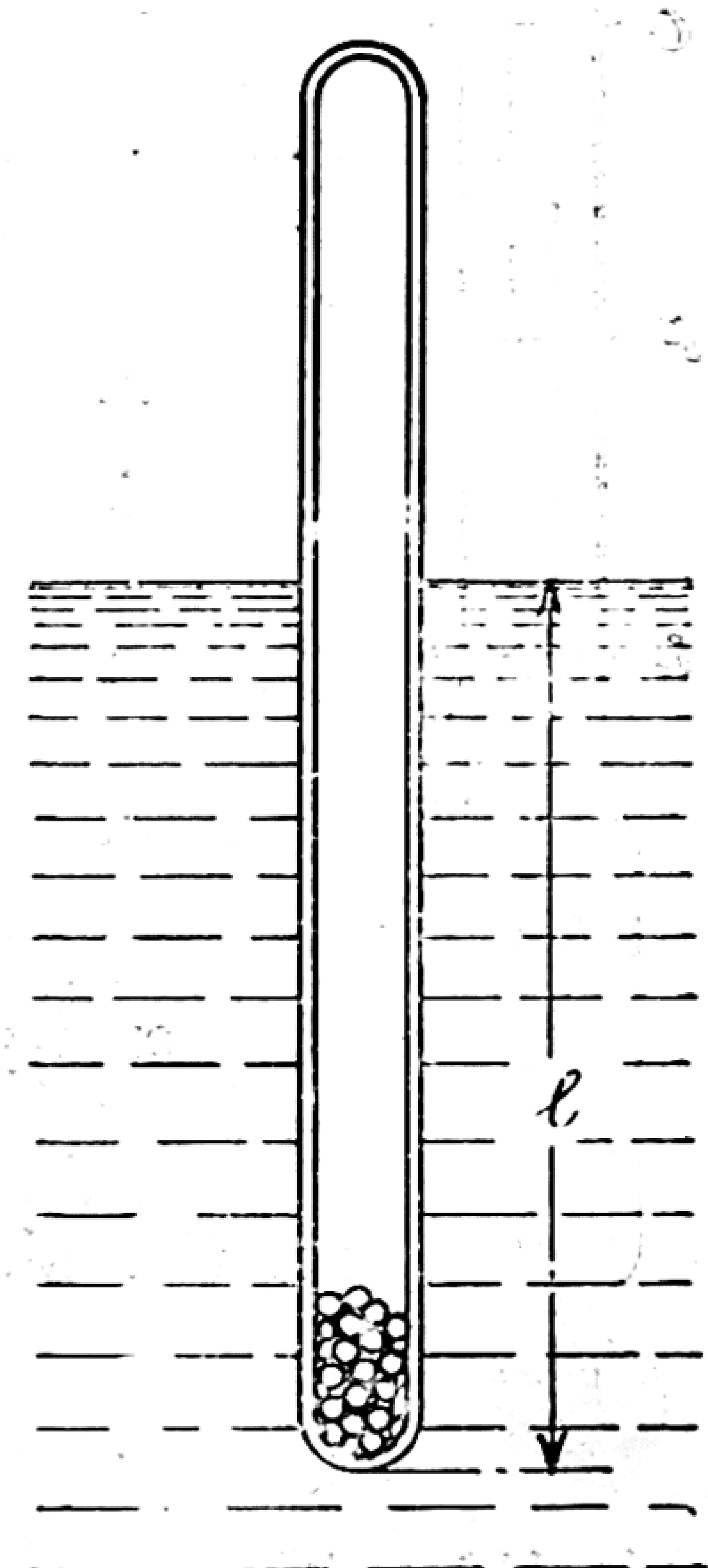

Fig. 109.

$$\text{*}\frac{\text{density of solid}}{\text{density of water}} = \frac{\text{density of solid}}{\text{density of liquid}} \times \frac{\text{density of liquid}}{\text{density of water}}$$
$$= \left(\begin{array}{c}\text{sp. gravity of solid}\\ \text{with respect to liquid}\end{array}\right) \times \left(\begin{array}{c}\text{sp. gravity of}\\ \text{the liquid.}\end{array}\right)$$

the densities are different the volumes are different. Let us suppose in one case the hydrometer sinks to depth l, and in the second case to l'. If a be the cross section of the tube, the volumes displaced will be equal to al and al' respectively. Let the densities of the liquids be d and d'; the masses of the liquids displaced will be equal to

$$ald, \text{ and } al'd',$$

As they are equal we get

$$ald = al'd'$$

$$\therefore \quad \frac{d'}{d} = \frac{l}{l'}.$$

If the first liquid be water, d will be equal to one, and therefore d' will give the specific gravity of the second liquid and it is inversely proportional to the lengths of the tube submerged.

In this form a hydrometer is not sensitive.

To increase the sensitiveness hydrometers are made with a long narrow stem ending on the lower side in two bulbs. The upper one is designed to increase the volume of the hydrometer whereas the lower one is filled with lead shot or mercury to float the hydrometer upright. (see Fig. 110). The narrow steam causes a considerable change in level when density of the liquid varies slightly.

Let us see how a hydrometer is graduated. Suppose the hydrometer is made for use with liquids of density greater than that of water. Let it sink to mark O in water ($\rho = 1{\cdot}00$) and to L in a liquid of known density ($= \rho$).

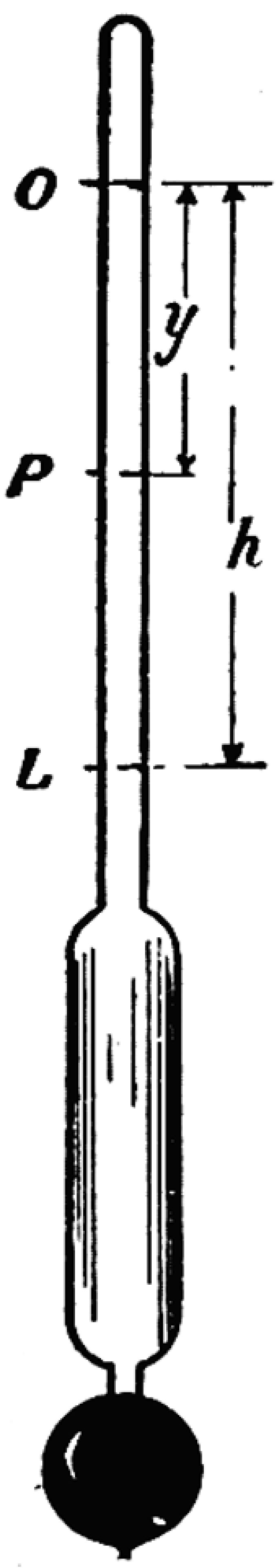

Fig. 110.

Let V = volume of the hydrometer upto the mark O,

v = the volume of 1 cm. length of the stem,

and $OL = h$ cm.

Since the mass of the liquid displaced in each case is the same

$$\therefore \quad V \times 1 = (V - hv)\rho$$

$$\text{or} \quad \frac{V}{\rho} = (V - hv)$$

$$\text{or} \quad V - \frac{V}{\rho} = hv$$

$$\therefore \quad h = \frac{V}{v}\left[1 - \frac{1}{\rho}\right]$$

If h be 10 cms. and ρ be $1{\cdot}50$ let us find what will be the density of the liquid in which the hydrometer sinks to P, half way between O and L.

Substituting 10 and $1{\cdot}5$ for h and ρ respectively in the relation above, we get

$$10 = \frac{V}{v}\left[1 - \frac{1}{1{\cdot}5}\right]$$

$$\text{and} \quad 5 = \frac{V}{v}\left[1 - \frac{1}{\rho}\right]$$

Dividing the first equation with second we get

$$2 = \frac{1 - \frac{1}{1{\cdot}5}}{1 - \frac{1}{\rho}} = \frac{0{\cdot}5/1{\cdot}5}{1 - \frac{1}{\rho}}$$

or
$$1-\frac{1}{\rho}=\frac{0 \cdot 5}{3}$$

or
$$\frac{1}{\rho}=\frac{2 \cdot 5}{3}$$

or
$$\rho=1 \cdot 20$$

This shows that the first 5 cm. of stem below O represent an increase of density from 1·00 to 1·20 while the next 5 cm. from P to L represent an increase from 1·20 to 1·50. The graduations in other words *get progressively closer together as we go down the scale.*

The method of graduation is not the same in all types of hydrometers. For instance, in Beaume's Hydrometer the scale is so divided that for liquids lighter than water, the density is given by the formula

$$D=\frac{144 \cdot 3}{134 \cdot 3+r}$$

where r is the reading on the scale, and for heavier liquids the density is given by the relation

$$D=\frac{144 \cdot 3}{144 \cdot 3-r}$$

In Twaddle's hydrometer, which is used for liquids heavier than water, the stem is graduated in such a way that the reading, divided by 200 and added to 1, gives the specific gravity of the liquid, *i.e.*,

$$D=1+\frac{r}{200}.$$

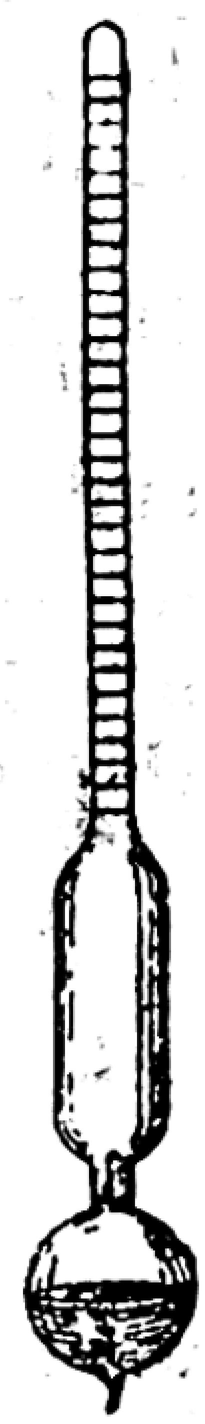

Fig. 98.

128. Lactometer.—The best-known constant-weight hydrometer is the lactometer which is used to test milk, the specific gravity of which varies from 1·029 to 1·035. The specific gravity of cow's milk is approximately 1·030. The scale of sensitive lactometers is divided into 20 parts and runs from 1·020 to 1·040. By floating the lactometer in milk to be tested, one can find whether it is pure or not; for the addition of water makes it lighter, and lowers its density. In cheaper types of lactometers the scale is marked W, 1, 2, 3, and M. These letters denote pure water, 1 part milk, 2 parts milk, 3 parts milk, and pure milk respectively.

A lactometer does not give a conclusive test as to the quality of the milk, for, on account of the removal of fat constituents, which are light, the skimmed milk is heavier than unskimmed milk, and hence by adding water to the skimmed milk we can make its density normal. Evidently the lactometer is unable to detect such a defect. It is only by determining both, the amount of fat and specific gravity that one can determine the real quality of milk.

129. Hydrometers of Constant Immersion but of Variable Weight.—In this case the hydrometer is made to sink just to the same level in different liquids by varying the weight. The most familiar

hydrometer of this type is Nicholson's Hydrometer (Fig. 112). It consists of a hollow metallic cylinder, from the top of which projects a stem made of a steel rod b, ending in a disc A. To the lower end of the cylinder is fixed a cone C loaded with lead in order to float the hydrometer vertically. A mark is made on the stem, and the hydrometer is always immersed to that level.

Fig. 112.

Let us see how we can use this hydrometer. Weigh it, and let its weight be H. Lower it into a cylinder full of water, and place weights in the disc or pan A so that it just sinks to the fixed mark on the stem. Let the extra weight be w. The weight of water displaced is $w+H$. Lower it next into a cylinder full of the liquid whose density is to be determined. Add extra weights so that it sinks to the same mark in this liquid also. Let the extra weights be l. Evidently the weight of the liquid displaced is $H+l$.

Since the volume is the same in both the cases, we get the following relation :

$$d \times V = H + w,$$

and $$d' \times V = H + l,$$

where d' stands for the density of the liquid, d for that of water and V for the volume of the hydrometer up to the mark and hence for the volume of the liquid or water displaced.

Therefore, $$\frac{d'}{d} = \frac{H+l}{H+w}$$

Since $d=1$, $$d' = \frac{H+l}{H+w}.$$

This hydrometer can be used to determine the density of solids as well. For instance to find the density of a piece of stone proceed as below. Make the hydrometer sink in water up to the mark with the help of extra weights ; suppose you require w grams. Remove these weights from the disc and put the solid on it and add extra weights to sink the hydrometer to the same mark. The total weight on the disc must be w. Subtract the weights added in the second case from w and you get the weight of the solid in air ; let it be a. Next, place this solid in the lower pan, the weight will be less now since the solid is immersed in water. Add more weights in the upper pan so that the hydrometer sinks again to the same mark. Let the weight of the solid in water be m. The loss in weight is $a-m$,

hence sp. gravity$=\dfrac{a}{a-m}$.

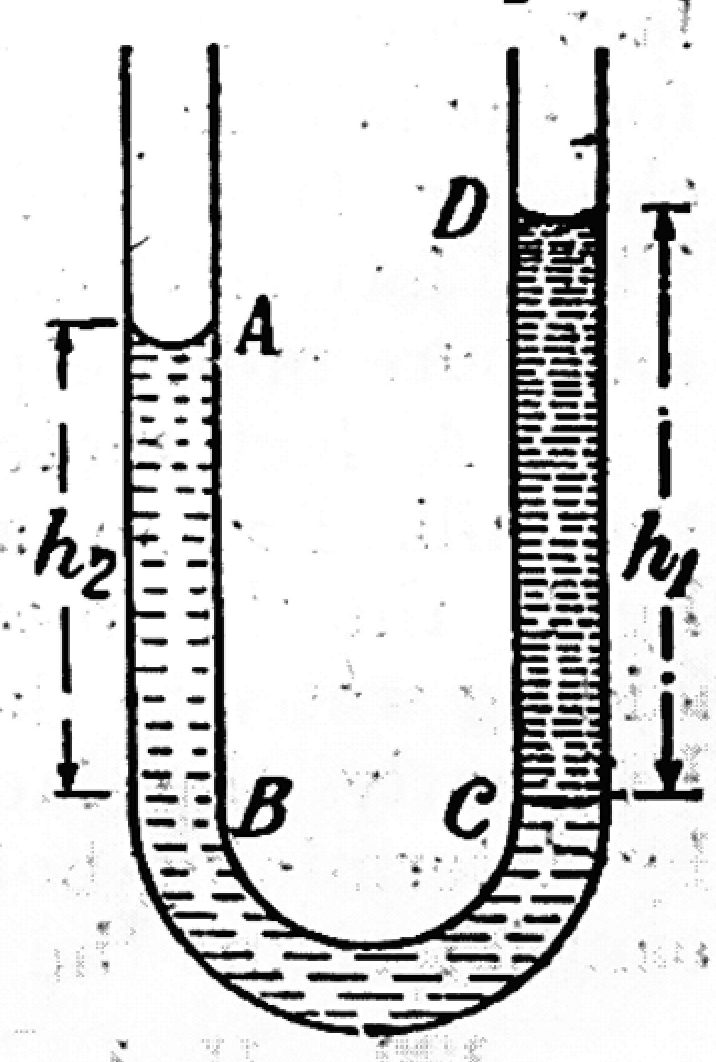

Fig. 113.

130. The U-Tube Method.—This method of comparing the densities of liquids is very simple and at the same time very instructive. If one of the liquids be water, the ratio of the densities gives the sp. gravity of the liquid*. Add first the heavier liquid to the U-tube

*This method can be used only if the liquids do not mix with each other. If they do so, introduce some mercury in the bend, and then add liquids, or use the inverted U-tube method.

and then the second liquid. Let them stand with the surface of separation at C; since the liquids are at rest, the pressure at the points B and C, which are at the same level, is equal. Let the heights of the liquid columns above C and B be h_1 and h_2, and let the densities be d_1 and d_2. The pressure at the point C, due to the column of liquid above it, is $h_1 \times d_1$. Similarly the pressure at B is $h_2 \times d_2$. But, as has been said above, the pressure is the same at B and C.

Hence $$h_1 d_1 = h_2 d_2{}^*$$

or $$\frac{d_1}{d_2} = \frac{h_2}{h_1}$$

i.e., the densities are inversely proportional to the heights.

If d_2 be the density of water, the ratio $\frac{h_2}{h_1}$ gives the sp. gravity of the other liquid.

Table of Densities

(in grams per c. c.)

Alcohol	... 0·79	Iron	... 7·8
Aluminium	... 2·70	Kerosene oil	... 0·80
Copper	... 8·9	Lead	... 11·3
Cork	... 0·24	Mercury	... 13·6
Diamond	... 3·53	Milk	... 1·03
Gasoline	0·63 to 0·72	Oak wood	... 0·8
Glass	... 2·6	Olive oil	... 0·91
Glycerine	... 1·26	Silver	... 10·5
Gold	... 19·3	Turpentine	... 0·87

EXERCISES

1. A piece of gold mixed with quartz weighs 6 grams and has a specific gravity 6·4. If the specific gravity of gold be 19·36 and of quartz 2·15, find the amount of gold in the piece.

Since in the C.G.S. unit the density=the specific gravity, therefore 6·4, 19·36, and 2·15 give the densities. Hence if we divide the mass in grams by these numbers we shall get the volume in c.c.

Let m be the mass in grams of gold in the piece, and $6-m$ of quartz.

Volume of gold$=\frac{m}{19{\cdot}36}$ c.c., and volume of quartz $=\frac{6-m}{2{\cdot}15}$ c.c.

Total volume of the piece$=\frac{m}{19{\cdot}36}+\frac{6-m}{2{\cdot}15}$ c.c.

Since mass of the piece=Volume × Density, we get

$$6=\left(\frac{m}{19{\cdot}36}+\frac{6-m}{2{\cdot}15}\right)\times 6{\cdot}4$$

$$\therefore \quad m=4{\cdot}48 \text{ grams.}$$

2. A tea-pot is filled with water until it just begins to drip from the sprout. The tea-pot then weighs 750 gm. 100 gm. of glass marbles

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*We have not taken into cosideration the pressure due to the atmosphere. Since it is common on both sides, it can be left out.

are then dropped into it displacing some of the water. The weight of the tea-pot is now 810 gm. What is the density of glass ?

Let the volume of 100 grams of glass marbles be v c.c. When they are dropped into the water in the tea-pot v c.c. of water will be pushed out. If no water had been pushed out the weight of the tea-pot, water and marbles would have been 750+100 gm. *i.e.,* 850 gm. Since the weight after v c.c. of water are pushed out is 810 gm., it is obvious that water pushed out weighs 40 gm.

Hence the volume of glass marbles is 40 c.c.

The density $=\frac{100}{40}=2 \cdot 5$.

3. If the height of a column of sulphuric acid in an inverted U-tube is 30 cm. and the height of water-column is 40 cm. find the specific gravity of sulphuric acid.

$$\frac{\text{Density of acid}}{\text{Density of water}}=\frac{\text{Height of water column}}{\text{Height of acid column}}=\frac{40}{30}=1 \cdot 33.$$

4. An alloy of gold and silver is found to have a specific gravity of 16. Find the percentage of silver in the alloy. The specific gravity of pure gold is 19·32 and of pure silver 10·5. *Ans.* 24·7 per cent.

5. Copper has a specific gravity of 8·9. How many grams of copper must be suspended in water in order that its apparent weight in water may be 100 gm. ? *Ans.* 112 66 (approx.)

6. A quart bottle of milk weighs 1,500 gm. The empty bottle weighs 390 gm. The bottle filled to the same height with water weighs 1463 gm. What is the denstiy of milk ? *Ans.* 1·034.

7. If the mass of a hen's egg is 52 grams and its volume is 50 c.c. find its density. *Ans.* 1 04 gm.

8. A cork is floating on the water which fills a graduated cylinder up to the 50 c.c. mark. If a 10 gram weight be placed on the cork what will be the reading of the level of water ? *Ans.* 60 c.c.

9. A closed cylindrical vessel 6 ft. in diameter, 15 ft. long, weighing 5,000 lb. is floating in fresh water with the axis of the cylinder in the plane of the water surface. What weight is it carrying ? *Ans.* 8259 lb.

10. Find the length and sp. gravity of a cylinder which floats in water with 2 inches of its vertical axis out of the water, and with 6 inches out of the liquid whose specific gravity is 1·5. *Ans.* 14 inches and 0·857.

11. The height of a mercury barometer is 760 mm. while glycerine barometer stands at 813 cm. If the specific gravity of mercury is 13 6, find the density of glycerine. *Ans.* 1·27 gm. per c.c.

12. The weight of a submarine boat, which lies damaged and full of water at the bottom of a sea, is 200 tons. If the specific gravity of the material is 7·8, find the total pull which must be exerted by the lifting crane in order to raise the boat from the bottom. (The sp. gravity of sea-water may be taken as 1·025.) *Ans.* 173·72 tons wt.

13. What is wrong with the following statement "Which is heavier, a pound of iron or a pound of cotton wool" ?

14. Two hydrometers of the same weight are to be made, one for use with liquids heavier than water and the other for use with liquids lighter than water. Would there be any difference in the size of the upper bulbs of the two hydrometers ?

Ans. Yes, the hydrometer for lighter liquids should have a bigger bulb.

CHAPTER XI

Liquids in Motion

131. The law of hydrostatic pressure, $p = h\rho g$, explained in the last chapter is found to hold good only when a liquid is at rest ; when it is in motion the pressure is no longer given by this expression. To find the value of pressure at a point inside a liquid in motion we have to take into account other factors as well, besides depth and density. To realise how difficult it is to deal fully with liquids in motion we have to picture to ourselves a river in flood and remember that water not only moves at different speeds across a section of it at a place but has eddies and whirlpools. To apply the second law of motion, $F = ma$, to each drop of water to explain the flow of river with whirlpools, and the difference in velocity from point to point requires the highest skill in mathematics. Whereas a detailed study of this subject is beyond the scope of this book, a study of it in broad outlines is essential on account its importance in life.

132. Steady Flow.—When a liquid flows through a pipe and as much of it leaves as enters the pipe per second the liquid is said to flow at a steady rate.

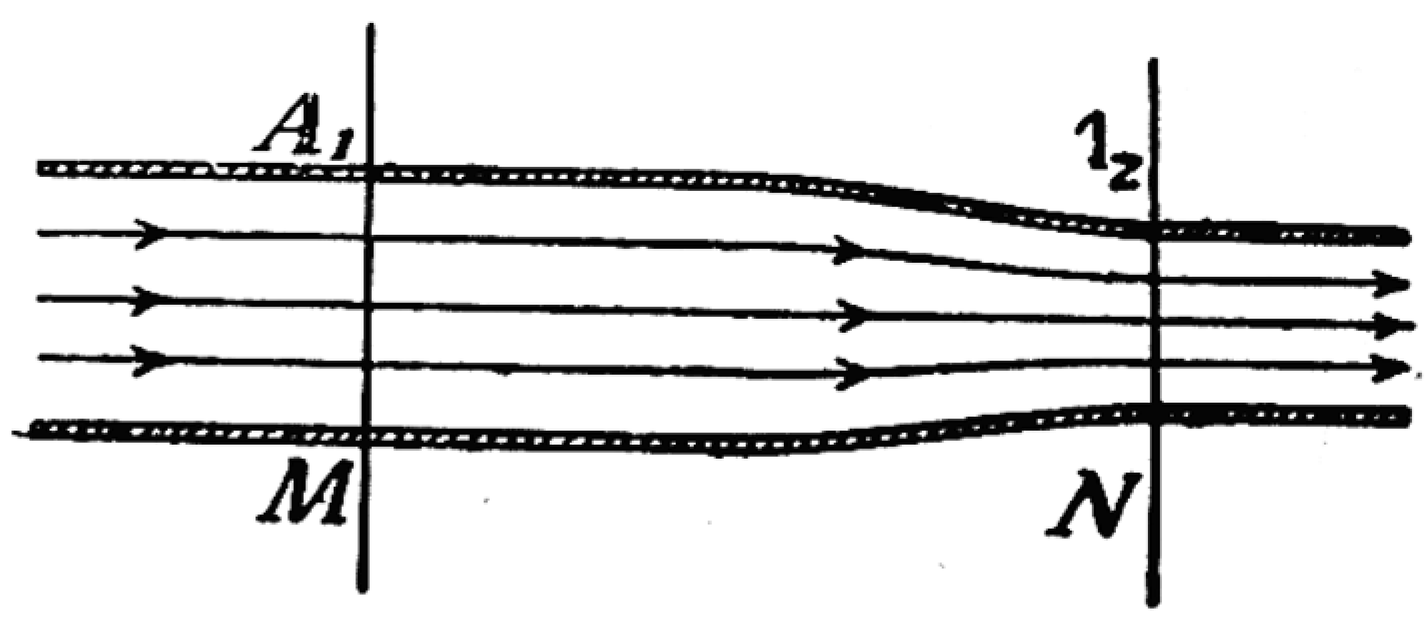


Fig. 114.

Fig. 114 represents a portion of the pipe in which the liquid is flowing from left to right. When the flow is steady every *particle* of the liquid passing a given point in the pipe has exactly the same velocity and therefore follows the same path as the preceding particles which passed that point. The path followed by a particle in a steady flow is called *a line of flow* or *stream line*. The tangent at any point on a stream line gives the direction of the flow of the liquid at the point. Three stream lines are shown in the figure. If the cross section of the pipe is not the same all over, the velocity of a particle will vary along a line of flow. Note the crowding together of the stream lines in the narrow portion of the pipe. In the region where the stream lines crowd together as in this part, the velocity of flow is increased.

In a real liquid because of its viscosity the velocity of flow is not the same at all points of a transverse section of the pipe. It is greater near the centre than near the walls. We shall, however, assume the liquid to be non-viscous and hence the velocity to be the same at all points of a transverse section.

Consider two typical areas A_1 and A_2 perpendicular to the stream lines at points M and N of the pipe. The quantity of liquid passing through area A_1 in t seconds is given by the expression

$$Q_1 = A_1 \rho_1 v_1 t$$

where v_1 is the velocity of the liquid at point M of the pipe and ρ_1 its density.

Similarly the quantity of the liquid passing through A_2 in the same time is given by

$$Q_2 = A_2 \rho_2 v_2 t$$

Since for steady flow these two quantities are equal, we have

$$A_1 \rho_1 v_1 t = A_2 \rho_2 v_2 t$$

or

$$A_1 \rho_1 v_1 = A_2 \rho_2 v_2 \qquad \ldots\ldots\ldots \quad (i)$$

This result is called the *equation of continuity of flow.*

If the liquid is *incompressible* *i,e.*, $\rho_1 = \rho_2$, equation (i) simplifies to

$$A_1 v_1 = A_2 v_2$$

or

$$\frac{A_1}{A_2} = \frac{v_2}{v_1} \qquad \ldots\ldots\ldots\ldots \quad (ii)$$

Equation (ii) states that the velocity of a liquid at any point in a pipe is inversely proportional to the cross sectional area of the pipe at the point. In other words the liquid moves slowly where the area is large and rapidly where the area is small.

Let us apply this result to the flow of water in a channel. At a place where channel is deep the cross-section is great and therefore the speed low. Hence the common proverb, "Still waters run deep".

The flow of the liquid is of the stream line type provided the velocity does not exceed a certain limit determined by the width of the pipe, and the bends or constrictions are not such as to cause the lines of flow to change their direction abruptly. If these conditions are not fulfilled the flow is *turbulent*.

132a. "Head".—A liquid cannot flow through a pipe unless something pushes it. The difference of level between the free surface of water in a tank and the delivery pipe in general supplies push and is called the *head.* It is measured in feet or metres ; the greater the height or head the greater the velocity of water leaving the pipe. We shall prove this result later on. At this stage we wish only to point out that it *cannot* be a mere difference in height that moves the water. It is the gravitational potential energy that does it. The term head is hence, merely a convenient measure of this energy.

The energy of a liquid in motion at any point can be separated into three types :—

(a) **Potential Energy.**—It is due to the height of water above an arbitrary level and for unit mass of liquid is equal to gH, where H is the height and g acceleration due to gravity.

(b) **Pressure Energy.**—It is due to the pressure p at the point. This form of energy at a point is given by $\dfrac{p}{\rho}$ per unit mass of liquid,

where p^* is measured in absolute units.

(c) **Kinetic Energy.**—It is due to the motion of the liquid and is equal to $\frac{1}{2}v^2$ per unit mass, v being the velocity of the liquid at the point.

These types are mutually convertible. The total energy of the liquid at a point is obtained by adding all these types. Thus

$$\text{Total energy at a point per unit mass} = gH + \frac{p}{\rho} + \frac{1}{2}v^2.$$

If we divide by g we get total head. Thus

$$\text{Total "head"} = H + \frac{p}{\rho g} + \frac{v^2}{2g}.$$

H is called potential or *elevation head*, $\frac{p}{g\rho}$, *pressure head* and $\frac{v^2}{2g}$, *velocity head.*

In these equations the value of p as said above, should be substituted in absolute units *i.e.* in poundals per sq. ft. or dynes per sq. cm. and not in feet or centimetres.

132b. Bernoulli's Equation.—It was Daniel Bernoulli (1700-1782) who first said that while a liquid flows along, its total energy remains unchanged. This result is called after him *Bernoulli's theorem or equation.* It is usually written as

$$gH + \frac{p}{\rho} + \frac{1}{2}v^2 = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad (iii)$$

This equation holds good only if (a) the liquid in motion is incompressible, (b) friction and viscosity are negligible, and (c) the flow is steady.

The application of this theorem leads to an important and paradoxical result that when the velocity of flow increases the pressure decreases. To demonstrate this result take a glass tube AB having a constriction at C (Fig. 115) and three narrow tubes fused to it at

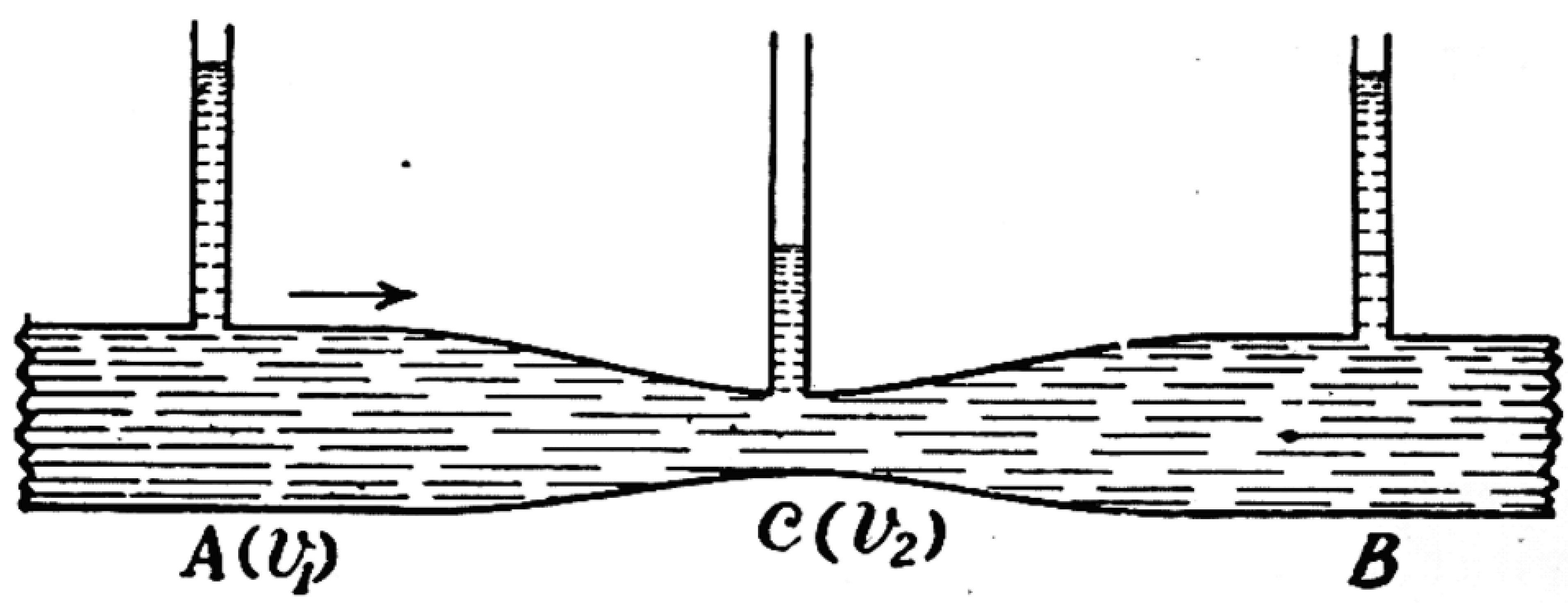


Fig. 115.

A, B and C to serve as manometers and fix it up *horizontally* and connect it at one end to a water tap by a rubber tubing. Open the tap and allow water to flow through the tube steadily. Taking water to be incompressible as much quantity of it must pass per second the section at C as the section at A. Since the cross section at C is smaller

* Water flowing in two pipes one connected to a tank placed at the roof of a double storey and the second placed at the first storey will have different potential energy but may have the same pressure energy.

the velocity here must be greater.

As the tube is horizontal H is same throughout, therefore, from equation (*iii*) we find that when v_2 is greater than v_1, p_2 is smaller than p_1.

We can obtain this result directly by applying Newton's second law of motion to the flow of liquid between A and C. Imagine a millimetre cube of water passing through the tube and let us follow its motion. At A its pressure energy is $m\frac{p_1}{\rho}$ and kinetic energy $\frac{1}{2}mv_1^2$, where m is its mass. At C its kinetic energy becomes $\frac{1}{2}mv_2^2$. Now so that its kinetic energy may increase a force must act on it. Obviously the required force comes from the difference in pressure acting on the cube while moving from A to C. In other words, the pressure at A is greater than pressure at C. Thus we conclude as before that at A, v_1 is smaller but pressure is greater than at C where v_2 is greater but pressure is smaller.

This paradoxical result that is where the velocity of a liquid is high the pressure is low and *vice versa* is called Bernoulli's principle.

132c. Applications of Bernoulli's Theorem to Liquids.—(1) We can derive the law of hydrostatic pressure from Bernoulli's equation in the following manner :—

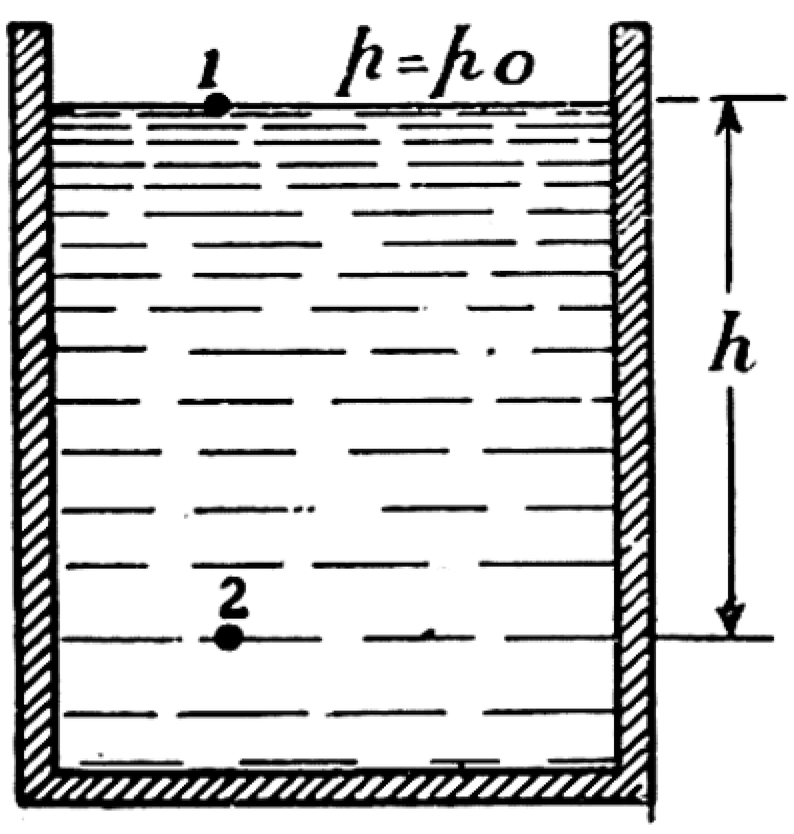


Fig. 116.

Consider two points 1 and 2 at different depths in a tank (Fig. 116) and let the height of point 2 above ground level be denoted by H, the difference in level of points 1 and 2 by h, and the pressure at point 1 by p_1 and at point 2 by p_2.

Since v_1 and v_2 are zero, from Bernoulli's theorem we have

$$gH+\frac{p_2}{\rho}=g(H+h)+\frac{p_1}{\rho}$$

or

$$gh=\frac{p_2-p_1}{\rho}$$

or

$$p_2-p_1=h\rho g$$

Remembering that p_1 is equal to atmospheric pressure (p_a) we get

$$p_2=p_a+h\rho g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (iv)$$

(2) **Torricelli's Theorem.**—Let us find the velocity of discharge of water from a tank through a hole h cm. below the surface of the liquid in the tank. Consider points 1 and 2 ; point 1 is h cm. below the surface and is at the same level as point 2. The pressure at point 1 is $p_a+h\rho g$, where p_a is the pressure at the free surface. At point 2 the pressure is p_a since it is at the centre of the hole and is therefore open to the atmosphere. Assuming that the tank always remains filled to height h cm. above the hole and the

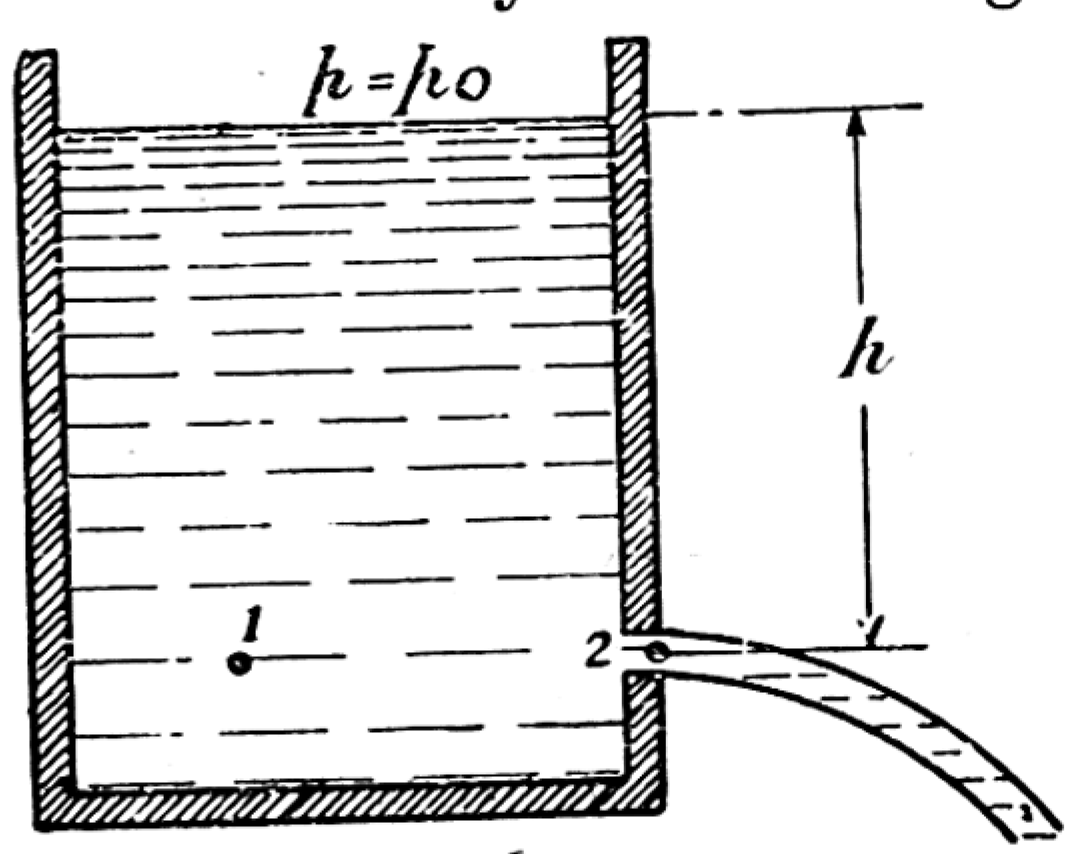


Fig. 117.

velocity of water at point 1 is zero, we get from Bernoulli's theorem,

$$\frac{p_a+h\rho g}{\rho}=\frac{p_a}{\rho}+\tfrac{1}{2}v^2$$

or
$$hg=\tfrac{1}{2}v^2$$

or
$$v^2=2hg$$

or
$$v=\sqrt{2hg} \quad \ldots\ldots\ldots \quad (v)$$

This relation was first proved by Torricelli and is hence known as Torricelli's theorem. If A is the area of the hole the discharge rate Q is given by the expression

$$Q=A\rho v$$
$$=A\rho\sqrt{2gh},$$

where Q is the mass of the liquid discharged per second.

Let us apply this result to calculate the quantity of water discharged in one minute from a tap with an opening half an inch in diameter connected to a water-main carrying water at a pressure of 62·4 lb. per sq. inch. To find velocity of water let us write Torricelli's theorem as

$$v=\sqrt{\frac{2p}{\rho}},$$

since
$$p=h\rho g.$$

Substituting values for p and ρ we get

$$v=\sqrt{\frac{2\times 62{\cdot}4\times 32\times 144}{62{\cdot}4}}$$
$$=96 \text{ ft/sec.}$$

Volume flowing out per minute

$$V=Avt$$
$$=\pi\left(\frac{1}{4}\right)^2\times\frac{1}{144}\times 96\times 60$$
$$=\frac{\pi}{16}\times\frac{1}{144}\times 90\times 60$$
$$=7{\cdot}85 \text{ cubic feet.}$$

While calculating the discharge we have substituted the value of the actual area of the hole but in practice it is found that on account of the convergence of the lines of flow of the liquid towards the hole the cross section of the discharge continues to diminish for a short distance even outside the hole. The area of *smallest cross section*, called *Vena Contracta*, should be used in the equation.

For a sharp edged circular hole, the area of the *Vena contracta* is 0·61 of the area of the hole and hence the actual discharge is 0·61 of the theoretical rate calculated above.

(3) **Filter Pump or Aspirator**, which is often used in laboratories to produce a partial vacuum required for the successful working of high vacuum pumps, is a simple application of the Bernoulli's principle. It consists of tube *A* ending in a jet and enclosed by an outer tube *B* having a tube *C* fused into its side as shown in the Fig. 118. A stream of water is admitted at *A* from a tap and flows through the jet into the outer tube *B*, which is connected by means of tube *C* to the apparatus or vessel to be exhausted. The small size of the nozzle causes the stream to issue from it with high velocity and hence at low pressure in accordance with Bernoulli's principle. On occount of the fall in pressure in *B*, air is sucked from the vessel through *C* and mingles with water and thereafter passes out at the lower end.

With the help of such a pump the pressure can be reduced in a short time to 2 cm. of mercury.

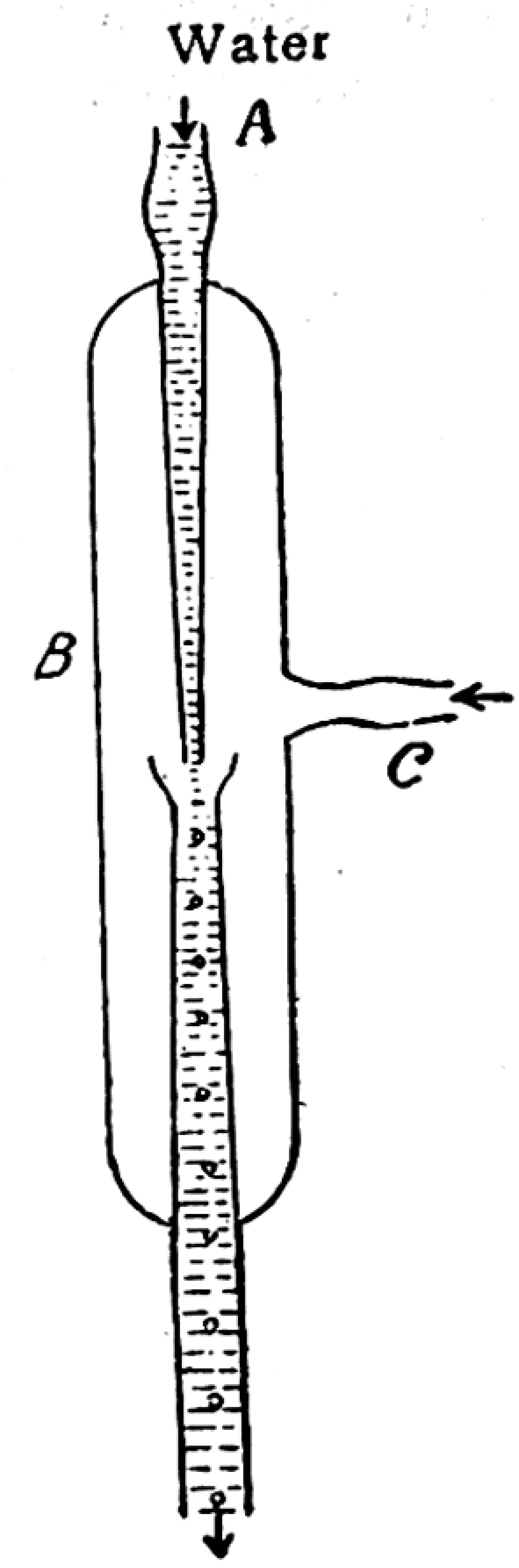


Fig. 118.
Filter Pump.

132d. Applications of Bernoulli's Principle to Gases. Although the mathematical treatment of the Bernoulli's theorem is somewhat more complicated in the case of gases than in the case of liquids on account of their compressibility nevertheless the general effect is the same as said before, namely that as the velocity of the air current increase the pressure decreases and *vice versa*.

(1) **Paint Sprayer.**—The modern paint sprayer is an illustration of the application of Bernoulli's principle. A stream of air is forced through a jet *D* (Fig. 119) with a high velocity. Just near the jet is placed the end of a tube *B* which dips into the paint to be sprayed. Since the pressure becomes low near the nozzle the paint is forced up the tube *B* by the atmospheric pressure acting on the surface of the paint in the vessel *C*. As soon as the paint reaches the top of *B* it is broken into spray by the impact of the air stream issuing out of the jet with a high velocity.

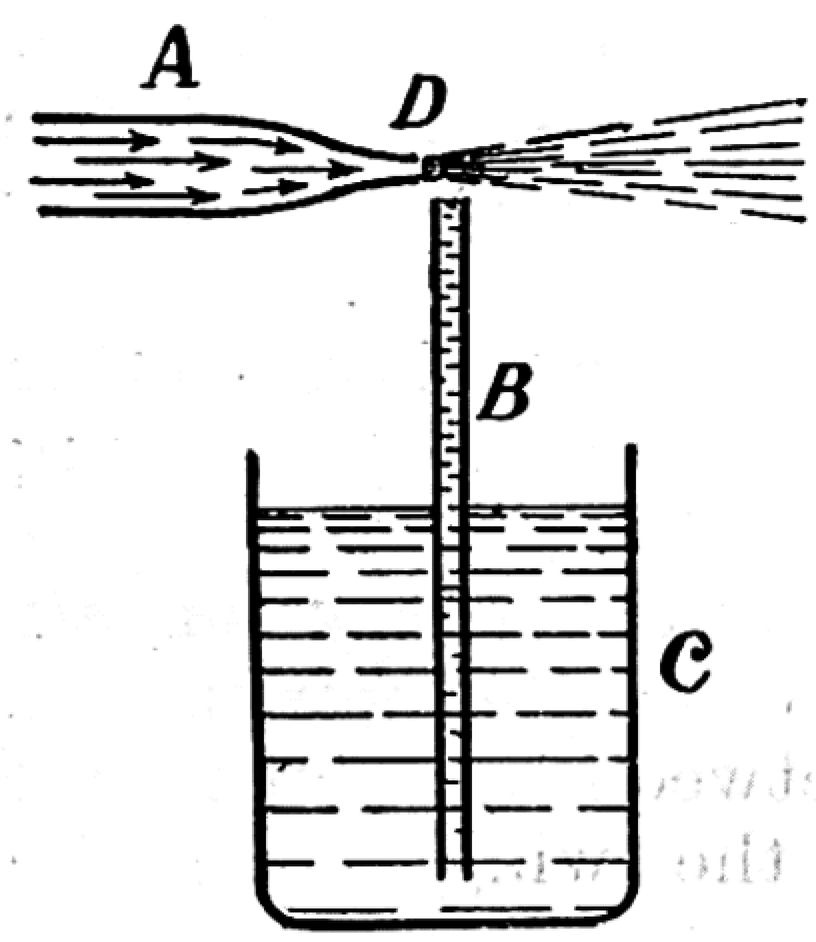


Fig. 119.

(2) **Bunsen Burner.**—In the Filter pump explained already we have seen that when a stream of water flows through a jet with a high velocity the pressure falls and the air from the vessel is sucked in and is carried off by the stream. The same thing happens when a stream of gas rushes out rapidly through a jet. It was Bunsen, who made use of this idea in the construction of a gas burner called Bunsen burner after him. It consists of the following parts :—

(1) An iron base (*a*) having a horizontal metal tube fitted into its side as shown in Fig 120. This tube communicates with a fine jet (*d*) fitted in the middle of the base. (See section of the burner shown.)

The gas is admitted into the burner through the side tube.

(2) A vertical metal tube (*b*) having holes at its lower end to admit air into the burner.

(3) A movable metallic collar (*c*) having holes of the same size as tube (*b*) to regulate the supply of air.

When gas rushes out of the jet, air is sucked into the tube through the holes. When a burning match stick is applied to the mixture at the top of the tube (*b*) it burns with a light blue flame which is extremely hot, provided enough air is admitted through the holes to permit complete combustion of the gas.

If too little air is admitted, the flame is luminous and sooty. On the other hand, if the air admitted is too much, the gas lights at the jet, *i.e.*, the flame "strikes back." Remember, the proper proportion of air to gas is 7: 4. When this proportion is maintained the flame obtained is non-luminous and is extremely hot.

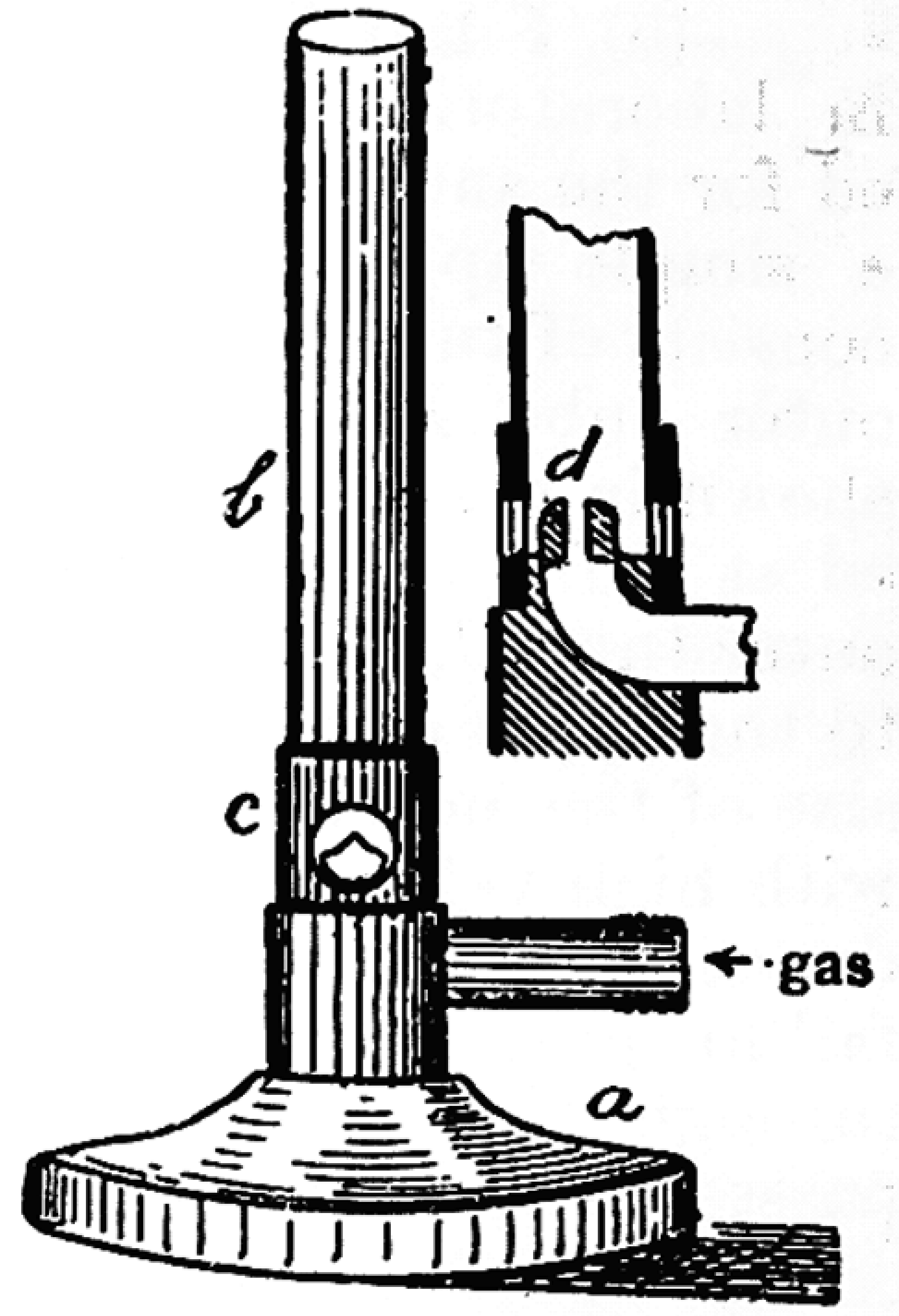


Fig. 120.
Bunsen Burner.

(3) **Flight of an Airplane.**—Let us see how Bernoulli's principle aids in supporting an airfoil or air-plane wing.

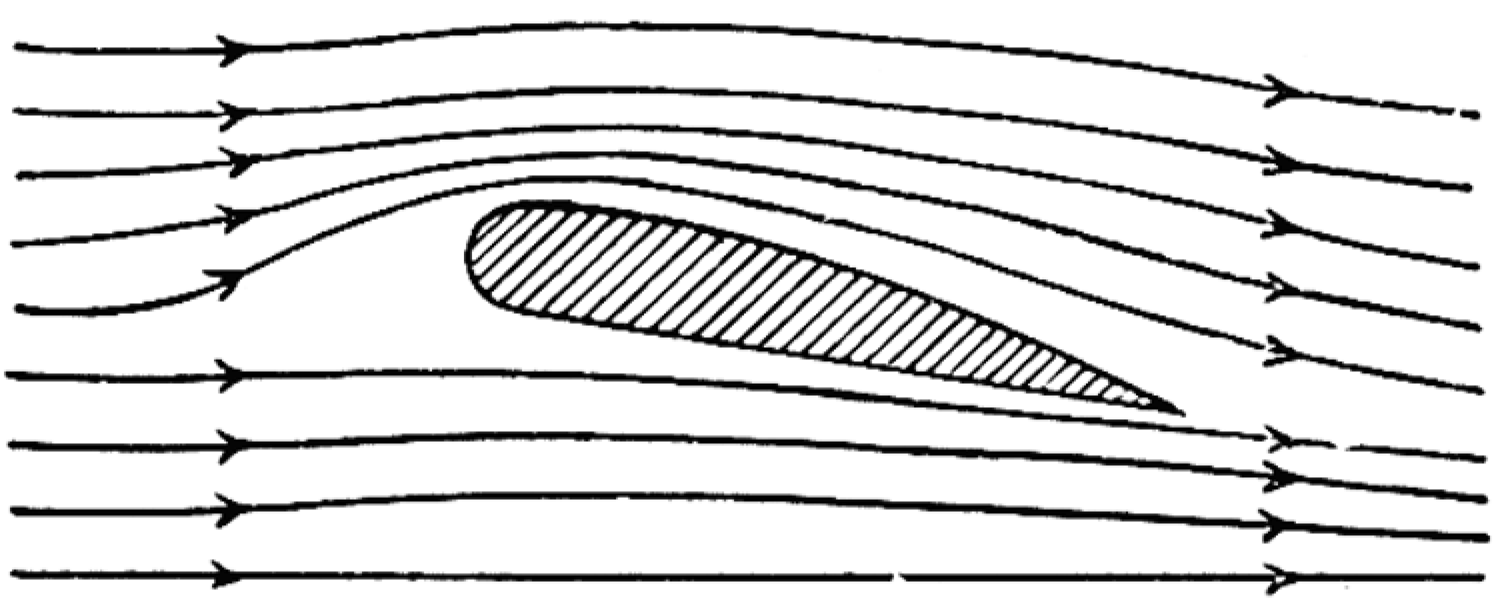


Fig. 121.

Fig. 121 represents a plane in level flight. On account of the shape of the wing the air current flows past the top surface with greater velocity than at the bottom. This is shown by the crowding together of the stream lines at he top surface very much like the crowding together in the narrow part of the tube *MN* in Fig. 114. Hence the region above the wing is one of increased velocity of air stream and of reduced pressure, while below the wing the pressure is almost atmospheric. It is this *pressure difference* between the upper and lower wing surfaces which gives rise to the lift on the wing when the plane is in level flight or is running on the ground.

Some idea of the magnitude of the lifting force produced due to difference in pressure on two sides of the wing may be obtained from the following example :—

The pressure on the top of the wing 40 ft. long and 8 ft. wide of an aeroplane in level flight was found to be 14·55 lb. per sq. inch while the pressure at the bottom was 14·70 lb. The difference of 0·15 lb. per sq. inch. will produce a lifting force$=0{\cdot}15\times40\times8\times144=6{,}912$ lb.

EXERCISES

1. Water flows through a horizontal pipe of varying cross section at the rate of 6·0 cubic feet per minute. Determine the velocity of

the water at a point where the diameter of the pipe is (*a*) 0·75 inch and (*b*) 1·5 inches.

Volume of water flowing per minute across the pipe at a point is *Avt*.

Substituting values we get

$$\pi\left(\frac{3}{8}\right)^2 \times \frac{1}{144} \times 60 \times v = 6$$

$$\therefore \quad v = \frac{6 \times 144 \times 64}{\pi \times 9 \times 60} = 32{\cdot}6 \text{ ft./sec,}$$

$$\pi\left(\frac{3}{4}\right)^2 \times \frac{1}{144} \times 60 \times v' = 6$$

$$\therefore \quad v' = \frac{6 \times 144 \times 16}{\pi \times 9 \times 60} = 8{\cdot}1 \text{ ft./sec.}$$

2. Kerosene oil flows through a pipe line 8 inches in diameter with a speed of 4 miles per hour. How many gallons of oil are delivered per hour by this pipe line ? (Given 1 cubic foot=6·24 gallons.)

Ans. 46021 gallons.

3. Describe a Bunsen burner and explain how air gets mixed up with a stream of gas.

4. The water-main in a part of your city is laid over a raised ground 50 ft. above the level of the pumping station. If the pressure at the station is 50 lb. per sq. in. calculate the pressure in the pipe at raised level, neglecting the frictional losses. *Ans.* 28·4 lbs per sq. in.

5. Explain what is meant by steady and turbulent flow of water.

What is the meant by a stream line ?

6. State Bernoulli's theorem.

Calculate the total energy possessed by one pound of water at a point where the pressure is 30 lb /in.^{2}, the velocity is 4 ft./sec and the height is 16 feet above ground level. *Ans.* 2735·4 ft. lbl.

7. An aeroplane weighing 12,000 lb. has a total wing area of 600 sq. ft. Find the pressure on the upper surface of the wing when the aeroplane is moving in a level flight at an altitude such that the pressure at the lower surface of the wing is 9·0 lb. per square inch.

Ans. 8·86 lb. per sq. in.

CHAPTER XII

The Pressure of the Atmosphere

133. It is found that 1,000 c.c. of dry air at 0°C, and a pressure of 760 mm. of mercury at sea-level in latitude 45°, weigh 1·293 grams. As one c.c. of water weighs one gram, the sp. gravity of air with respect to water is 0·001293. Since air is so light, it is considered by many to be weightless. But when we deal with big volumes the weight is by no means negligible. For instance, the weight of 1 cubic foot of air is approximately $\frac{1}{13}$ of a lb., or of 13 cubic feet about 1 pound. The weight of air in a class room measuring 20′ × 20′ and 20 feet in height will be about 612 pounds.

Since air has weight, it must, like a liquid, exert pressure on all surfaces in contact with it. To show experimentally that it exerts pressure, perform the following experiment:—

Place a toy balloon full of air under the receiver of an air pump and exhaust the air in the receiver. As air is pumped out from the receiver the balloon is found to swell more and more on account of the pressure of the air inside till it bursts.

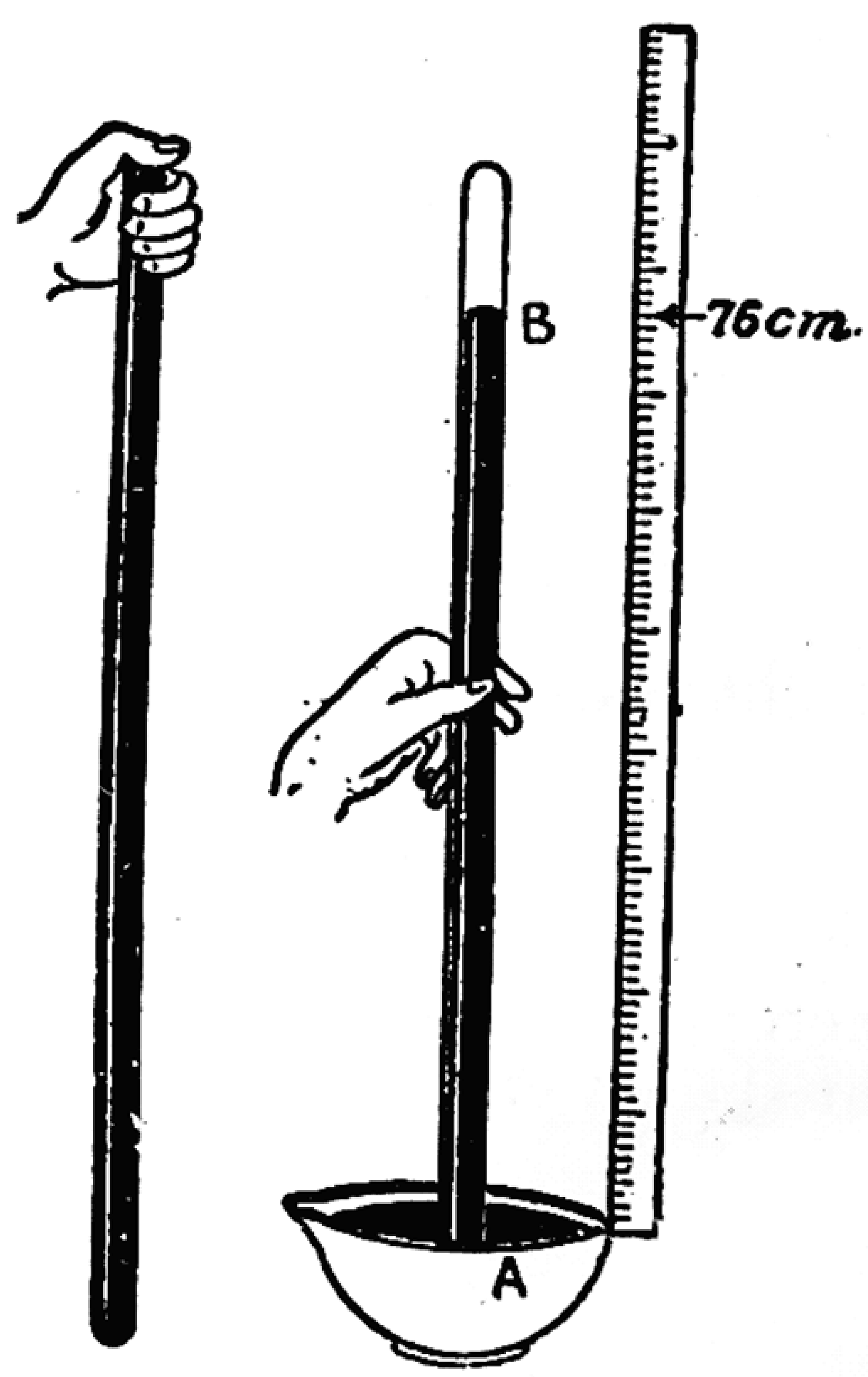


Fig. 122.

134. Torricelli's Experiment.—In 1643 Torricelli, a pupil of Galileo, took a glass tube closed at one end, about 1 metre long and 1 cm. in diameter, and completely filled it with mercury (Fig. 122). He closed the open end with the thumb and inverted the tube into a cup of mercury. On removing the thumb he found that the mercury descended leaving a clear space at the top. After a few oscillations the column of mercury became stationary, with its height 76 cm. above the surface of mercury in the cup. Since the pressure over a horizontal surface of liquid at rest is the same at all points, the pressure at A (inside the tube) must be the same as the pressure outside the tube at the free surface of mercury. The pressure at A is due to the column of mercury BA, therefore, the pressure exerted by atmosphere must be equal to it.

Pascal, taking Torricelli's experiment as his basis, argued that since mercury is 13·6 times as heavy as water, the atmosphere should support a 34 ft. high column of water. He repeated Torricelli's experiment with a 46 ft. long tube, and actually obtained a column of water 34 ft. in height.

To completely establish the fact that it was atmospheric pressure which supported the liquid column Pascal said that since at the top of a mountain the pressure is less, the height of the column inside the tube must also be less there. He asked his brother-in-law, who lived at the top of a mountain nearly 1,000 metres high, to repeat the experiment there. His brother-in-law found that the column of mercury supported was actually about 8 cm. less in height. Hence it was inferred that the height of the mercury column measured the pressure of the atmosphere.

If the tube be furnished with a scale to read off the height of the mercury column, it could be used to measure the atmospheric pressure. Such an apparatus, which enables us to measure the pressure of the atmosphere, is called a **barometer.**

The space above B is almost a perfect vacuum. It is called *Torricellian Vacuum.* Strictly speaking, it contains a very minute quantity of mercury vapour, but it is so small that we can ignore its pressure*. If the tube be lowered or inclined so that the top of the mercury column strikes the tube, a characteristic clik is heard, showing thereby that there is no air above the mercury column.

The barometric height depends upon the climatic conditions and the altitude of the place. But generally, by the term *pressure of one atmosphere* we understand the pressure equal to the weight of a column of mercury 1 sq. cm. in cross section and 760 mm. high. It is sometimes also called *the standard* or *the normal* pressure. This corresponds to a pressure of 14·7 pounds per sq. inch. or 1·033 kilograms per sq. cm., or $1{\cdot}013 \times 10^6$ dynes per sq. cm.

134a. Air Pressure and the Human Body.—The total surface of an average man's body is 14 square feet and hence the total pressure acting on him is $\dfrac{14{\cdot}7 \times 14 \times 144}{2240}$ or about 13 tons. The reason why we do not feel uncomfortable under this tremendous pressure is because it acts on us from all sides. We have become so much used to it that we do not feel it at all. As a matter of fact when the outside pressure is reduced to any considerable extent, we begin to feel uncomfortable. Men who fly at great heights or climb very high mountains not only experience a difficulty in breathing but also suffer from bleeding at the nose because the outside pressure is lower than the pressure of the blood.

135. Probable Height of the Atmosphere.—If the density were not to decrease with the height, the weight of the air column considered above would correspond to a height of five miles. But we know that as we go up the density decreases. It can easily be shown that after a rise of every $3\frac{1}{2}$ miles in the atmosphere the density becomes half of what it was before. For instance at a height of 7 miles the density is one-fourth and at $10\frac{1}{2}$ miles it is one-eighth of what it is on the surface of the earth and so on. It is impossible to lay down a definite limit beyond which the atmosphere does not extend. Even at a height of 125 miles air is dense enough to offer resistance to the

*At ordinary temperatures it is about 0·001 cm.

passage of meteors and hence heat them to incandescence. It is quite safe to state that the atmosphere extends up to a height of more than 200 miles.

The lower portion of the atmosphere say up to a height of about 8 to 10 miles is called **Troposphere**. It is in this part that the temperature falls as we go up and the winds blow and the clouds are formed. Above this part the temperature remains constant and the air remains calm and quiet. This upper part is called **stratosphere**. The temperature of stratosphere has been determined by self-registering balloons. Strange as it may appear, the stratosphere is coldest (about −80°C) over the equator and warmest (about −40°C) over the poles.

We shall now consider the two types of barometers that are generally used to read the pressure of the atmosphere.

136. Fortin's Barometer.—In a simple barometer like the one shown in Fig. 122, whenever the atmospheric pressure increases, mercury is forced up into the tube, and the level of mercury in the cup changes. The reverse happens when the pressure decreases. Although by making the cross-section of the tube small in comparison with the cross-section of the cup the change in the level of mercury in the cup can be made small, yet for accurate work this must be taken into consideration. In Fortin's Barometer (Fig. 123) this difficulty is removed by using a leather bag as the cup, the bottom of which can be raised or lowered with the help of a screw. The zero point of the scale coincides with the tip of an ivory point P, and each time a reading is taken the surface of mercury is first made to touch the ivory point. Near the top in the copper tube enclosing the glass tube there is a slot in which a vernier slides up and down. At the back also there is a slot in which a brass plate connected with the vernier slides. the lower edge of the back plate is just at the same level as the lower edge of the vernier. To take a reading the vernier is moved till its lower edge, the lower edge of the back-plate, the top of the meniscus of the mercury column, and the eye are all at the same level. It gives reading accurate to $\frac{1}{20}$ of a mm., or $\frac{1}{500}$ of an inch.

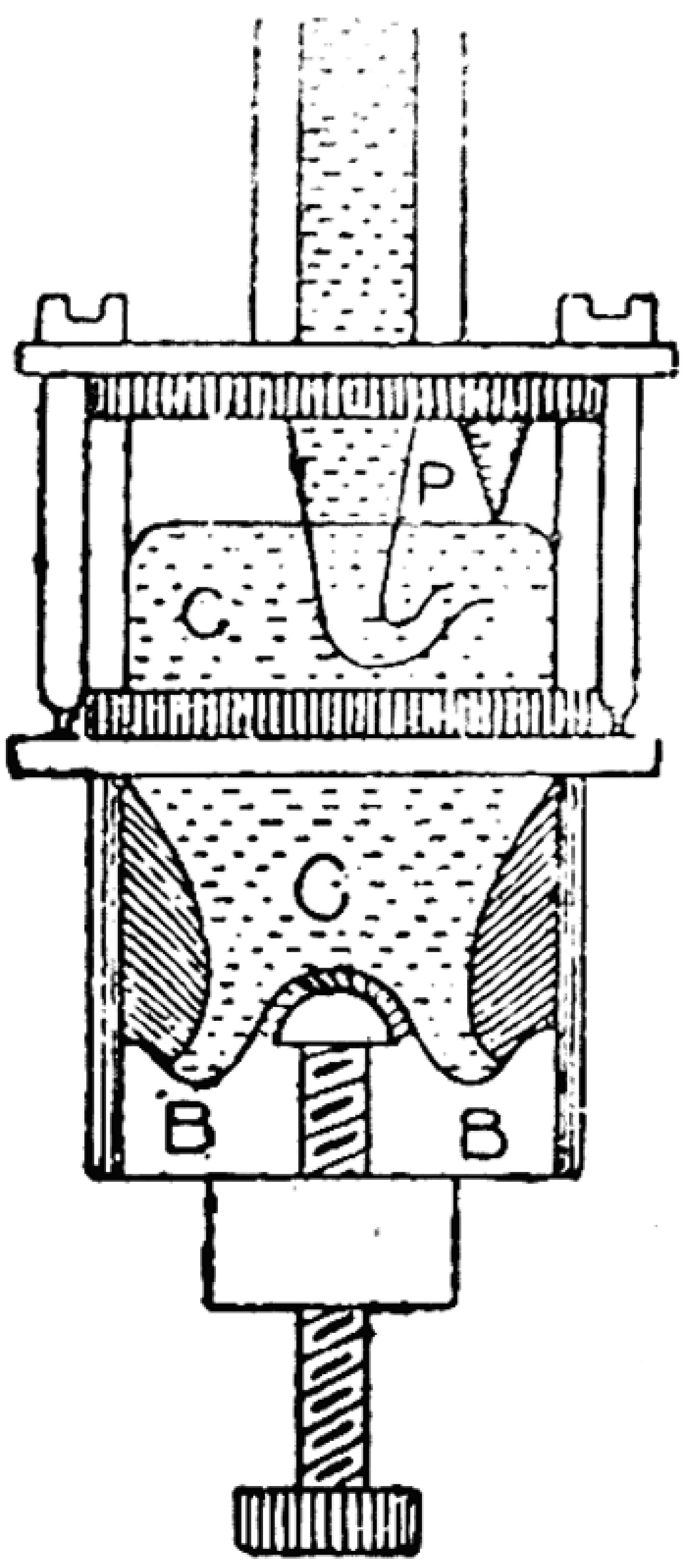


Fig. 123.

Fig. 124. Aneroid Barometer.

137. Aneroid Barometer.—The trouble with the Fortin's Barometer is that it cannot be carried easily from place to place. If, for instance, we want to measure the height of a place with the help of

a barometer, Fortin's barometer is practically useless. For such a purpose an Aneroid* barometer (Fig. 124) is used, which is so called because, no liquid is used in its construction.

It consists of a shallow cylindrical metal box B [Fig. 125] the lid of which is made of thin corrugated sheet of metal. The air inside the box is exhausted by means of an air-pump, leaving more or less a perfect vacuum. The top of the box would collapse on account of the atmospheric pressure from above but for the pull of a stout spring S. When the pressure increases the top yields a little and as a result of it the spring moves down. This movement is multiplied manifold by a system of levers which in turn moves a pointer over a circular scale. With a decrease of pressure the top moves up a little and so does the spring thereby making the pointer move in the opposite direction. The scale is graduated to read pressures in inches or centimetres of mercury by comparison with a Fortin's barometer. A good aneroid barometer is so sensitive that it shows a change of pressure even when it is lowered from the top of a table to the ground.

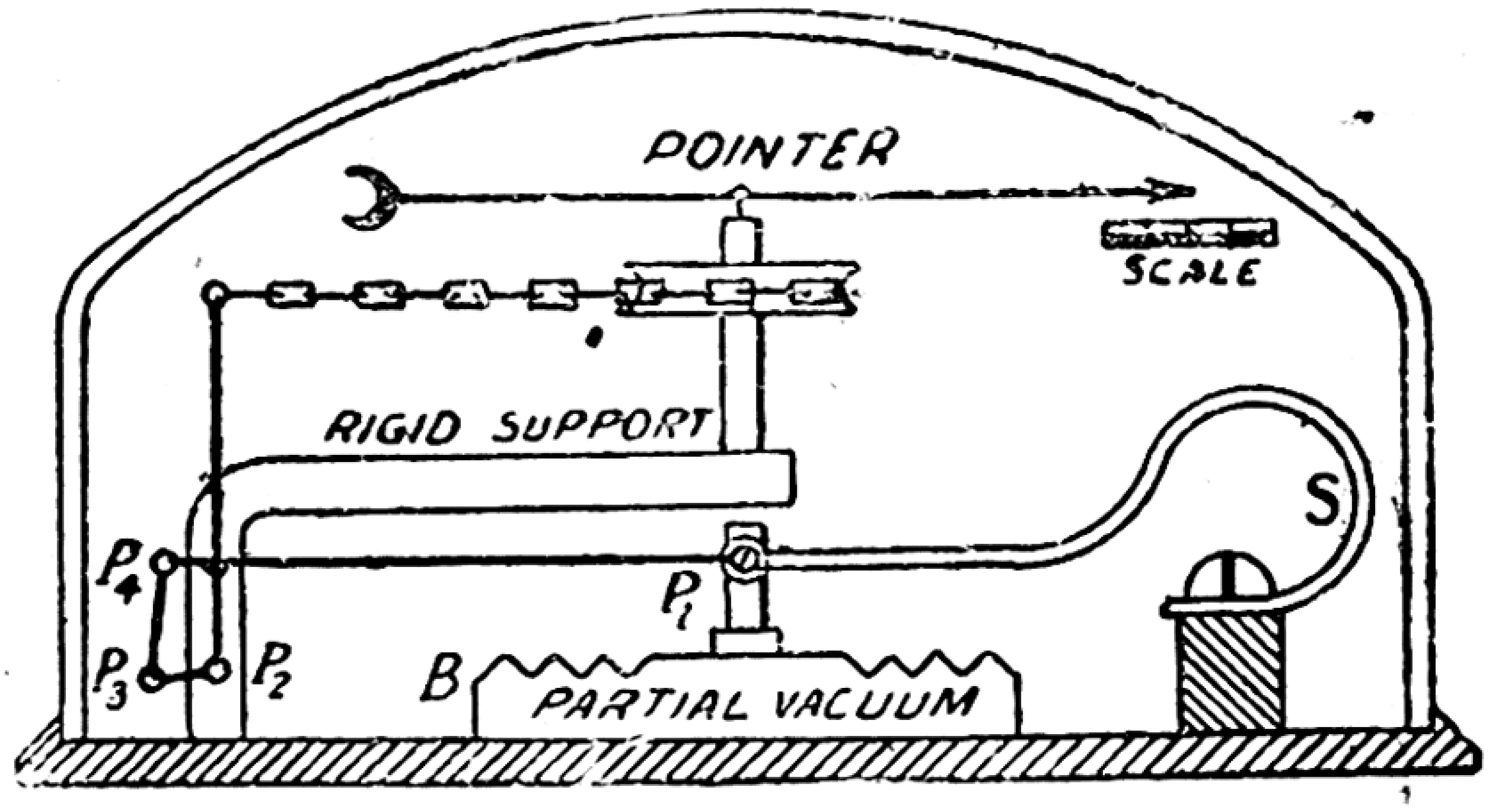


Fig. 125.

To determine the heights accurately with the help of a barometer a complex formula has to be used, but for rough calculations the student should remember that the pressure changes by about an inch of mercury column for the first 900 ft. above sea-level, by another inch for the next 1000 ft., and by an inch again for the next 1,100 ft., and so on.

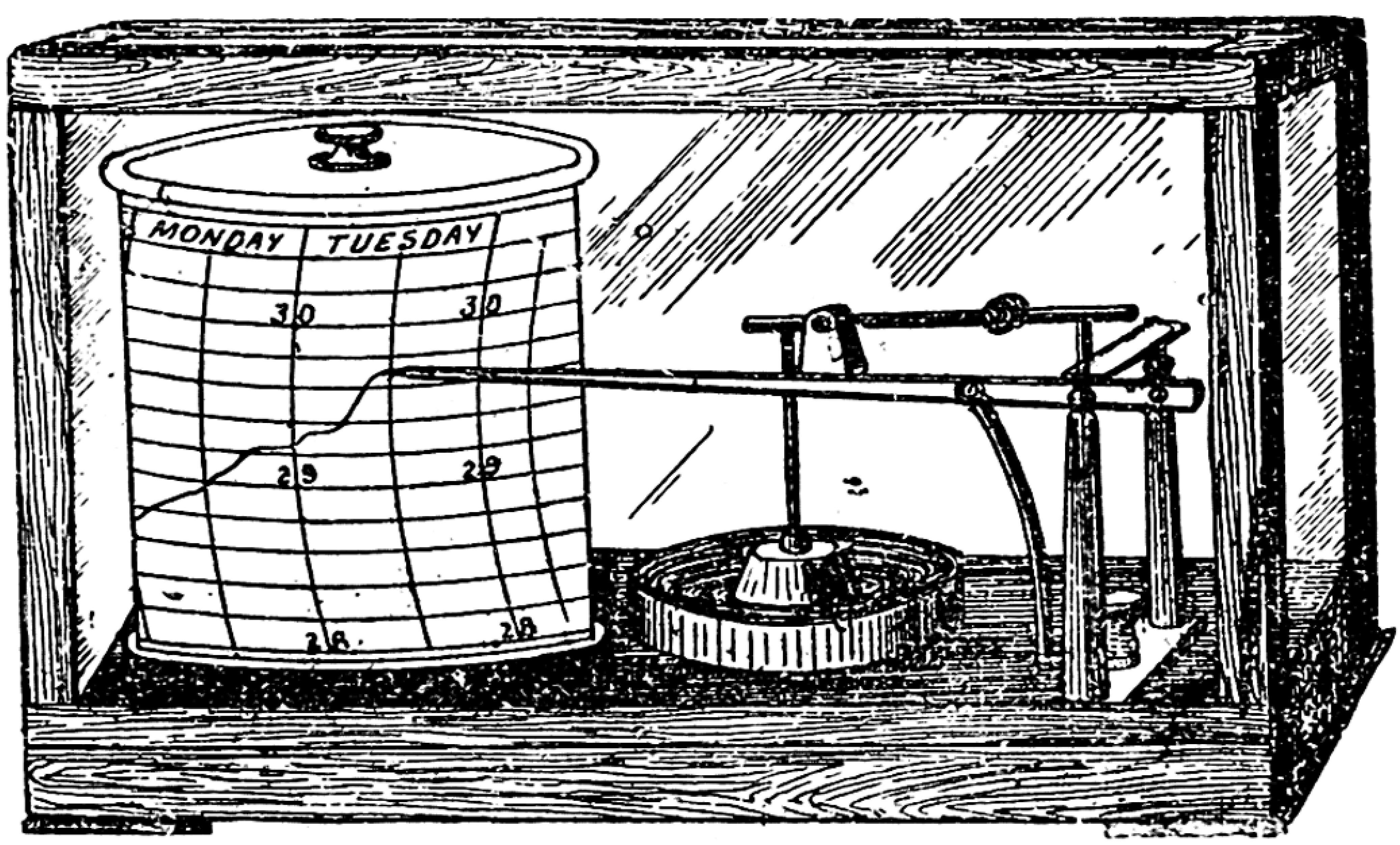


Fig. 126.

For aeroplanes and balloon flights self-recording aneroid barometers are used which are calibrated to give direct readings of the height

*Aneroid means no liquid.

above the station at which adjustment was last made. They can be made of quite small dimensions so that they may be sent up in sounding balloons for exploring the upper atmosphere.

138. Barograph.—It is a self-recording barometer (Fig. 126). The usual form consists of an aneroid barometer carrying a long pointer which traces in ink a continuous record of the atmospheric pressure on a sheet of paper marked with horizontal lines to indicate pressures and vertical lines to show the days and hours of the week. The sheet is wrapped round a revolving drum which is turned by a clockwork through one complete revolution in one week. As the drum rotates the pointer traces a continuous line that shows the pressure at any time during the week.

139. To Forecast Weather.—The barometer is used by the meteorologists to forecast changes in weather.

The principle underlying the science of Meteorology is that damp air is lighter and hence exerts less pressure than dry air, the temperature etc., being the same. This fact is contrary to the popular belief that moist air is heavy. Moist air may be oppressive but it is really less dense than dry air.* This being so, it is clear that the more the water vapour in the air, the lower will the barometer stand.

Now evidently when there is a lot of water vapour in the air, the chances of rainfall are greater. Hence falling of the barometer indicates the coming of rain.

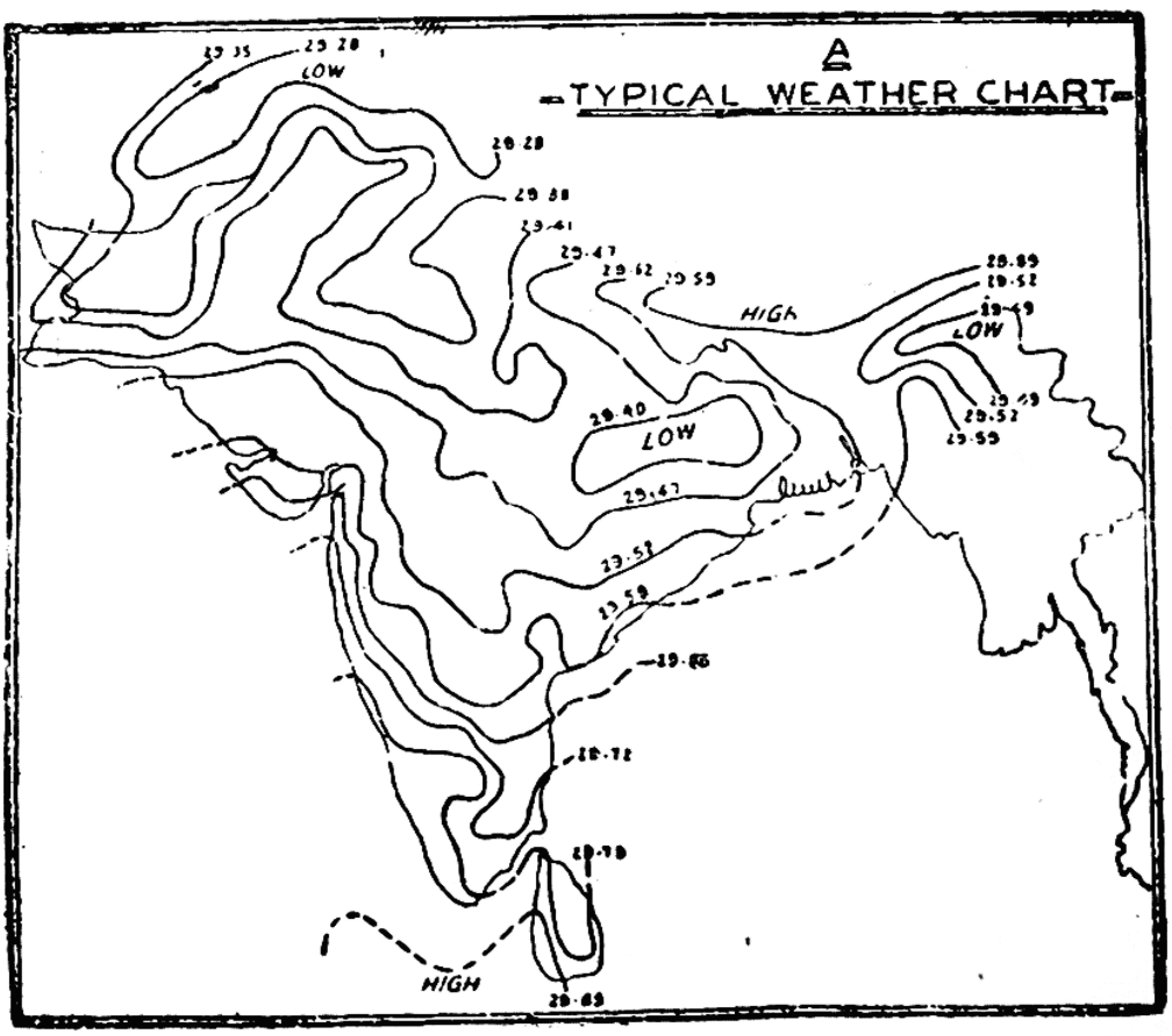


Fig. 127.

On the other hand, when the barometer stands high, it indicates that there is very little water vapour in the air, and hence very little,

*That moist air is lighter than dry air at the same temperature and pressure we shall show in Example 3 at the end of Chapter VIII of Heat.

or practically no chance of rain. In other words, a high barometer indicates fair weather.

To sum up we can say that

(*i*) A falling barometer indicates the approach of rain or storm.
(*ii*) A rising barometer indicates the approach of fair weather.
(*iii*) An unchanging high barometer indicates settled fair weather.

In a rough sort of way what we have said above is all right. But it does not mean that every time a barometer falls a storm is on its way. Consequently the readings of a barometer taken by a person are not a very reliable guide to the future weather. It is only experts who, when supplied with sufficient data from a number of stations, can make weather forecasts of any value. They issue daily charts containing information regarding air pressure, temperature, wind, etc. They tabulate on a map of the country the barometric readings taken simultaneously at different stations and draw lines through the stations which have the same pressure. These lines are called **Isobars** (Fig. 127). At some places the isobars enclose an area marked "Low" and at other places a region marked "High". The low pressure region is called *cyclone* or depression and the high pressure region is called *anticyclone*. The cyclones and anticyclones are not, as a rule, stationary. Their direction and speed of movement can be found by comparing the weather maps for different periods of the day. The arrival of a cyclone or anticyclone can be predicted up to a day or two in advance. The arrival of an anticyclone heralds a period of fine weather whereas a cyclone brings storm or rain with it.

140. Boyle's Law.—Solids and liquids undergo practically no change in volume when subjected to a change of pressure. Gases do so to a considerable extent even with small changes of pressure. It was Boyle who, in 1662, first discovered the law connecting the pressure and the volume of a gas. It is called after him Boyle's Law. It states *that the volume of a given mass of gas varies inversely as the pressure, the temperature remaining constant.*

Mathematically, it can be writen as

$$P \propto \frac{1}{V}, \text{ or } PV = \text{constant.}$$

To prove the law experimentally take the apparatus shown in Fig 128. It consists of a glass tube *B*, about 20 cm. long and 1 cm. wide, closed at one end by a ground glass stop-cock, and drawn out a little at the other end so that a rubber tube can be slipped on to it. A second piece of glass tubing *A* is attached to the other end of the rubber tube. The tubes *A* and *B* can be clamped in any position.

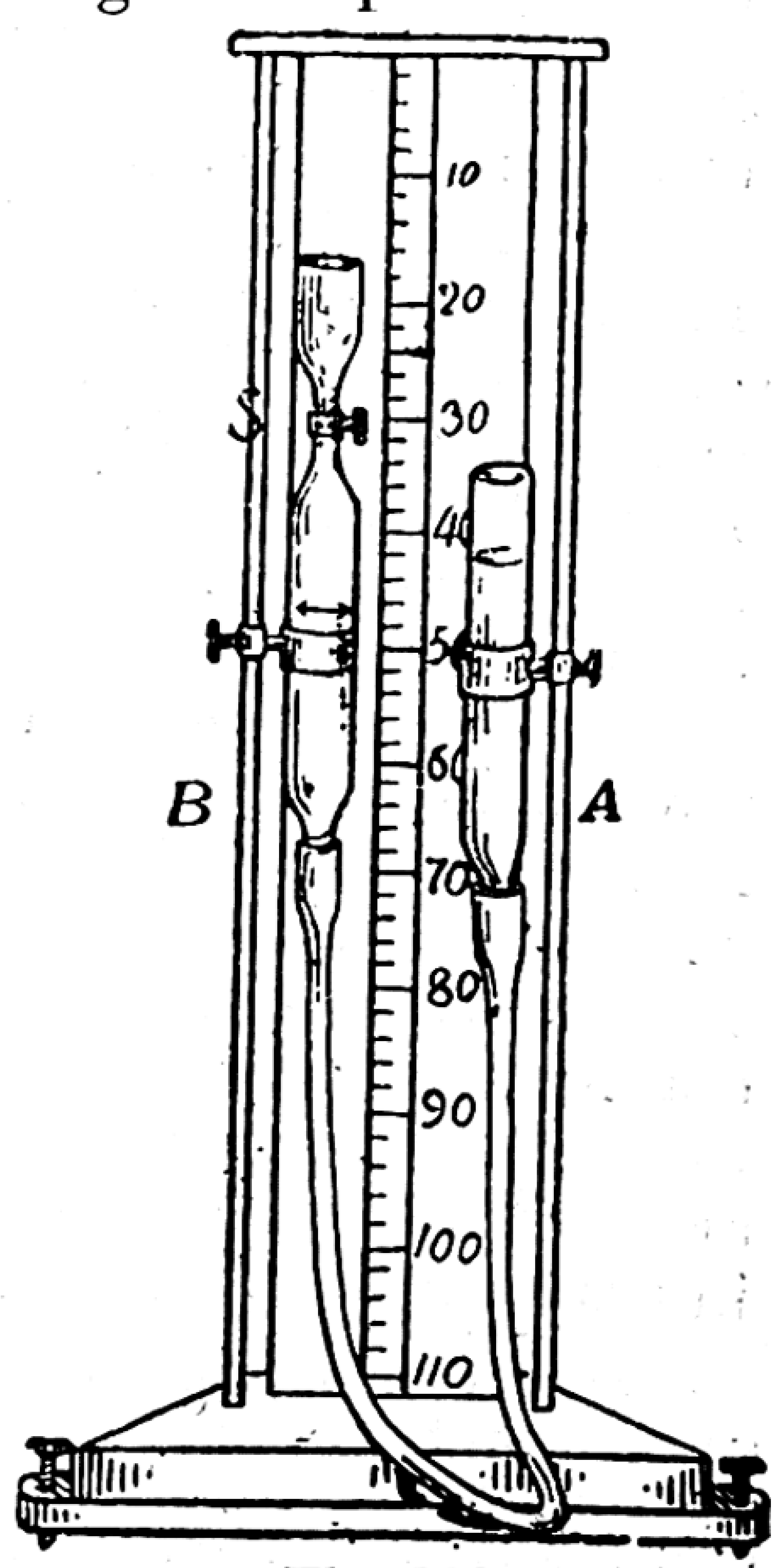


Fig. 128.

Open the stop-cock *S*, and pour mercury down the tube *A*. It will stand at the same height in both the tubes *A* and *B*. Close the

stop-cock, enclosing thereby a quantity of gas at atmospheric pressure in the tube B. Note the length of the enclosed column of air and read the barometer. Now raise the tube A, keeping the tube B at the same level. The mercury will be seen to rise in both the tubes, but more rapidly in A than in B. Note the difference in level between mercury surfaces in the two tubes; let it be h cm. If the atmospheric pressure be P cm. of mercury column, the pressure on the enclosed gas is $P+h$ cm. Measure the length of the enclosed column of air in the tube B, making due allowance for the irregular shape of the upper end. The length of the air column will be proportional to the volume of the enclosed air. Hence for volume we can consider length only. If the tube A be lowered below the level of the tube B so that the difference in level between the mercury surfaces in the two tubes be h' the pressure on the gas in the tube B will be $P-h'$ cm. Take several readings for the pressure and volume of the enclosed gas. It will be found from the observations that the product of pressure and volume (or length) has the same value in each case.

Boyle verified the law for a small range of pressures. The investigation was extended further by Amagat, who used pressures upto 3,000 atmospheres. He found that the gases do not obey this law rigorously over a wide range of pressures, but for ordinary pressures the law* is quite accurate.

If the pressure be kept constant and the temperature be changed, it is found that the volume of a gas changes considerably. The quantitative relation between the increase in volume and the rise in temperature or the decrease in volume and the fall in temperature was first discovered by Charles. He found that *the volume of a given mass of a gas increases by* $\frac{1}{273}$ *of its volume at* $0°C$*, when it is heated at constant pressure through* $1°C$. This is known as **Charles's Law.** We shall consider this law in detail in §171.

Before we explain the working of some pneumatic appliances like air pumps, water pumps, airships and æroplanes, we shall explain briefly one of the most interesting examples of Boyle's law which we come across in our daily life, *i.e.*, 'breathing.' Every person breathes without interruption from birth until death but few ask how they breathe. Some think that they "swallow" air when they breathe but in reality they do nothing of the sort. They simply enlarge the volume of the chest-cavity when they want to breathe in by the movement of the ribs and diaphragm and thereby reduce the pressure inside the lungs. The pressure of the air outside forces air down the wind pipe until the air pressure inside the lungs becomes equal to the pressure outside. When they relax the muscles of the ribs and diaphragm they allow them to return to their original position and thus make the chest-cavity smaller, and thereby force out the used air. It is obvious from this that breathing in and out is simply an illustration of Boyle's Law.

*In the case of permanent gases like hydrogen, nitrogen, etc., the divergence from the law is very small, but in the case of gases like carbon dioxide, or sulphur dioxide, which condense at ordinary temperatures by the application of pressure alone the deviations are considerable.

141. Air Pump.—An air pump is used for exhausting air. Formerly it was used only in laboratories, but nowadays it is extensively used for commercial purposes, as for instance, in exhausting the bulbs of electric lamps, X-ray tubes, etc.

A simple form of an air pump is shown in Fig. 129.

It consists of a barrel in which an air-tight piston moves up and down. By means of tube *T* the barrel is connected to the vessel to be exhausted. There are two valves *A* and *B*, as shown in Fig. 129. Normally both of them remain closed on account of their own weight, but they open upwards when pressed from below and close all the more tightly if pressed from above.

To understand the action of the pump let us start with the piston at the bottom of the barrel. When it is raised up, a partial vacuum is created in the barrel. The valve *B* is pressed down by the atmosphere, whereas valve *A* is forced open by the air in the vessel *R* and

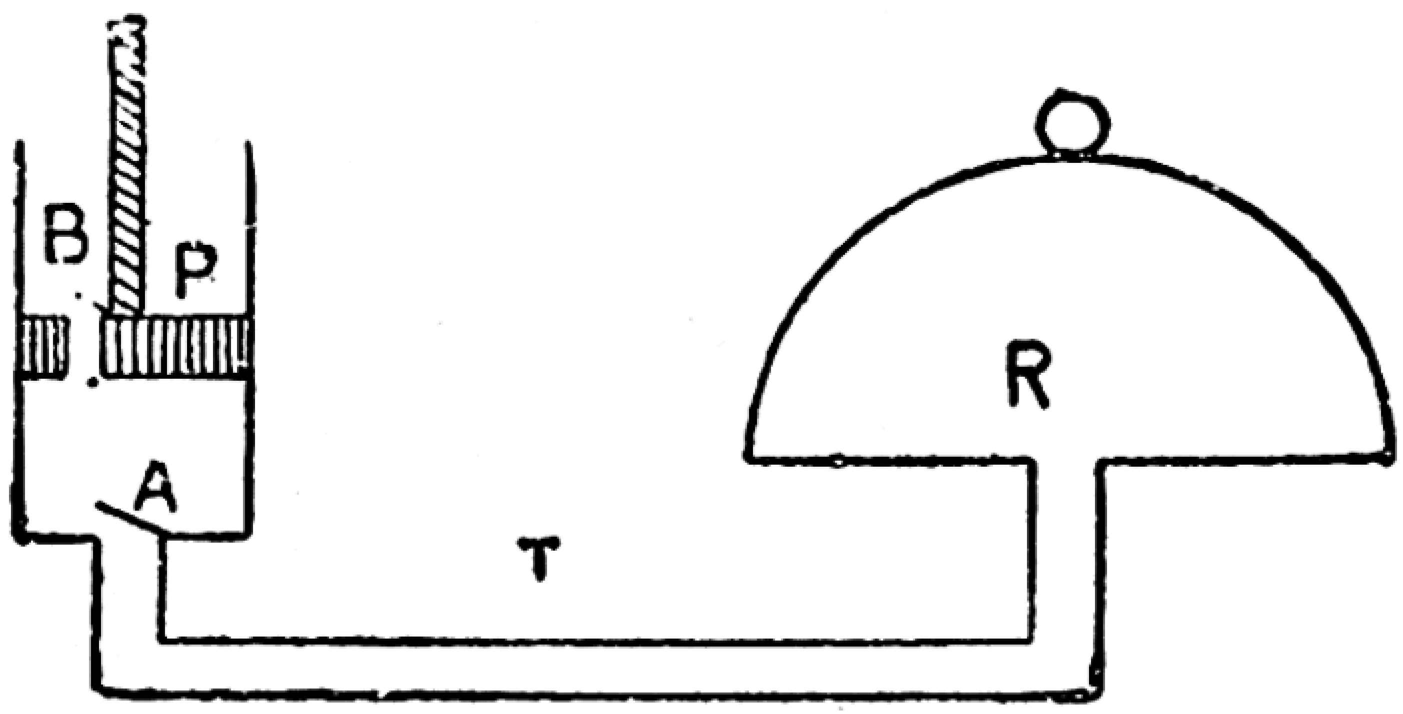


Fig. 129. Air-Pump.

the tube *T*. When the piston is at the top of the stroke, the air which at first filled the vessel *R* and the tube *T* now fills the barrel as well.

During the down-stroke, the air between the piston and the valve *A* becomes compressed, and on account of increased pressure the valve *A* closes tightly, whereas *B* opens upwards and the air is forced out into the atmosphere. The next upstroke of the piston causes *B* to close and *A* to open as in the first stroke, with the result that again some air is sucked from the vessel *R*. During the down-stroke the air in the barrel is thrown out. Every double-stroke removes some air, until finally the pressure of the air left in the vessel is too low to raise the valve *A*.

142. Even if the valve *A* were weightless, it would not be possible to attain a perfect vacuum with a mechanical pump. Let us see why? Suppose the volume of the vessel *R* and the tube *T* up to the valve *A* is V, the volume of the barrel swept through by the piston in one upward stroke is v, and the initial pressure is P.

Let us start with the piston at the bottom At the end of the first upstroke the volume increases from V to $V+v$; and the pressure falls from P to P_1, given by the relation

$$P_1(V+v)=PV \quad \text{or} \quad P_1=P\frac{V}{V+v}.$$

During the downstroke v c.c. of air are expelled and V c.c. of air at pressure P_1 are left in vessel *R* and tube *T*. At the end of the next upstroke, the volume of air in the vessel and the tube *T* again increases

to $V+v$ and the pressure falls from P_1 to P_2 where

$$P_2=P_1\left(\frac{V}{V+v}\right)=P\left(\frac{V}{V+v}\right)^2$$

After n strokes the pressure of gas inside R and T will be

$$P_n=P\left(\frac{V}{V+v}\right)^n$$

Since the fraction $\left[\frac{V}{V+v}\right]^n$ can never become zero for any finite value of n, the pressure inside the receiver can never become zero, which means that we can never attain a perfect vacuum with a finite number of strokes, or what is the same thing, that perfect exhaustion would require infinite time.

143. Rotary Oil Pump.—It would appear from the formula proved above that by increasing n we should be able to atttain as low a pressure as we please except perfect vacuum. But, as a matter of fact, this is not the case, for as soon as the pressure of the air left in the vessel is too low to raise the valve A, further exhaustion is impossible. It is found that with the mechanical air-pumps of the above type we can reduce the pressure in the vessel to about 1 mm. of mercury. With the best type we can get about $\frac{1}{25}$ of a mm. To produce higher vacuum than this we use pumps called *high vacuum pumps* to distinguish them from the ordinary pump described above. A common type of a high vacuum pump is the motor driven rotary oil pump (Fig. 130). It consists of an outer cylinder A, an inner cylinder B (called rotor) mounted eccentrically on its axis, a partition C (called vane) held tightly against the rotor in all its positions by a spring (not shown in the figure), inlet i, and outlet O. The vessel to be exhausted is

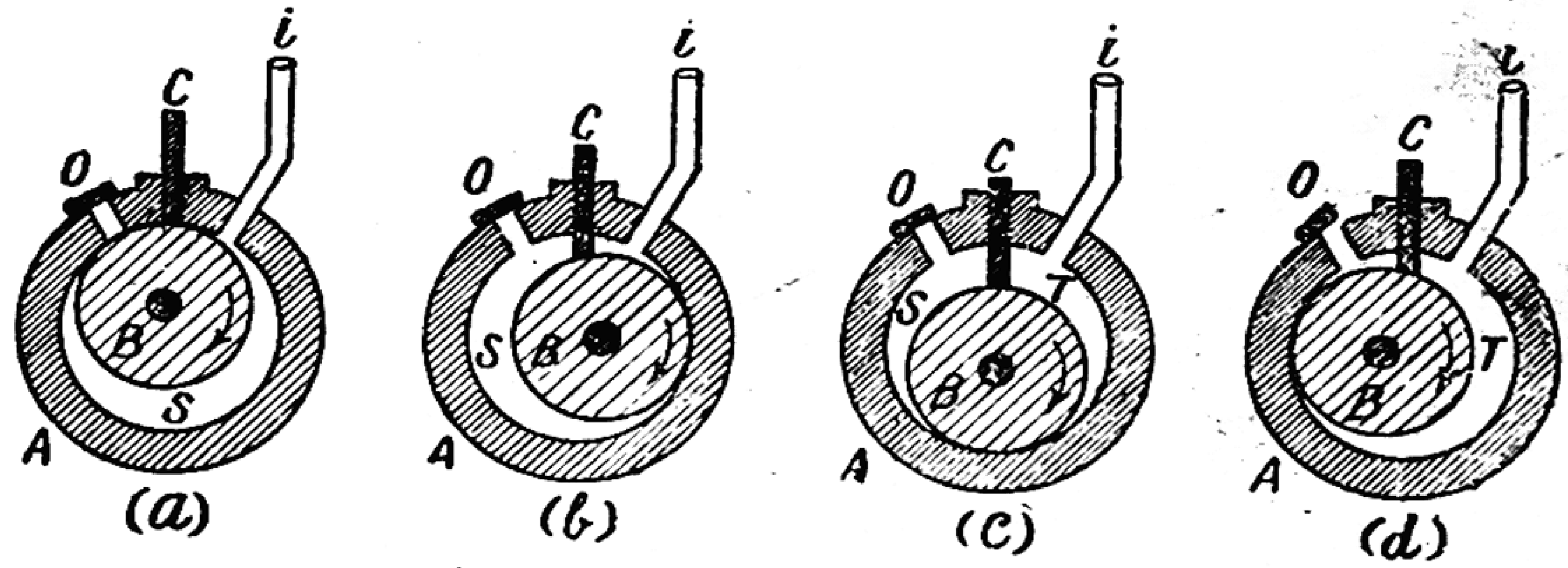


Fig. 130. Rotary oil pump.

connected to inlet i. When the rotor is in position (a) the space between A and B is in communication with the vessel to be exhausted. As the rotor turns the space becomes divided into two parts by the vane C, as shown in Fig. 130 (b). These parts are called S and T. In position (c) of the rotor the space S is reduced in volume and consequently the air in it is compressed. When the pressure of the gas in S becomes sufficient to raise the valve [see Fig. 130 (d)], the air is forced out of the outlet O. In the meanwhile the space T increases in volume and air flows into it from the vessel. The above cycle is then again repeated with the air in T.

To eliminate leakage the pump is immersed in oil. Such pumps under favourable conditions give pressures as low as 10^{-4} mm.

of mercury.

144. Diffusion Pump.—For obtaining vacuum higher than 10^{-4} mm., mercury vapour pumps are commonly used. A simple form of such a pump is shown in Fig. 131. When the mercury is boiled in vessel A, a strong stream of mercury vapour passes up the tube B and comes out into a bigger tube from the nozzle N. It is then condensed and returned to the vessel A. As these heavy molecules rush out of the nozzle they strike against the air molecules which diffuse into their way from the vessel to be exhausted and connected to the pump at i and drive them forward. Very quickly such a pump carries away the last trace of gas from a vessel.

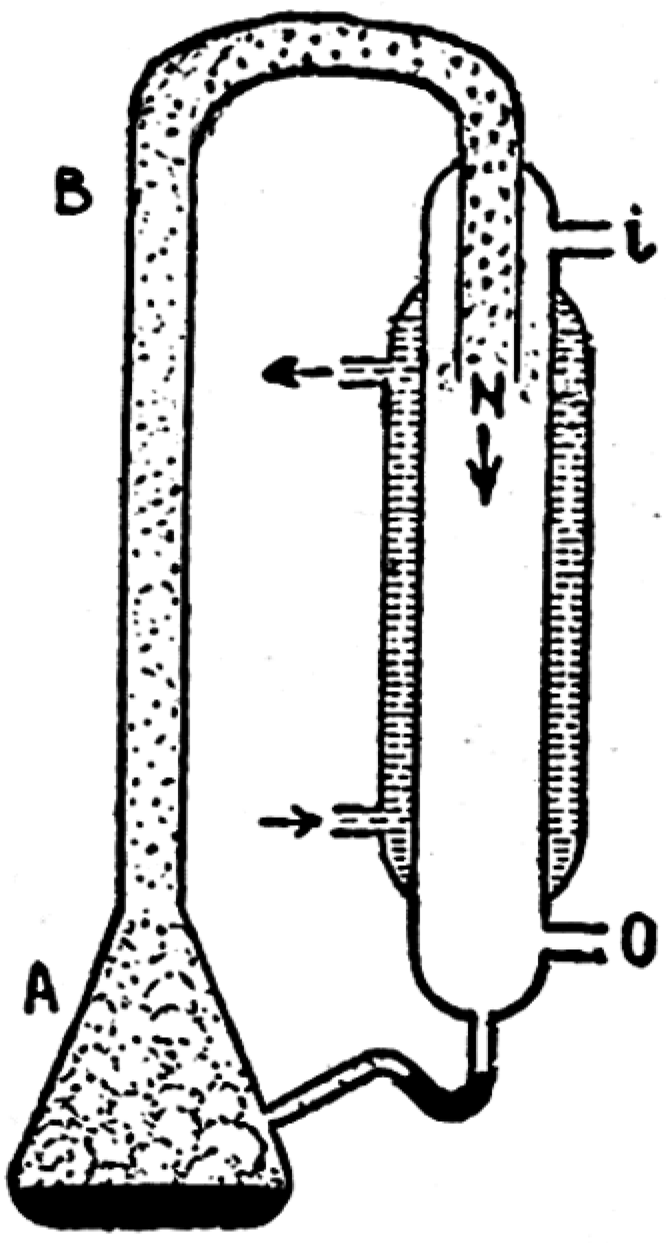
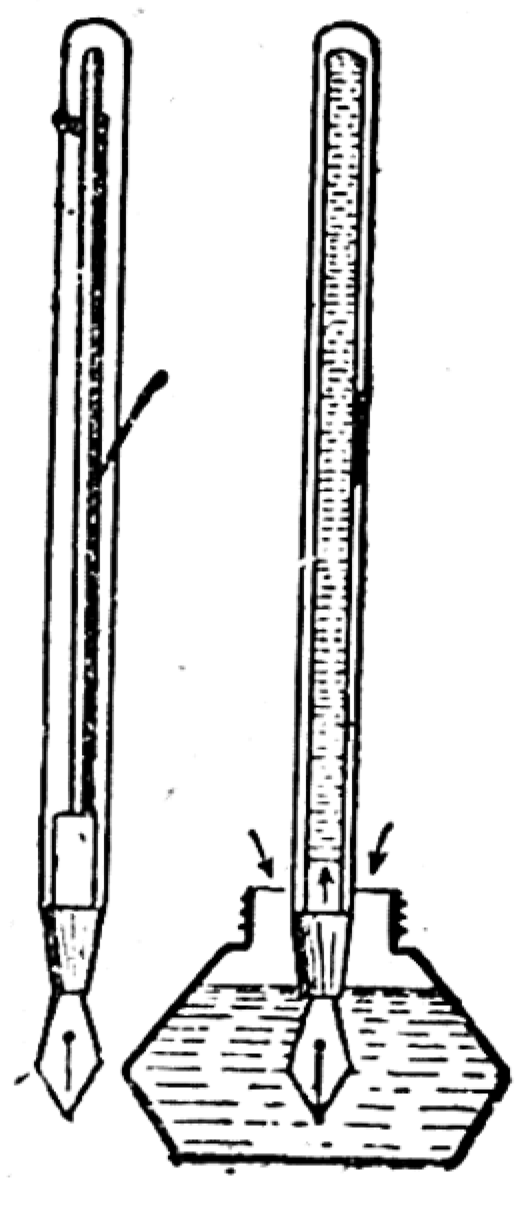


Fig. 131.
Diffusion Pump.

Such a pump will not work against high pressure at the exit O, as then the pressure will force the air molecules back. Hence usually a rotary pump or a filter pump described already is used in conjunction with a diffusion pump to give a fore-vacuum at the exit. The rotary pump so used is called a backing pump.

Do you know that when you fill your self-filling fountain pen with ink you are really first exhausting air? The diagram will help you to understand as to how it is done. Inside these self-filling pens which are filled with the help of a lever fixed in the side there is a rubber tube. It is flattened by moving the lever as shown in part (a) of Fig. 132. and thereby the air is forced out. When the lever is released with the nib under the surface of the ink the atmosphere pushes the ink up into the tube as shown in Fig. 132(b).



(a) (b)
Fig. 132.

145. Compression Pump.—The object of a compression pump, or a compressor, is to pump air into a given vessel, so as to increase the pressure within it. The mechanism of such a pump is practically the same as that of an air pump discussed in §141 *except that the valves open in the reverse direction, i.e.*, downwards instead of upwards. The use of compressed air in industries makes the compression pump very important.

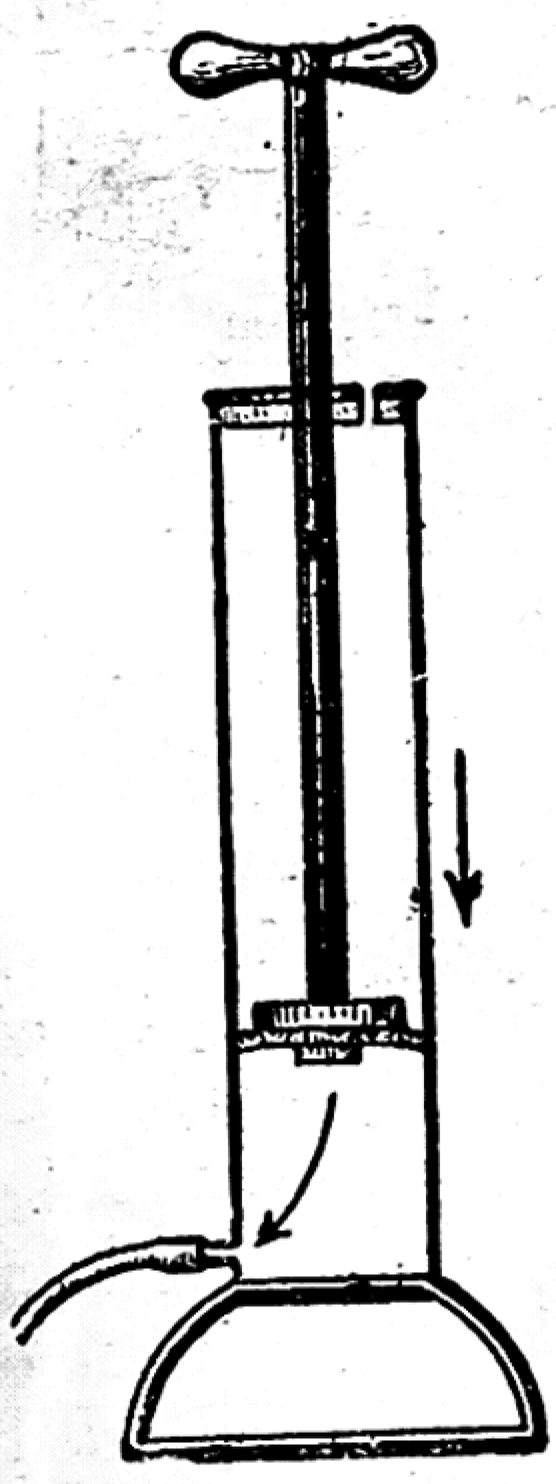


Fig. 133.
Bicycle Pump.

To explain its principle we shall consider a bicycle pump which is perhaps the most familiar compression pump. It consists of a barrel in which a piston moves up and down. A cup-shaped washer, made of a leather piece slightly larger than the diameter of the barrel, is fixed to the disc of the piston with a nut. The rim of the washer is turned towards the bottom of the barrel. The washer acts as a valve corresponding to valve B in the piston of the air-pump. When the piston is raised up, air rushes

down the space between the rim of the washer and the walls of the barrel into the space below. During the downstroke the increased pressure of the air below the piston presses the rim of the washer against the walls and opens the valve of the valve-tube through which the compressed air makes its way into the rubber tube of the bicycle.

Note the second valve is in the valve-tube. This too, opens downwards.

The compressed air in the bicycle or motor car tyres forms an elastic cushion to absorb the shocks due to rough roads. There are several appliances in which compressed air is used. A familiar appliance of this type is an oil stove. We shall briefly describe its construction and principle of action.

146. Oil Stove.—The oil stove consists of a reservoir of metal *A*, (Fig. 134) to which are fitted a pump *B*, and a tube *D* (which terminates at a distance of a centimetre or less from the bottom of the reservoir). It has two holes at the top, *C* and *F*. *C* can be closed with a lid and is used for filling the reservoir with kerosene oil. On the second hole *F* is firmly screwed a vaporising tube *K* having a small hole *H* at the top. Inside tube *K* is inserted a piece of wire gauze rolled up into a cylindrical roll.*

In addition to the above parts the stove has a metallic cup *G*, fitted near the lower end of the vaporising tube *K*, a burner *L*, and a burner plate *M*, near the top of the burner.

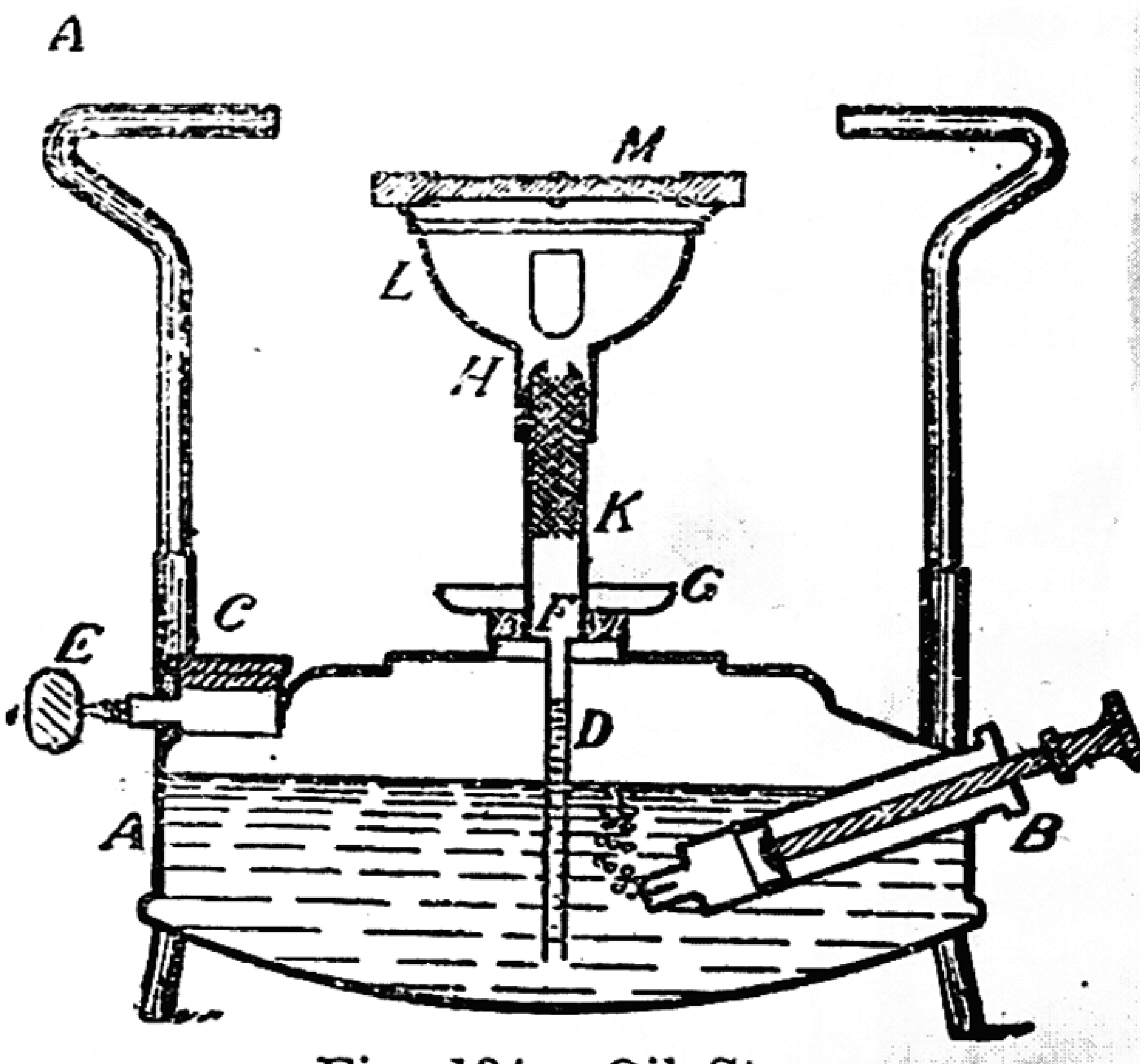


Fig. 134. Oil Stove.

To start the stove working, first unscrew the lid closing the hole *C*, and pour into the reservoir *kerosene oil sufficient to fill it about two-thirds.* Close the hole *C*, by screwing on the lid tightly, and open the air valve *E*, attached to its side.

For preheating the vaporising tube fill the cup *G* with methylated spirit and apply a burning match to it. By the time the spirit in the cup burns out, a tiny flame will be noticed at *H*. Now close the air-valve and give a few full strokes to the piston of the pump†. The stove will be seen to be lit up with a blue flame.

Working Principle. When the air pump is given a few strokes the air on the surface of the oil gets compressed. This increased pressure on the surface of the oil forces the latter up the tube *D*. As soon as the oil reaches the bottom of the vaporising tube *K*, it changes into gas. In the gaseous condition the fuel issues through the small opening *H*, mixes with air and burns with an intensely hot blue flame. This flame

*The object of the gauze is to stop the dust particles and other suspended impurities in the oil to come to the top of the vaporizing tube and thereby choke the hole *H*.

†The nozzle of the pump is fitted with a valve such that it does not allow oil to enter the barrel of pump.

keeps the vaporising tube sufficiently hot to convert the oil into the gaseous state as it passes up.

To extinguish the stove, open the valve *E*. The air in the reservoir is no more compressed and hence the oil is not forced up the tube.

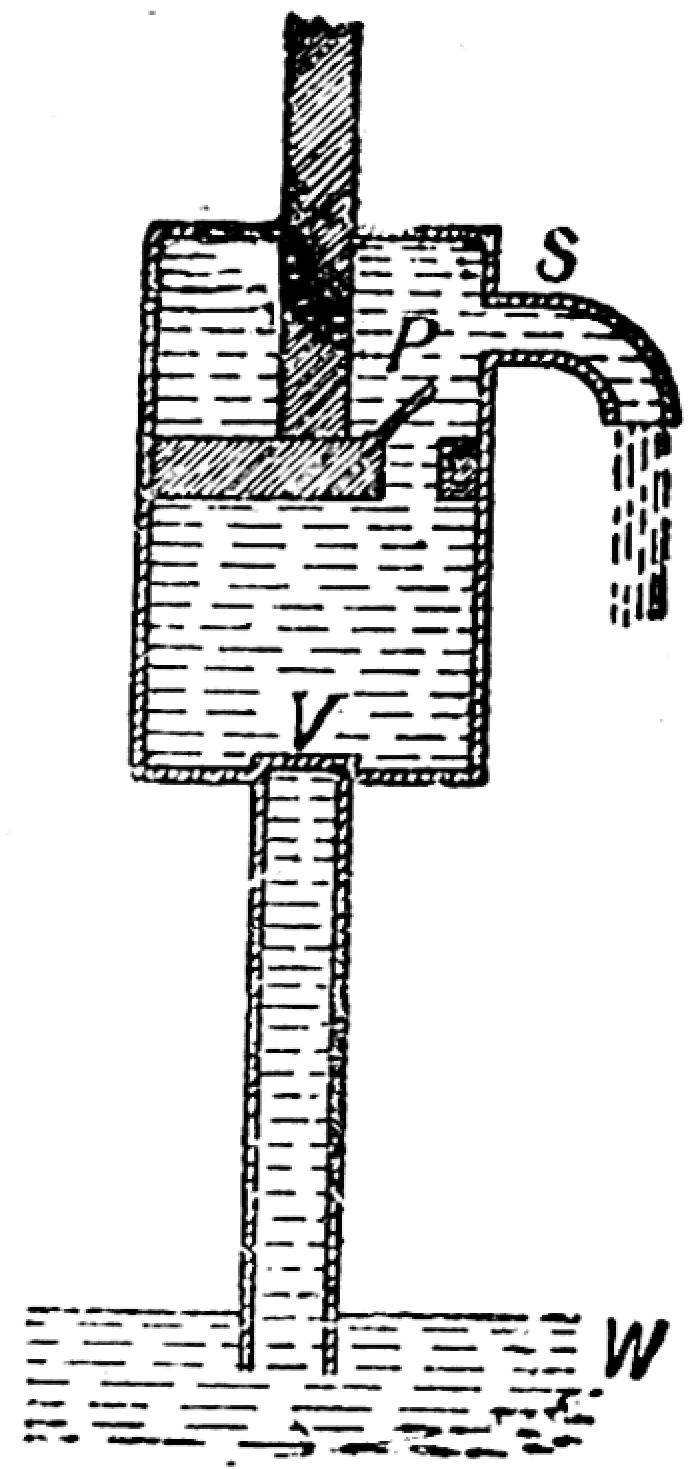


Fig. 135.
Water Pump.

147. Water Pump.—It is a machine used for raising water. It consists of a long vertical pipe of iron, the lower end of which dips in water *W*. At its upper end there is a valve *V* which opens upwards into a barrel. There is a second valve *P* in the piston, which also opens upwards. *S* is the outlet by which water passes out (Fig. 135).

A water-pump in the first few strokes acts like an air-pump and exhausts air from the pipe below the valve *V*. To see how it acts, let us start with the piston at the bottom of the barrel. When it is raised up, a partial vacuum is created above *V*. Now since the pressure on the lower surface of *V* is greater than that on the upper surface, the valve opens upwards, allowing the air in the pipe to enter the barrel. Due to this expansion the pressure of air in the pipe is reduced. This atmospheric pressure on the water at *W* consequently forces some water up the pipe. During the down-stroke the air in the barrel below the piston is compressed, and on account of the increased pressure *V* closes and *P* opens upwards, allowing the air in the barrel to escape into the atmosphere. During the next up-stroke some more water is forced into the pipe, and so on, till all the air in the pipe is removed and water is pushed through the valve *V* into the barrel therefrom outside into the air.

Since water is raised in the pipe by the atmospheric pressure it is clear that the pipe must not be longer than 34 ft. (the barometric height of water column), otherwise water will not reach the valve *V*. Owing to mechanical imperfections it is not possible to raise water from a depth more than 29 or 30 ft. Pouring in a little water at the top, or as it is often called "priming" the pump helps it to start by swelling the valves and leather of the piston and thereby making them air-tight. Water pump is also sometimes called **Suction** or **Lift Pump.**

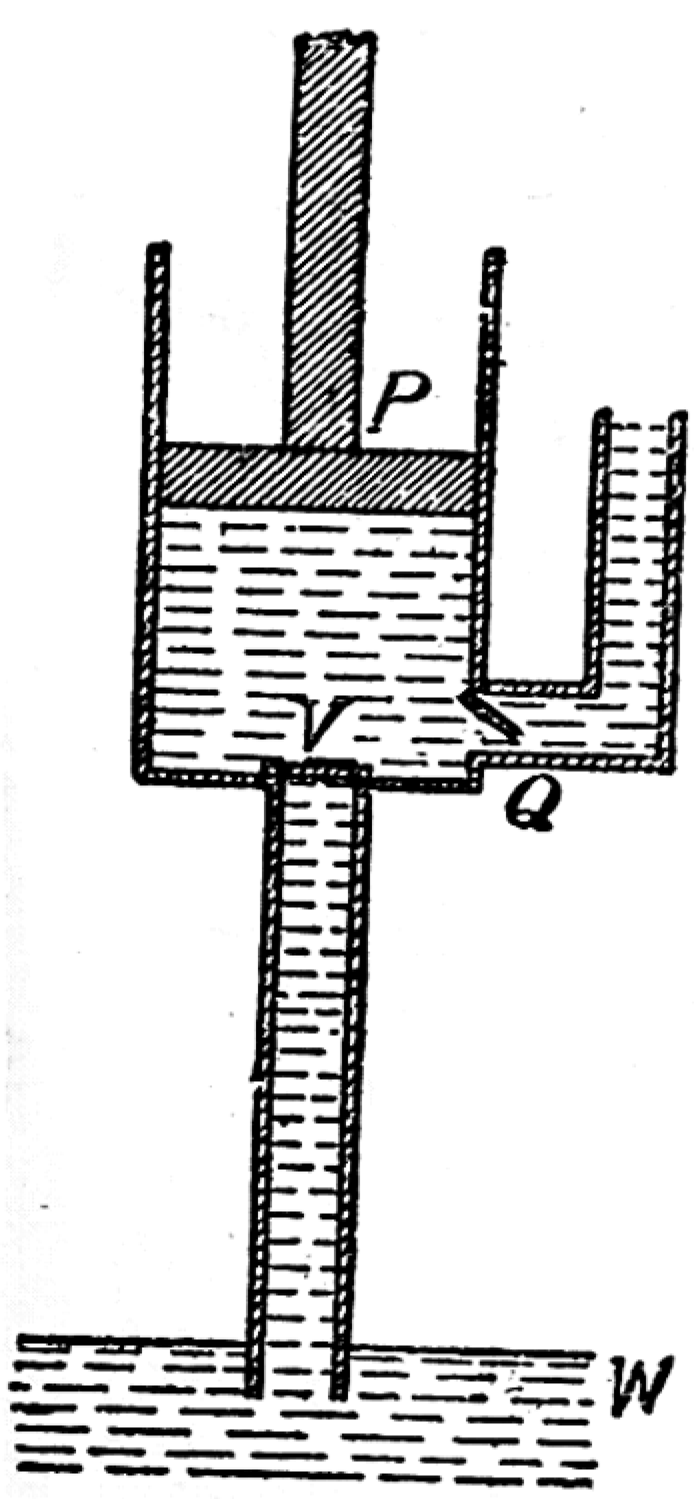


Fig. 136.
Force Pump.

Force-Pump is slightly different in construction (Fig. 136). The side valve opens outwards into a side tube *Q*, fitted near the bottom of the barrel, instead of in the piston. The action up to *V* is just the same as in the water-pump, and hence is subject to the same limitations. After the water has entered the barrel it is forced up the side tube by the force of the piston during the downward stroke, the valve *V* remaining closed and *Q* open.

With the help of this pump, water may be forced to any height

above the level of the pump, but, on account of the limitations referred to above, the valve V at the bottom of the barrel must be within 30 ft. from the surface of water at W.

Perhaps the most familiar example of this type is the **Fire-Engine.**

148. Petrol Pump.—The petrol pump has become a very familiar sight these days. It is used to raise petrol from an underground storage tank into a graduated glass container wherefrom it is delivered to motor cars and trucks.

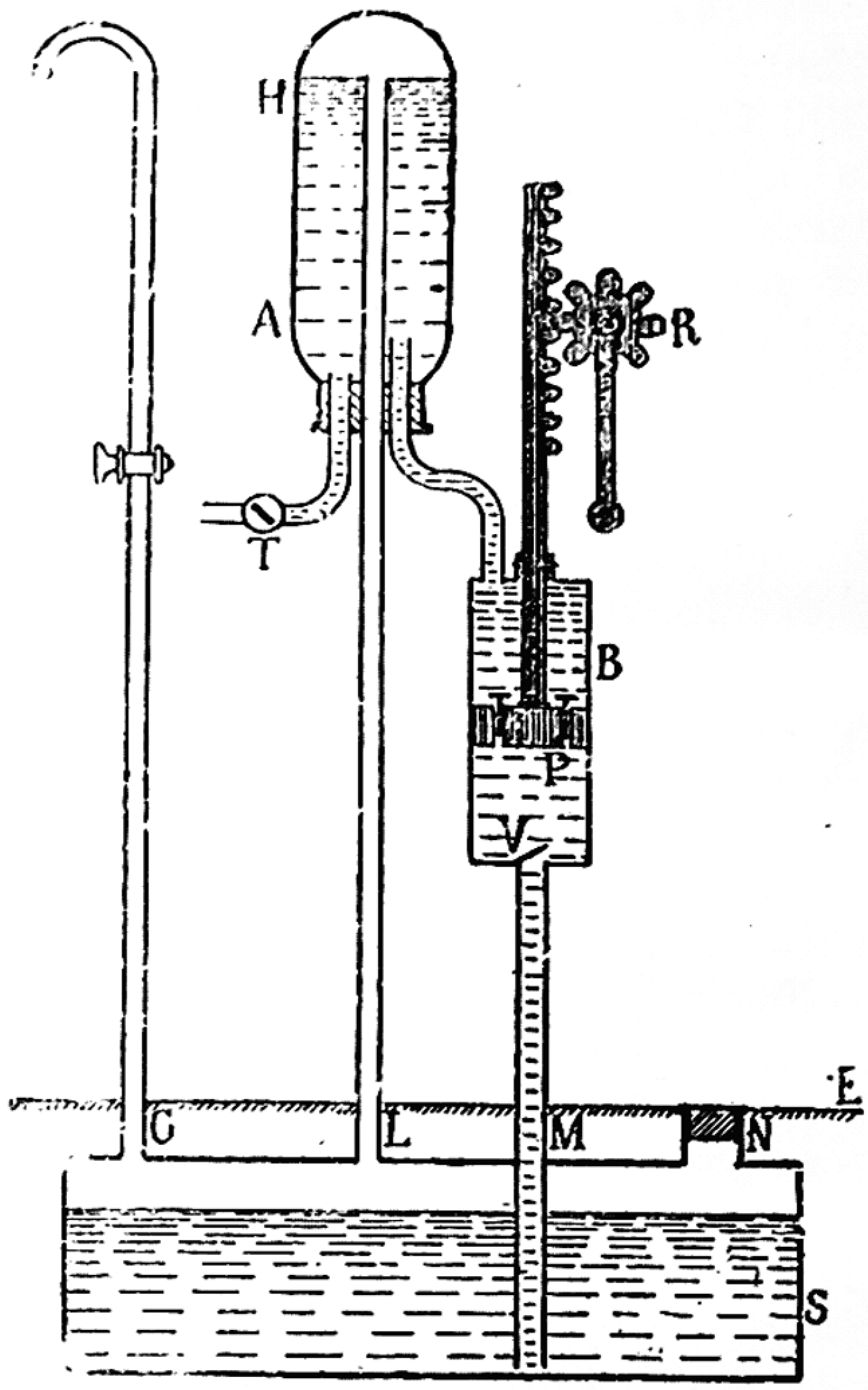


Fig. 137.

In Fig. 137, B is the barrel of the pump, P the piston which is worked with the ratchet R, S the underground tank and A the glass container.

When the piston P is raised by means of the ratchet R, the valves in the piston remain closed and the valve V opens, so that petrol rises into the barrel B.

When the piston is pushed down the valve V is closed and the valves in the piston open and the petrol is forced into the glass container A. The air in the container escapes through the pipe L into the tank and from there into the atmosphere through the pipe O. When A is filled up to height H, the excess petrol pumped into it escapes down into the tank through the pipe L, which acts also as an over-flow pipe.

The petrol in A is then delivered to a car through the tap T.

Some containers have a capacity of ten gallons and some of one gallon only.

The tank is filled with petrol through the pipe N, the mouth of which is covered with an iron plate and is kept under lock and key.

149. Flying Machines.—The possibility of ærial flight was foreseen by the writers of the fourth century, but the problem remained practically unsolved till very recent times.

The first instrument which enabled man to rise into air like a bird was a balloon.

At first balloons were filled with hot air. It was Professor Charles of Paris, who substituted first hydrogen for hot air, and thereby not only removed the danger of serious accidents to which the hot-air balloons were liable, but also increased their weight-lifting capacity. A balloon of the capacity of a room (12′ by 10′ and 12 ft. high) will require 8 lb. of hydrogen to fill it at 0°C. The hot air required to fill the same balloon at the same pressure will weigh 44 lb. The weight of the same volume of air, at the same temperature and pressure, will be about

109 lb. Hence the balloon will lift a weight of 101 lb. in all (including its own weight) when filled with hydrogen as against 65 lb. when filled with hot air.

One of the most remarkable ascents was made in 1931 by Piccard of Belgium. He attained a height of 9·6 miles above the earth. The pressure recorded at that height was about 5 inches and the temperature, $-50°C$. Anderson and Stevens in 1935 reached a height of 13·7 miles.

A free balloon unprovided with any means of propulsion is at the mercy of the wind and completely beyond the control of the æronaut so far as direction is concerned. He may, by rising or falling* to another level, find a favourable wind which will carry him in the desired direction, but under ordinary circumstances the probability of his reaching the destination is very small. Steering rudders cannot be of any help *for there is no relative motion between the balloon and the air.* So it was recognized at quite an early date that if a sufficiently light and powerful engine capable of driving the balloon through the air in which it floated could be obtained, the æronaut could go in the direction desired. All the remarkable advances that have been made during the recent years in ærial navigation have been mostly due to the development of such an engine—the internal combustion engine.

To sum up, there are two primary questions involved in the problem of ærial navigation.

(1) *The upward movement of the machine against the action of gravity.*

(2) *Its propulsion through air.*

In addition to these two primary questions there are a good many

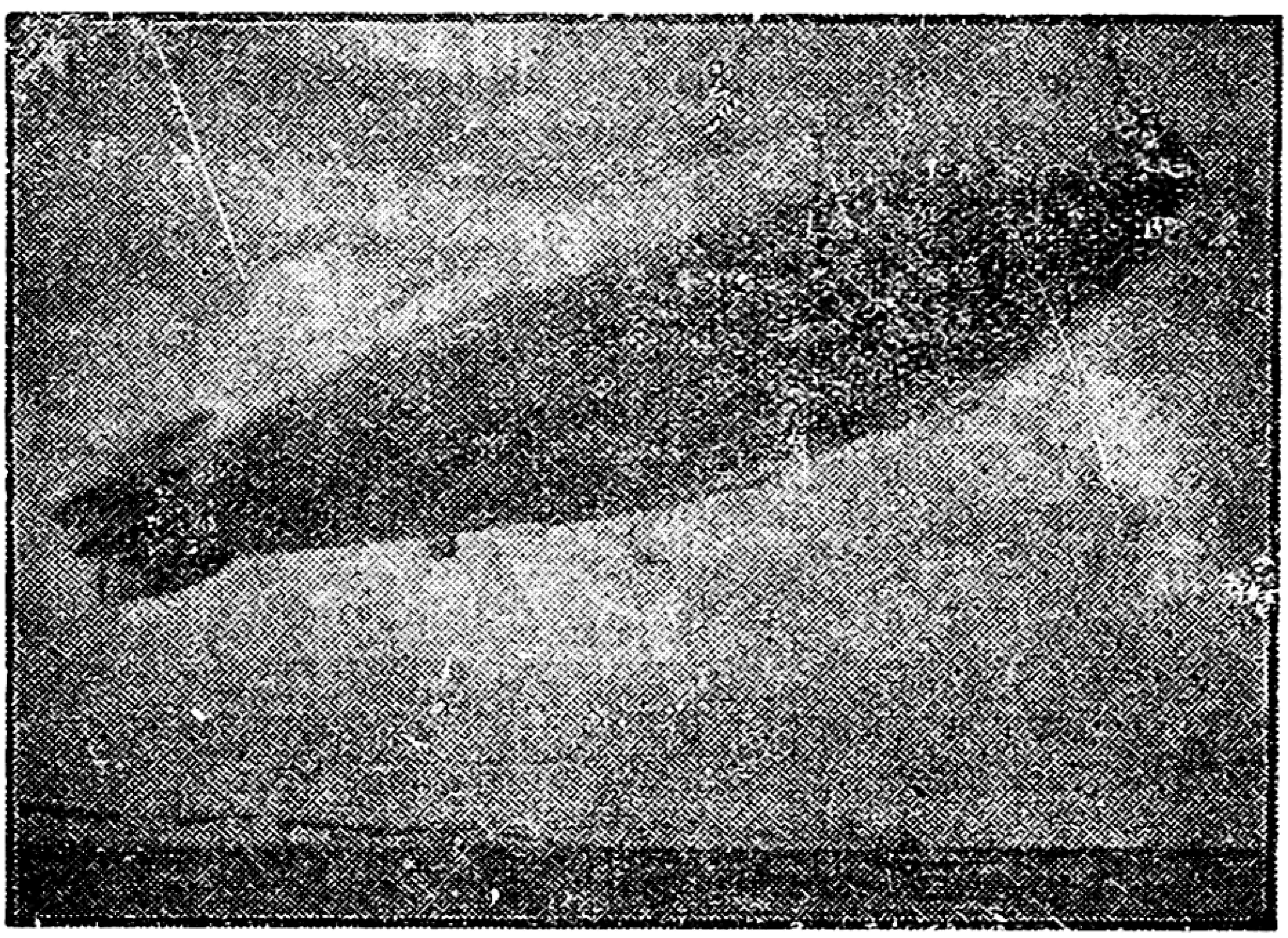


Fig. 138. Airship.

secondary questions, such as stability, control, etc., but we shall confine ourselves to the two primary questions only.

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*If the æronaut wishes to descend, he opens a valve with the result that the gas inside escapes and the balloon sinks. If he wants to make the speed slower or wants to rise again, he does so by emptying out the bags of sand which are carried by him in the balloon.

The flying machines of to-day are of two kinds :

(*a*) *Lighter-than air*, or *airships* ; and

(*b*) *Heavier-than-air* or *æroplanes*.

We shall first briefly refer to airships, and then explain the principle of æroplanes.

150. Airships.—They are huge cigar-shaped balloons (Fig. 138) provided with light engines of high power. The frame-work is made of aluminium or one of its alloys and is covered on outside with a waterproof fabric. It is divided into a large number of compartments, each containing a gas bag filled with hydrogen or helium* which gives the necessary buoyancy. The engines drive the propellers.

Description of two airships will be given here, just to enable the student to form some idea about the machines used.

(*a*) The *R* 34, which was the first airship to cross the Atlantic Ocean, was 672 ft. long, 90 ft. high and 79 feet in diameter. It had 18 gas bags of total capacity 2,000,000 c. ft. Its engines developed nearly 1400 h.p.

(*b*) The *R* 101 shown in (Fig. 138) was a British airship. It was 732 feet long and 132 feet in diameter ; and was fitted with five Diesel engines developing nearly 2600 h. p. The gas-bags had a total capacity of 5,000,000 cubic feet. Its maximum lift was 150 tons.

151. Aeroplanes.—The æroplanes, as has been said above, are heavier-than-air machines. Before we explain the principle on which they are based, let us first understand how a kite flies.

If a kite is balanced in a wind it speedily attains a certain height, at which it remains so long as the wind does not drop. Let us see what forces are acting upon it when it is balanced. They are shown diagrammatically in Fig. 139.

(*i*) The weight of the kite.

(*ii*) The pressure of the wind on the lower surface of the kite.

(*iii*) The pull of the string.

Forces (*i*) and (*iii*) pull the kite downwards, whereas force (*ii*) tends to lift it. The force (*ii*), *viz.*, the pressure of the wind, is really a combination of two forces:—(*a*) the *drift* tending to move the kite in the direction of the wind, and (*b*) the *lift*, tending to raise the kite vertically up. When the kite is in equilibrium, it is evident that the *lift* acting on it must be equal and opposite to the sum of the vertically downward forces, *i.e.*, the weight and the vertical component of the pull of the string; and the drift must be equal and opposite to the horizontal component of the pull.

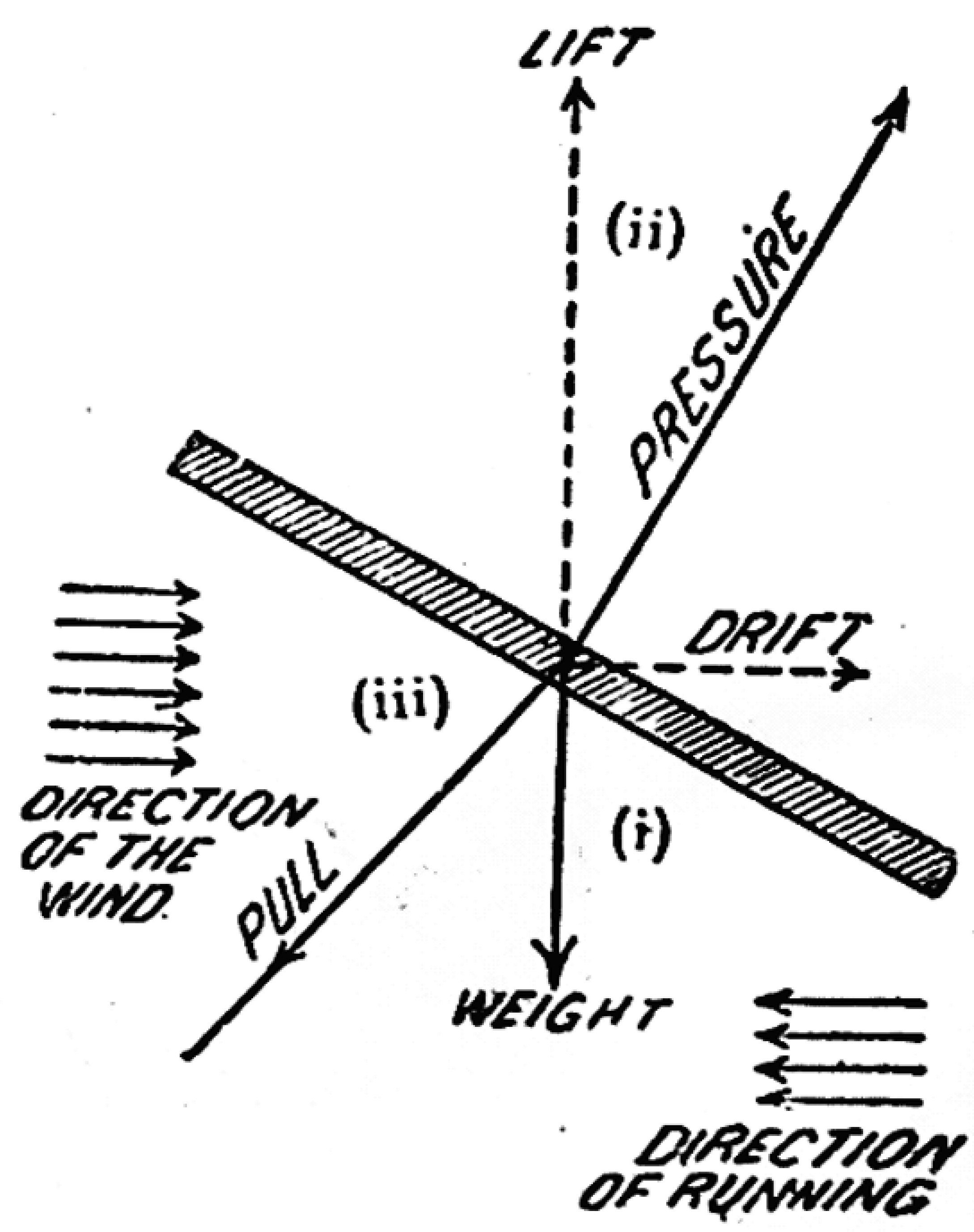

Fig. 139.

*Helium has the advantage in that it is not inflammable and its lifting power is almost as great as that of hydrogen. It should be clearly understood that the lifting power depends not on the density of the gas filling the gas-bags but on the difference between its density and the density of air.

But when the vertical component of the pressure of the wind becomes greater than the sum of the force of gravity and the downward component of the pull of the string, it lifts the kite.

Now let us explain the principle of an æroplane. Suppose a plane is held inclined to the wind. If the angle of inclination between the direction of the wind and the plane is small, the thrust (which is almost normal) in kilograms due to the wind is equal to

$$kSv^2\theta$$

where S denotes the surface of the plane in sq. metres, v the velocity of the wind in metres per second, θ the angle of inclination (in degrees), and k a constant depending on the shape of the plane. Its value varies from 0·005 to 0·007. As in the case of the kite, the thrust can be resolved into two components, one tending to move the plane in the direction of the wind, and the other tending to lift it vertically up.

It is clear from the expression given above that in order to increase the pressure and therefore the lift the speed of the wind should be increased and the greater the speed, the greater the lift. Since by the speed of the wind we mean the relative speed between the plane and the wind, if the speed of wind is not sufficient we can move the plane and increase the relative speed and thereby the lift. In normal flight an æroplane makes an angle of 3° to 6° with the horizontal, and is driven forward by propellers which produce a wind. The effect is the same as if a strong wind were blowing against the plane and the plane were standing still. This is the reason why the æroplane is made to run along the ground before flying, because it will rise only when a certain speed* is reached. It should be noted that the æroplane rises only when the engine works. As soon as it stops, the æroplane begins to move down.

The pressure due to wind is not the only factor which is responsible for supporting the æroplane in the air. There is another factor. As the plane moves forward the air slides along the surfaces of the plane. But it moves much more rapidly over the upper surface than along the lower surface as said in §132*d* (Bernoulli effect) with the result that the pressure above the plane becomes lowers than the pressure on the lower surface. The difference in the pressure helps to support the æroplane.

As a result of experimental work it has been found that the lift due to the difference of the pressure on the two sides is twice that due to the pressure of wind on the lower surface of the plane. It is on account of this reason that much attention has been paid to the form of the plane. It is found that by making the plane thick near the front edge, and tapering near the other edge and by slightly curving it as shown in Fig. 121 the lift is greatly improved.

It should be noted that while the æroplane runs along the ground the angle of attack *i.e.*, the angle of the plane with the horizontal is very small and hence the entire lift is due to Bernoulli effect. It is only after the plane leaves the ground that the angle of attack is increased and the lift due to pressure on the lower side of the plane becomes added to the lift due to Bernoulli effect.

*Of course the speed depends upon the weight to be lifted.

The æroplanes may have one, two, or three planes ; in the first case they are called monoplanes, in the second biplanes, and in the third triplanes. In biplanes (and triplanes) the planes are kept apart

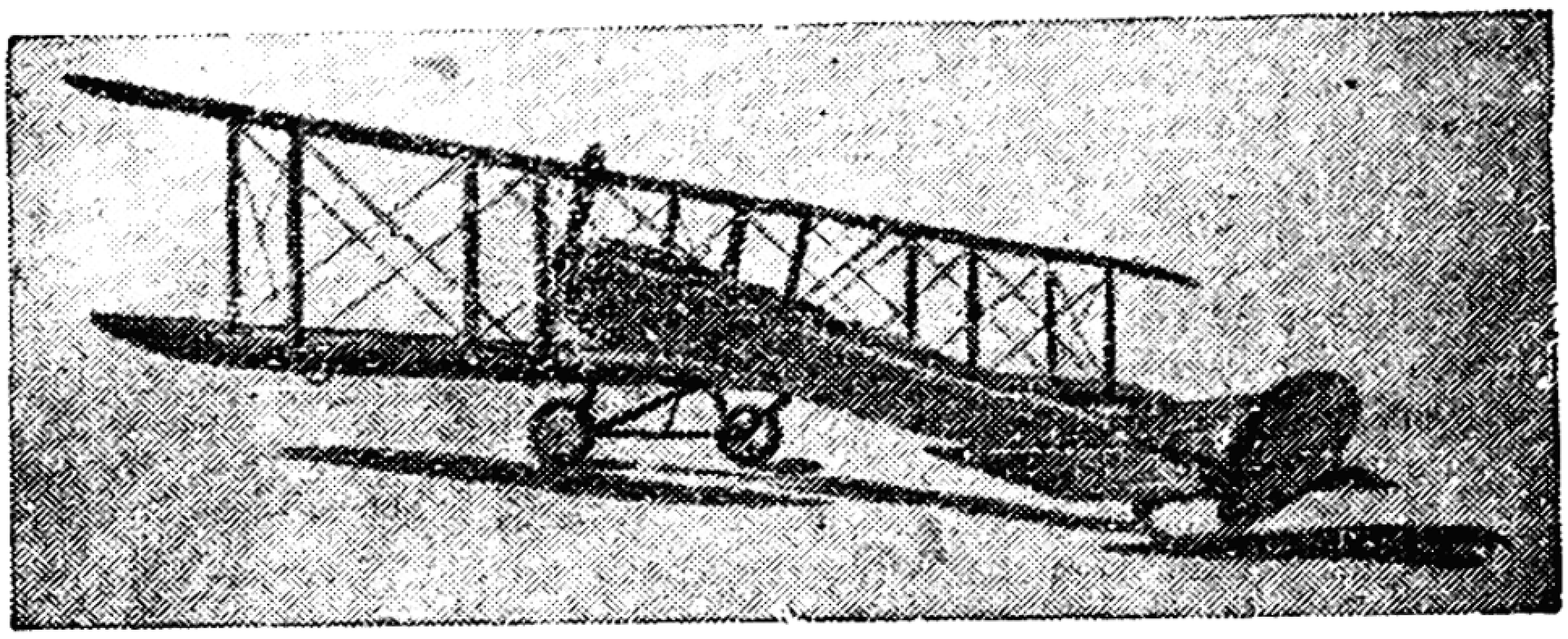

Fig. 140. Aeroplane.

by wooden struts which are about a foot wide ; the struts have the maximum thickness (about 3 inches) along the middle line in the vertical plane, and taper towards the edges in order to reduce the resistance of the air to the motion of the æroplane.

The propeller blades are generally made of wood and are two in number. The diameter varies usually from 5 to 10 ft., and the width is about a foot. They are generally placed at the front. The blades are curved in such a manner that when they revolve, the air pressure pushes them from behind. The horse power of the engine which revolves the propellers depends on the size of the æroplane.

There is a tail at the other end of the æroplane. It is made up of three parts, one vertical plane in the middle, and two horizontal wings just at right angles to the central vertical plane (Fig. 140). Each of the horizontal wings is made up of two parts ; one remains fixed, whereas the angle of inclination of the other with the horizontal can be varied. At first this second part is inclined a little below the horizontal. For rising this part is tilted up, and for descending it is tilted down. For going to the right the central vertical plane is tilted to the right, and for turning to the left the vertical plane is tilted to the left.

To start the machine, the engine is started and two men rotate the propeller blades. When they begin to move rapidly the men get aside and remove the obstacles placed before the wheels. The æroplane then runs forward and as soon as the lift is sufficiently strong to overcome the pull of gravity on the machine it begins to rise up. In the case of an ordinary military æroplane, a speed of 40 miles an hour produces a lift on the plane sufficiently strong to raise the machine high up in the air.

EXERCISES

1. Explain the principle of an aneroid barometer. Can you use it to measure heights and foretell weather ? Give reasons for your answer.

2. State Boyle's Law. How will you verify it experimentally ? The length of the Torricellian vacuum in a barometer tube was 25 cm. when the reading was 75 cm. When 11 c.c. of air at atmosphe-

ric pressure were admitted, the reading was 40 cm. Calculate the diameter of the tube. *Ans.* 7 mm.

3. Explain the principle of kite-flying. Could it be flown from a running motor car when there is no wind ?

4. The total lift of an airship filled with hydrogen is about 70 tons. Helium is twice as heavy as hydrogen. What will be the total lift of the airship if helium is used ? *Ans.* 93 per cent.

5. Why must an æroplane run forward before flying upward ? Does an airship also run forward ? If not, why not ?

6. Describe as fully as you can the working of an air-pump.

After 4 strokes the density of the air in the receiver of an air-pump is found to bear to its original density the ratio of 256 to 625. What is the ratio of the volume of the barrel to that of the receiver ? *Ans.* $\frac{1}{4}$.

7. An accurate barometer reads 30 inches when one containing air above the mercury column reads 24 inches. If the tube of the latter be raised 3 inches, the reading becomes 25 inches. Find what length of the tube the air would occupy if brought to atmospheric pressure. *Ans.* 2 inches.

8. The space above mercury in a faulty barometer measures 12 cm. and the mercury column extends 72 cm. above mercury in the cistern. On depressing the tube into the cistern, the mercury stands at 70 cm. and the space above measures 7·2 cm. What is the atmospheric pressure in cm. of mercury ? (*P. U. 1932*). *Ans.* 75 cm.

9. It was said during the war that the "Germans have discovered a gas which has *several times* the lifting power of hydrogen." Could this be possible ? Give reasons for your answer.

10. Explain the action of a medicine dropper.

When you drink lemonade through a straw does the liquid go up into your mouth on account of your effort or because of the push of the atmosphere on the liquid ?

PART II

HEAT

CHAPTER I

Thermometry

152. Take a kettle containing cold water and place it over fire. Dip your hand in water from time to time. You will find that at first it becomes lukewarm, then warm, hot, very hot and so on till at last it begins to boil. To account for the change that takes place in its condition (from cold to hot) we say that something has passed from the fire to the water. This something which is responsible for the sensation of warmth is called **Heat.** In other words, *Heat is the agent which produces in us the sensation of warmth.*

No doubt, by the sense of touch we can know that the water in the kettle is getting hotter and hotter, but *how much* hot it is at a particular moment we cannot say. This means that our sense of touch does not enable us to form a quantitative estimate of hotness, *i.e.*, of the *degree of hotness.* This *degree of hotness* is called **Temperature** in Physics. It is very important to understand the difference between temperature and heat. To make the difference clear let us consider an example. Take two exactly similar Bunsen burners and connect them to gas taps and adjust their flames so that they are of the same height and character. Keep over one a kettle containing water, and over the other an iron ball. It will be noticed that the ball becomes red-hot very soon, whereas the water in the meantime is hardly affected. Since flames are similar, they have given out the same amount of heat but the degree of hotness or temperature of the ball and the water is widely different ; it is very much higher in the case of the iron ball. On introducing the ball into the water the ball parts with its heat, becoming colder thereby and making the water hot. The flow of heat supplies us with a method to find out which of the two bodies is at higher temperature. A body which parts with its heat when put in contact with another body is at a higher temperature and the body which receives heat is at a lower temperature.

To sum up *heat is the agent which makes a body hot or cold, whereas temperature is the state or condition of a body on which its power to communicate heat to, or to receive heat from, another body placed in contact with it depends.*

The following analogy will help the student to understand clearly the distinction between heat and temperature. Take two vessels of unequal size fitted with taps. Fill them half with water. The quantity of water in the bigger vessel will evidently be greater than that in

the smaller vessel. Place the bigger vessel at a low level and the smaller vessel at a high level and connect them with a rubber tube. Open the taps. The water will be seen to flow from the smaller vessel to the bigger one, in spite of the fact that the quantity of water in it is smaller. As a matter of fact, *the quantity* has nothing to do with the flow of water, otherwise rivers would never flow down mountains to the oceans, for springs contain much less water than oceans. It is, as a matter of fact, *the level*, which determines the direction of the flow of water, and not the quantity. The role which is played by level in the case of the flow of water is played by temperature in the case of the flow of heat. In other words the level corresponds to temperature, and the quantity of water to the quantity of heat.

153. Effects of Heat.—When a body is heated, the following effects are produced :

1. Rise of temperature.
2. Increase in volume.
3. Change of state.
4. Chemical action.
5. Change of physical properties.

We shall briefly consider these effects one by one.

(1) **Rise of Temperature.**—We have said above that the temperature of a body rises when it is heated. It is true so long as the physical state of the body does not change.

(2) **Increase in Volume.**—In general, all bodies, expand when heated though to different extents. As a rule, liquids expand more than solids, and gases more than liquids.

(*i*) *Expansion of Solids.*—Take *Gravesand's Ring and Ball* (Fig. 1) and notice that the ball passes through the ring freely. Heat the ball and place it on the ring. It will be found to lie there without passing, showing thereby that it has expanded. If the ball is allowed to remain there, it contracts as it cools, and the ring, on the other hand, expands a little, due to the contact of the hot ball, with the result that after some time the ball drops through.

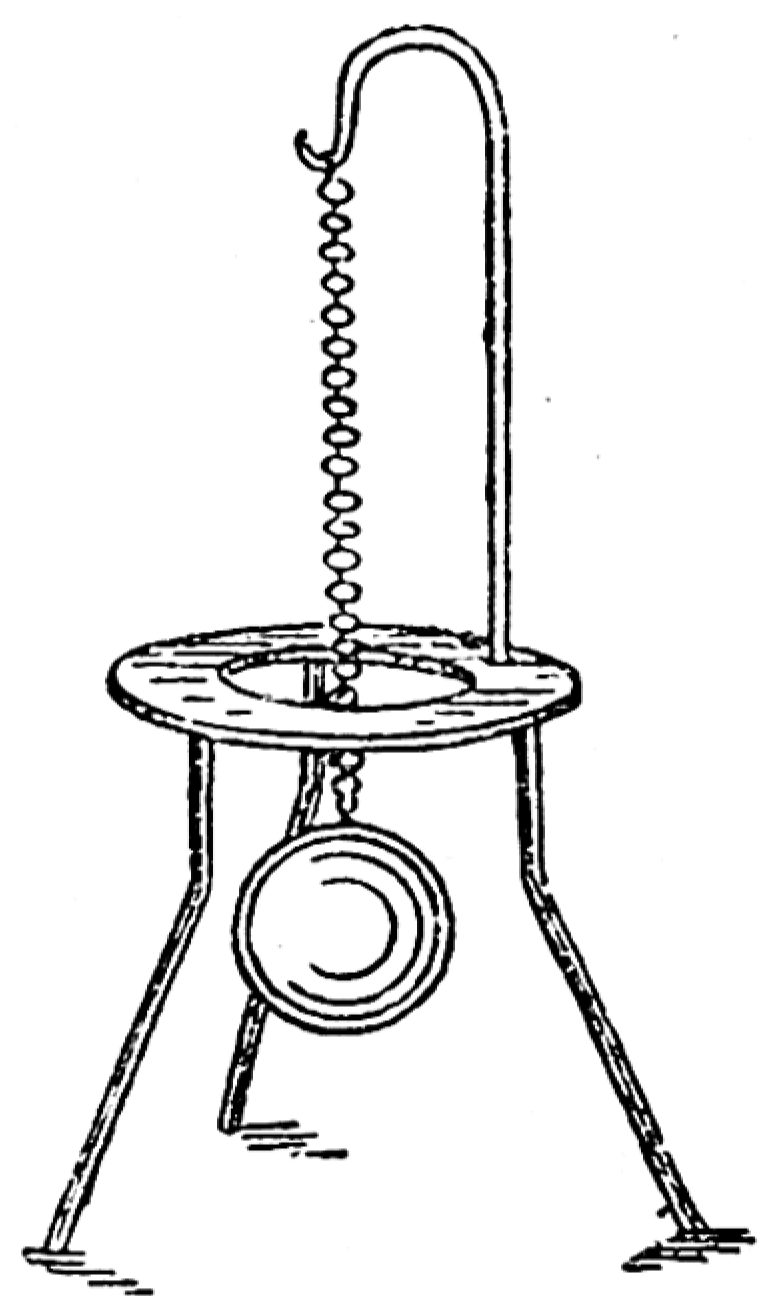

Fig. 1.
Gravesand's Ring and Ball.

(*ii*) *Expansion of Liquids.*—Take a flask, and fit it up with a cork through which a glass tube is passing (Fig. 2). Fill the flask with coloured water and force the tube down so that the water stands in the tube a little above the level of the cork. Heat the flask over a Bunsen burner. The column of coloured water will be seen to rise, proving that liquids expand when heated.

(*iii*) *Expansion of Gases.*—Take a flask fitted with a cork and a glass tube as in the previous experiment. Instead of filling it up with water, keep it empty. Hold it upside down and let the open end of the tube dip in coloured water in a beaker. Heat the flask gently with a burner.

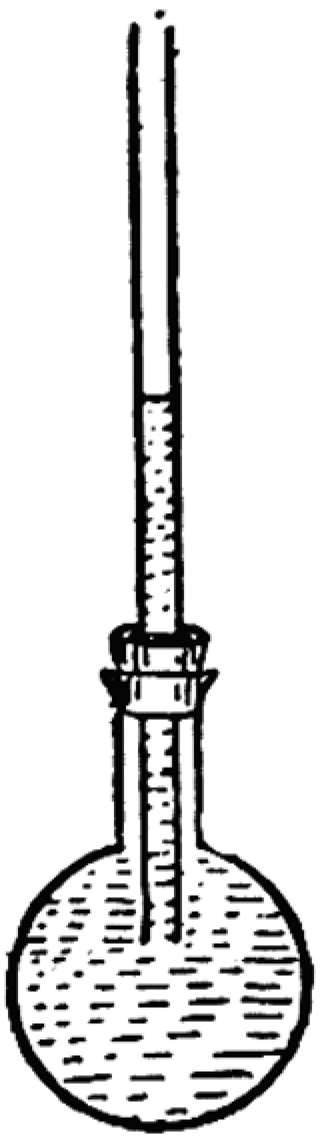

Fig. 2.

Bubbles will be seen to rise through the coloured water. Since the air in the flask on being heated increases in volume it escapes through water in the form of bubbles.

(3) **Change of State.**—It is a matter of common observation that ice when heated turns into water *viz.*, from the solid state it changes into the liquid state. This is true not only in the case of ice but in the case of other solids as well.

By further application of heat a liquid can be changed into a gas. For instance, water if heated for a sufficient time begins to boil and to change into steam, which is the gaseous form of water.

(4) **Heat accelerates Chemical Action.**—It is a well-known fact that substances combine with oxygen more easily when heated than when cold : for instance steam which has practically no action on iron filings at room temperature, gives up its oxygen to heated iron filings when passed over them forming iron oxide and leaving hydrogen free. Substances like magnesium, sulphur, wood, and coal combine vigorously with oxygen when heated, and while doing so produce a large amount of heat. This process of oxidation is usually called *combustion*. Another example of the acceleration of chemical action is met with in dissociation at high temperatures of salts like ammonium chloride which splits into ammonia and hydrochloric acid when heated.

(5) **Heat modifies Physical Properties of Substances.**—Hardness, rigidity, elasticity etc., change with the rise of temperature ; for instance, zinc is brittle and hard when cold, becomes soft and flexible when heated. Some substances undergo a change in colour when heated ; for example, mercuric iodide, which at ordinary temperatures is red, changes into yellow on being heated. The conductivity of metals for heat and electricity is also affected by increase in temperature.

154. Thermometry.—We have said in the beginning of this Chapter that by the sense of touch we can compare two bodies and say which of them is hotter than the other. But it must be remembered that it is true only in a general way, for the sensation may sometimes mislead us as shown below. Take three beakers, one containing cold water, the second tepid water, and the third hot water. Put one hand in cold water, and the other in hot water ; and after some time put both in tepid water ; to one hand the tepid water will seem cold and to the other hot. Thus we get cold as well as hot sensation from one and the same body. What sensation we get depends upon the previous state of the hand.

Next, let us suppose that on a hot day while cycling on a road we touch the handle of a bicycle instead of the grips. The handle will seem to be hotter than the grips. At first sight it will appear that the handle is really at a higher temperature than the grips, but it is not so. The handle is a good conductor of heat, and when we touch any part of it, heat flows to the hand, not only from that part, but also from the neighbouring parts ; whereas in the case of grips the heat flows to the hand from only that part which is touched. Hence the handle seems hotter, not because it is at a higher temperature, but because it is a good

conductor of heat, and the flow of heat to the hand is greater and more rapid than in the case of grips.

Both these examples show that *the sensation cannot be relied upon as a safe guide for determining the degree of hotness of a body.* Besides the above difficulty, the sensation of warmth does not enable us to tell by *how much* one body is hotter than the other. In other words, it does not enable us to make any quantitative comparison of temperatures.

For this purpose the effects produced by heat are more useful than the sensation of hotness. The effect that is generally made use of is the *expansion* of bodies with rise of temperature. We know that the length of a solid (like a metal rod) or the volume of a liquid or a gas depends upon the temperature ; as the temperature increases, the length or volume increases. Solids expands by very small amounts ; therefore they can be used to measure large intervals of temperature only. Gases, on the other hand, expand enormously even with a slight increase of temperature and hence can be used conveniently to measure small intervals of temperature only. Further since the volume of the gas depends, in addition to temperature, upon its pressure, the use of gases in actual practice is rather difficult. On account of these reasons the liquids are, on the whole, best suited for this purpose, their expansion being moderate and sufficiently regular.

Mercury is specially convenient, for it is not only easily obtainable in pure state and is clearly visible in capillary tubes, but has a uniform expansion with increase of temperature, and remains liquid over a wide range of temperatures. *An instrument that is used for measuring temperatures is called a Thermometer* and if mercury be used as the thermometric substance the instrument is called a mercury thermometer. It may be remarked here that the best thermometric substance is air (or hydrogen), because it expands very regularly with increase of temperature and does not change its state over a very wide range of temperature. But since an air thermometer is not very convenient to work with, it is only used to standardise other thermometers.

155. Construction of a Mercury Thermometer.—A mercury thermometer generally consists of a bulb and a long capillary stem of uniform bore. On account of the contained air and the fineness of bore the bulb cannot be filled directly by pouring down mercury. For filling, generally the capillary tube is made to end in a funnel. Pure mercury is placed in it. The bulb is then heated, the air expands and passes out. As the bulb cools the air contracts : some mercury is forced down the tube into the bulb by the atmospheric pressure. Repeating this process of alternate heating and cooling, the bulb is filled with mercury which is next boiled to drive off the contained air and moisture. While the mercury is still at a high temperature—higher than the highest temperature which the thermometer is intended to read—the tube is sealed off. A thermometer sealed off in this manner is said to be **hermetically sealed**. After this the thermometer is put aside for about two weeks* to allow it to cool slowly and recover its original size.

*Accurate thermometers are put aside for as long a period as a year or two before they are graduated.

Now let us see how such a thermometer can be used for measuring temperatures. We require first of all a standard temperature and then a scale, by means of which we might say that the temperature of the given body is so many degrees higher than the standard temperature.

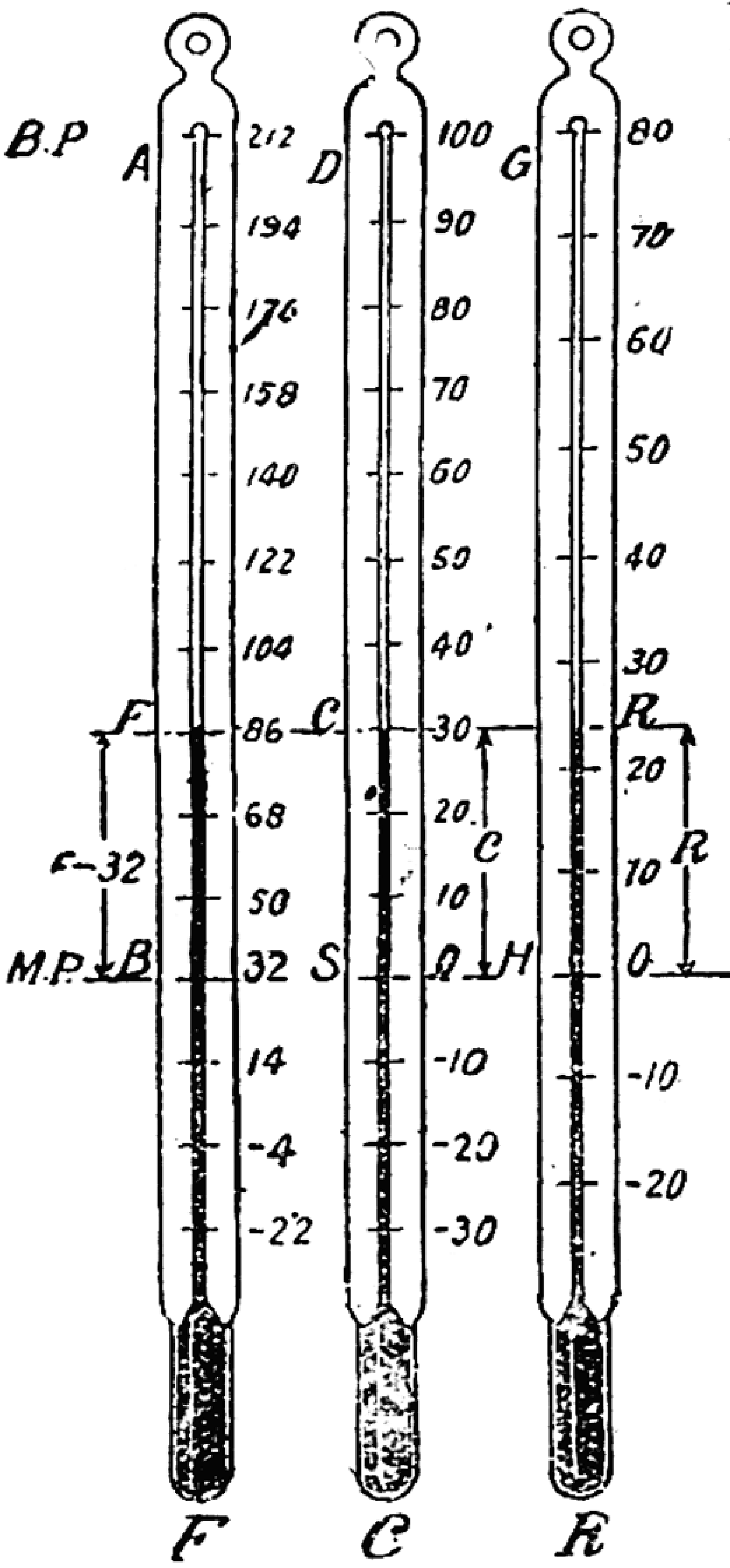

Fig. 3.

While selecting a standard temperature we make use of the fact that the change in the physical state of a body takes place at a fixed temperature, and that, so long as the state does not completely change, the temperature remains constant. Ice is a very common substance, hence the temperature at which it melts is taken as the standard temperature. Next we require a scale. For this purpose we require another standard temperature or fixed point on the thermometer. The second fixed point corresponds to the temperature at which water boils under normal conditions (760 mm. pressure, sea-level and 45° latitude). These two fixed points are very convenient standards of reference, for ice and water can be easily obtained in pure state. The interval between these fixed points is divided into a given number of parts, for instance, in the case of the **Centigrade Scale** the lower fixed point S is called 0, the upper fixed point D, 100, and the length between the two fixed points (*i.e.* DS) is divided into 100 equal parts. If the upper point is called 80 instead of 100 and the interval is divided into 80 equal parts, we get **Reaumur Scale**. If, on the other hand, the lower temperature is called 32 and the upper 212, and the interval is divided into 180 equal parts, we get **Fahrenheit Scale**. To convert a reading on one scale into a reading on another, remember that 0° Centigrade corresponds to 32° Fahrenheit and 0° Reaumur and that the interval between the fixed points is equal. If we call a particular reading C, F, and R on the respective scales corresponding to the *same* temperature we have the following relation :

$$\frac{C}{100}=\frac{F-32}{180}=\frac{R}{80}.$$

This relation enables us to convert a temperature reading on one scale into a reading on the other scales.

Determination of the Fixed Points.—(1) **Lower Fixed Point.** Surround the bulb and the part of the stem containing mercury by ice, which has been well washed with distilled water and has been broken into small pieces. Mark the position at which the column remains stationary for about ten minutes. This gives the lower fixed point (or 0° on the Centigrade scale). It is important to remember that the

bulb of the thermometer must be immersed in a mixture of ice and water, and not amongst fragments of ice from which all the water has been drained away, nor in water with a few pieces of ice floating in it.

It is of extremely great importance to note that the ice must be pure, and that water used to wash it must be distilled.

(2) **Upper Fixed Point.**—To determine this point, place the thermometer in a **Hypsometer** (Fig. 4) and adjust its position so that steam can play upon the whole of the stem. The steam while rising up the inner cylinder plays upon the stem, goes down the space between the inner and outer cylinders, and escapes through the outlet. It is important to remember while adjusting the position of the thermometer that the bulb must not dip in the boiling water, for the boiling point of water is somewhat higher if it contains impurities. Mark the level of the column of mercury when it remains stationary for about ten minutes. At the same time read the barometer for the temperature of the steam or the boiling point of pure water depends upon pressure, as the following table shows :—

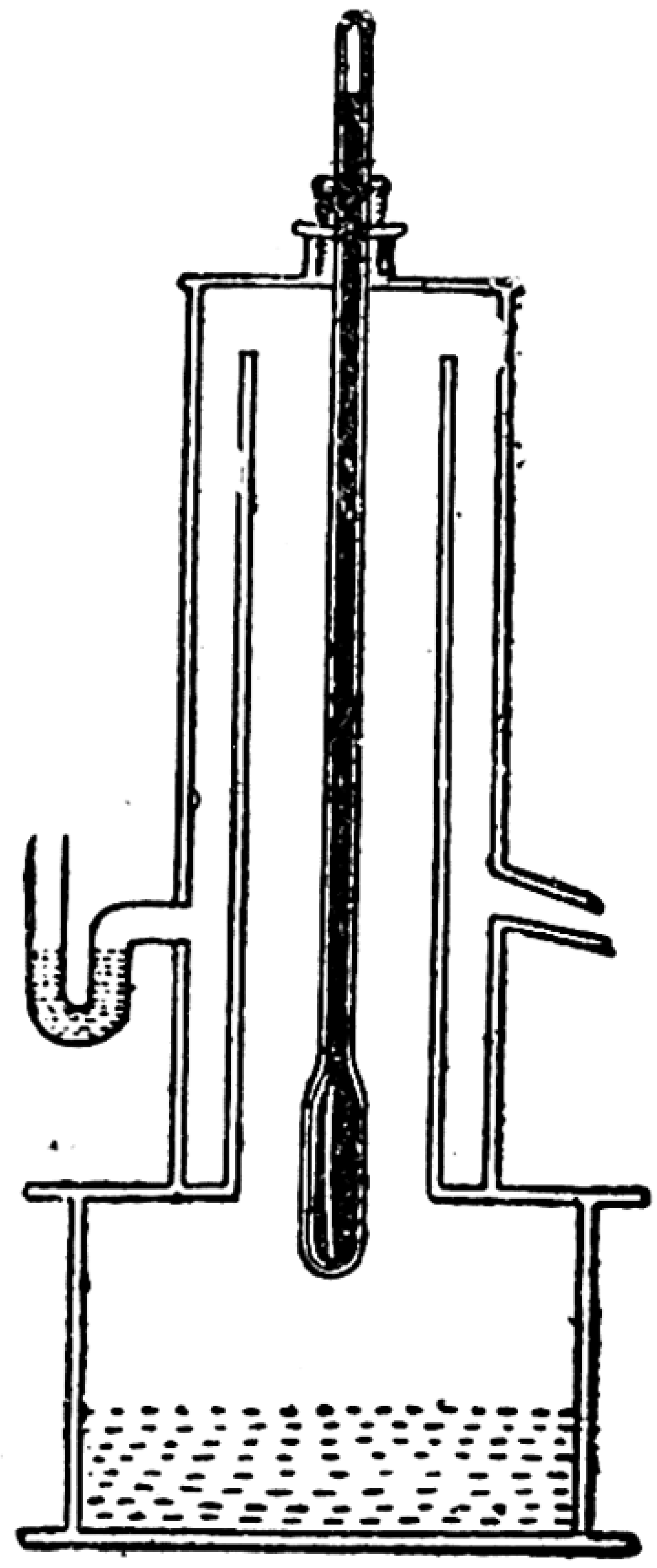

Fig. 4.
Hypsometer.

Pressure.	Temperature,	Pressure.	Temperature.
700 mm.	97·71	750 mm.	99·63
720 mm.	98·49	755 mm.	99·82
735 mm.	99·07	760 mm.	100·00
740 mm.	99·25	765 mm.	100·18
745 mm.	99·44	770 mm.	100·36

Knowing the pressure we can find the temperature of the steam or boiling point of water from the table. Thus we know both the fixed points.

We can verify in the manner explained above the fixed points of a mercury thermometer and determine the errors, if any.

Errors of a Mercurial Thermometer.—(1) **Change of Zero.**—We have already remarked that a thermometer after being hermetically sealed is set aside for about two weeks to allow it to regain its original size. If a thermometer is graduated before its bulb contracts fully, the size of the bulb will go on decreasing even after zero had been marked with the result that after the bulb regains its original size, the column of mercury will stand higher up than at 0°C. when immersed in ice. Suppose it stands at 0·8 °C. Evidently to get the true temperature we should subtract 0·8 from all readings. This error can be easily determined by placing the thermometer in pure ice washed with distilled water.

(2) **Recent Heating.**—In this case the bulb increases in size, and consequently the column of mercury stands lower down than at 0°C. when immersed in ice. Suppose it stands at —0·6°C ; to get the true

temperature we must add 0·6 to all readings. This error will occur after the thermometer has been heated to a high temperature. It is temporary only.*

(3) **Exposure of the Stem.**—While determining high temperatures the whole of the stem cannot be immersed in the substance whose temperature is to be read (say oil which is being heated over a flame). Part of the stem is exposed to the atmosphere, and hence its temperature is too low, with the result that reading on the whole is low. In order to avoid this error, either the whole of the stem should be immersed in the substance or correction applied to the reading.

(4) **Inequality of the Bore.**—As explained already, the stem is graduated by dividing the distance between the two fixed points into a given number of equal parts. This method is accurate only if the bore is uniform, otherwise a thread of mercury should be detached and its length determined at different points; corrections obtained therefrom should be taken into account while reading temperatures with the thermometer.

(5) **Position and Pressure.**—The thermometer should be held in the position in which it is graduated, for due to the hydrostatic pressure there will otherwise be some difference. For instance, a thermometer which is graduated in the horizontal position will give rather low readings when held vertically.

156. Limitations of the Mercury Thermometer.—Since mercury freezes at $-38·8$°C., a mercury thermometer is not used to measure temperatures below -32°C. For measuring lower temperatures an alcohol thermometer is used which can go upto -130°C. If we want to measure still lower temperatures we use a platinum resistance thermometer or a gas thermometer.

Mercury boils at 357°C., therefore, a mercury thermometer is not suitable for measuring high temperatures. As a matter of fact it cannot be used above 250°C., on account of rapid evaporation of mercury above this temperature.

If the upper part of the stem, however, is filled with an inert gas like nitrogen under pressure, temperatures as high as 500°C. may be read with a mercury thermometer. For high temperatures a gas or a platinum resistance thermometer or a thermo-couple is used.

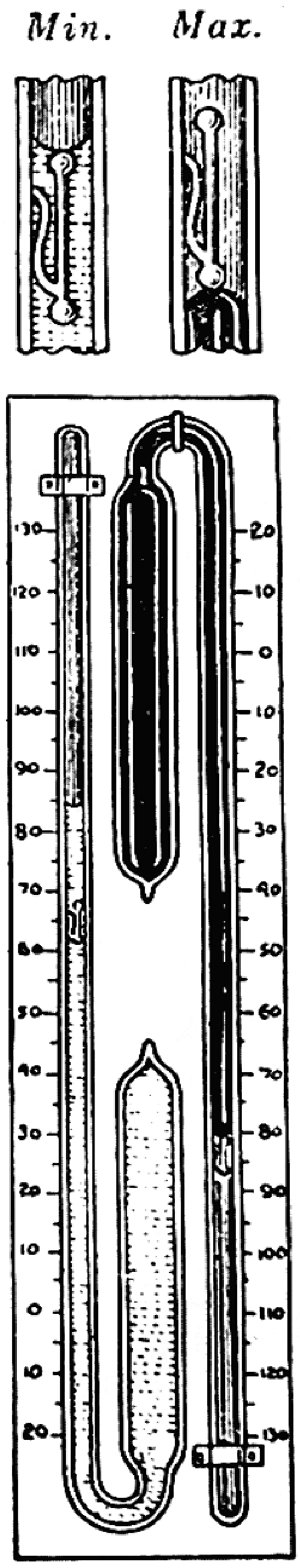

Fig. 5
Max. and Min. Thermometers.

157. Maximum and Minimum Thermometers.—When we require to ascertain the highest temperature reached during a given time, we use a

*It is for this reason that the freezing point is marked before the boiling point while constructing a new thermometer. If the boiling point be determined first and then the freezing point soon after, the zero point marked would be too low.

Maximum Thermometer. It is an ordinary mercury thermometer, containing a small, light, steel* index. With the help of a magnet it is moved down so that it just touches the surface of the mercury. As the mercury column moves up, it pushes the index before it. When the temperature falls, the mercury column retreats, leaving behind the index, which is prevented from slipping back by a spring [Fig. 5 (*a*)]. The index thus *registers the highest temperature reached.*

When, on the other hand, we require to ascertain the lowest temperature reached, we use a **Minimum Thermometer.** It is just like a Maximum Thermometer, with the difference that in this case *alcohol* is used in place of mercury. The concave surface of alcohol [Fig. 5 (*b*)] due to the surface tension, drags the index back as it falls, but leaves it unaffected while expanding. Thus *it registers the lowest temperature reached.*

Six's Combined Maximum and Minimum Thermometer.—Although it is not an accurate instrument but since it is commonly used by gardeners and nursery men, we shall describe briefly its construction and action.

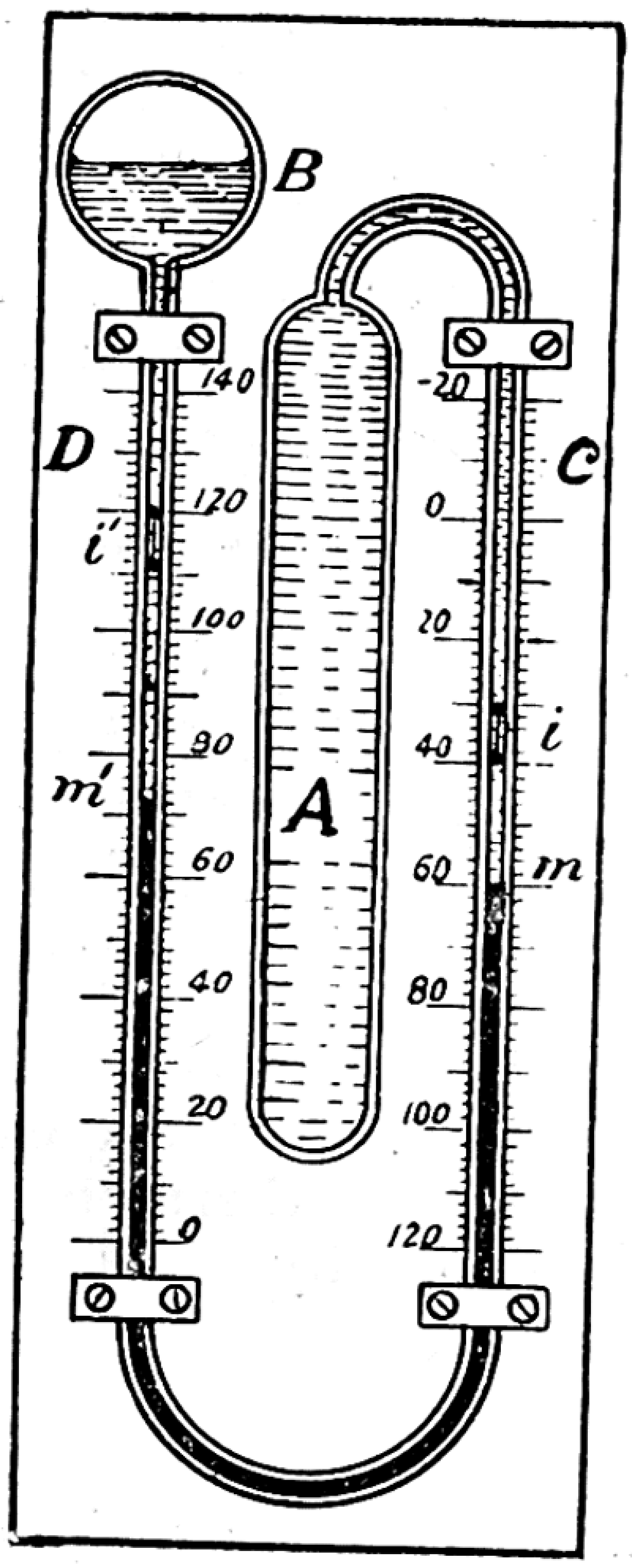

Fig. 6.

It consists of two bulbs A and B connected together by a capillary tube CD. The bulb A and capillary tube upto m are filled with alcohol. From m to m' the tube is filled with mercury. Above m' the tube as well as part of bulb B are filled with alcohol.

Two small steel indexes i and i' are placed above the free ends of the mercury column. They are fitted with light springs to prevent them from slipping downwards on account of their weight.

The alcohol in bulb A and capillary tube (or stem) upto m serves as the thermometric liquid. The column of mercury mm' in the capillary tube acts as a thermometric substance only when the temperature is rising whereas it acts as an index when the temperature falls.

To set the thermometer for the day the indexes i and i' are pulled down with a small magnet, until they rest on the mercury surface at m and m'. When the temperature rises, the alcohol in A expands pushing the mercury column down in arm C of the tube and upwards in arm D. The index i' is pushed by the mercury column in front of it. The lower end of i' thus registers the maximum temperature. When the temperature falls the alcohol in A contracts and the mercury column is pushed, from left to right by the weight of alcohol and the pressure of the alcohol vapour and air in bulb B with the result that the mercury

*In some cases glass index is used. In such cases the thermometer is used in the horizontal position and we have only to tilt it to make the index touch the mercury column.

column not only remains in contact with alcohol in arm C of the tube but pushes in front of it the index i, the lower end of which registers the minimum temperature.

The temperature scales on the two sides are not exactly equal, since the mercury column mm' also expands with increase of temperature and thus acts as thermometric substance in addition to alcohol as said above whereas when temperature falls it is only the alcohol in A that acts as thermometric substance.

Clinical Thermometer.—It is a sensitive, quick-acting and short range ($95°F$—$110°F$) maximum thermometer. It is used to measure the temperature of the human body, which, in the normal conditions, is at $98{\cdot}4°F$. In this thermometer in place of a steel index we have a constriction in the stem near the bulb.

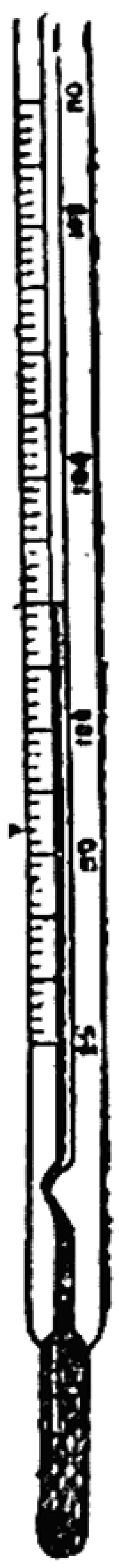

Fig. 7. Clinical Thermometer.

When the thermometer is placed in the arm-pit or below the tongue of a person the force of the expanding mercury pushes the mercury thread past the constriction but when it is removed from there the column breaks as there is vacuum above the mercury column and hence there is no force to push it past the constriction. The only force that tends to pull the mercury column is the force of cohesion of the mercury molecules but this is too weak to pull the column past the constriction. Therefore the column beyond the constriction stays where it was. The temperature of the patient hence can be read at leisure.

To get back the mercury into the bulb and make the column continuous the thermometer is jerked twice or thrice.

To make the thermometer sensitive the capillary is made so fine that if a thermometer breaks the bore can hardly be seen. To make the mercury column easily visible the stem is so shaped as to magnify the column as well as the scale when seen from the front. Besides each degree is divided into 5 parts. To make the thermometer quick-acting the bulb is made extremely thin. This is why a clinical thermometer is so fragile.

EXERCISES

1. When would the two thermometers Fahrenheit and Centigrade read the same temperature ?

Let that reading be x.

$$\therefore \qquad \frac{x}{100} = \frac{x-32}{180}.$$

or $$18x = 10x - 320.$$

or $$x = -40°.$$

2. How are the Fahrenheit and Centigrade scales connected with each other ?

At what temperature is the Fahrenheit reading twice as great as the corresponding Centigrade reading ? *Ans.* 320°F.

3. What temperature on the Centigrade scale is expressed by a number three times as large as that expressing the same temperature on the Fahrenheit scale? *Ans.* $-21\frac{9}{11}$°C

4. A thermometer with an arbitrary scale of equal parts reads 14·6 in melting ice and 237·9 in water boiling under standard pressure. Find the Centigrade temperature indicated by the reading 97·1 on this thermometer. *Ans.* 36·95°C, ; 89·3°C.

5. At what temperature has the Fahrenheit thermometer the same reading as the Reaumur thermometer? *Ans.* —25·6°.

6. A thermometer stands in a test tube containing hot water. A second thermometer is taken out of a vessel containing melting ice in which it has been standing and is placed in the test tube beside the other thermometer. What will be the behaviour of the two thermometers? How do you explain the result? (*P. P. Exam.*)

7. Prove the following formula for the conversion of Fahrenheit temperatures into Centigrade temperatures :

$$C+40=\tfrac{5}{9}\,(F+40).$$

8. If a Fahrenheit thermometer reads 70° when a standard Centigrade thermometer reads 20°, what is the error in the Fahrenheit thermometer? *Ans.* +2°F.

9. Describe the construction of a clinical thermometer. Explain the conditions which make it sensitive and quick-acting.

10. What is the temperature of a healthy man on the centigrade scale? The temperature of a swallow is 8°F above the normal temperature of a man. What will it be on the Centigrade scale? *Ans.* 36·9°C., 41·3°C.

CHAPTER II

The Expansion of Solids

158. It was mentioned in the preceding chapter that as a rule all* substances increase in size when heated. A few experiments were also given there to illustrate this effect of heat. We shall now deal with this effect in detail.

That solids expand when heated and contract when cooled is a well known fact. We see daily numerous practical applications of this fact. By way of illustration we mention here only a few of them.

The rails on a railway track are laid with a small gap between them so that due to expansion with a rise in temperature in summer they may not bend.

The iron tyre of a cart wheel is always made a bit smaller in diameter than the wooden wheel. After making the tyre red hot it is slipped on to the wheel, and water is poured over it. On cooling the tyre contracts and holds the wooden parts firmly together.

To loosen a glass-stopper which has stuck fast in the neck of a glass bottle we heat the neck a little by turning it round in the flame. Due to the expansion of the neck the stopper can be easily pulled out.

Rivetting is another familiar example. Two plates which are to be fastened together are held fast whilst ahole is drilled through them both. A red-hot rivet is passed through the hole and is hammered until both ends have heads closely gripping the plates. The contraction of the rivet as it cools binds the plates together with a great force.

Solids, when heated, expand in all directions; for example, if a metal cube be heated, all its sides become longer, with the result that on the whole it becomes a bigger cube. The increase in length which each side undergoes is spoken of as *Linear Expansion*. The increase in area or surface of each face is called *Superficial Expansion*.

The increase in volume is called *Cubical Expansion*. Generally solids expand equally in all directions. It is only to such bodies that we shall confine our discussion.

159. Linear Expansion.—Take a thin rod of a metal and heat it. Its expansion will depend upon;

(1) *the rise in temperature i.e.*, the greater the rise in temperature the greater the increase in length; for instance if the increase be $\frac{1}{20}$ of a mm. for 5° rise of temperature, it will be $\frac{1}{4}$ of a mm. for 25° rise of temperature.

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*Some substances like iodide of silver below 140°C., or water below 4°C. contract when heated.

(2) *the original length*, *i.e.*, the greater the original length, the greater the increase in length ;

(3) *the nature of the material*, some substances expand more than others.

If L_o is the length of a rod at 0°C., L_t its length at t°C., and α the constant depending upon the nature of the material, the increase in length, $L_t - L_o$, is equal to $L_o\alpha t$. We can write this result as an equation,

$$L_t - L_o = L_o\alpha t, \qquad (1)$$

or

$$\alpha = \frac{L_t - L_o}{L_o t} \qquad (2)$$

The constant α is called the **coefficient of linear expansion.** It is clear from relation (2) that the *coefficient of linear expansion is the increase in length per unit length of a rod when its temperature is raised 1°C.* We may also define it as follows :

It is the ratio of the increase in length produced by 1°C. rise of temperature to the original length.

The formula (1) can be written as

$$L_t = L_o + L_o\alpha t,$$

or

$$L_t = L_o(1 + \alpha t) \qquad (3)$$

This formula enables us to find the length of a body at any temperature provided we know its original length, the rise in temperature, and the coefficient of linear expansion.

The coefficient of linear expansion of a few substances is given in the following table :—

Coefficients of Linear Expansion

Glass	... 0·0000087	Brass	... 0·000019
Platinum	... 0 0000089	Silver	... 0·000019
Iron	... 0·000012	Zinc	... 0·000026
Copper	... 0·000017	Porcelain	... 0·000003

It was discovered by Guillaume, a French metallurgist in 1904, that the coefficients of expansion of the alloys made of nickel and steel varied very much with the percentage of nickel. He found that invar, which consists of 36% nickel and 64% steel, has as low a coefficient of linear expansion as 0·0000009, whereas the coefficient of expansion of nickel steel (45% nickel and 55% steel) is 0·0000089, which is just the coefficient of expansion of platinum or very nearly equal to the coefficient of expansion of glass. Hence wires of this material are fused in electric bulbs.

To enable the student to visualise the magnitude of the quantity he is dealing with we might add that a brass rod 1 metre in length will expand by 0·019 mm. when heated through 1°C. This illustration will enable him to realise why temperature was mentioned in defining the standard metre or yard. At 20°C. a brass rod 1 metre in length will be longer by 0·38 mm. than at 0°C.

166. Determination of the Coefficient of Linear Expansion.—(*a*) To measure the linear expansion experimentally, a rod which has two vertical needle-points fixed to it near its ends is taken and the distance

between the needle-points is measured. It is placed on rollers in a trough, and is surrounded by ice, and two vernier microscopes are focussed upon the needles. The trough is heated so that the water is at a given temperature. With the help of the microscopes the displacement of each needle point is measured accurately. Adding up the displacements we get the total increase in length of the rod. The object of using the rollers is to let the rod expand freely. Knowing the increase in length, the original length, and the rise in temperature, we can determine the coefficient of linear expansion with the help of formula (2).

(*b*) A simple method which is in common use in laboratories makes use of the apparatus shown in Fig. 8.

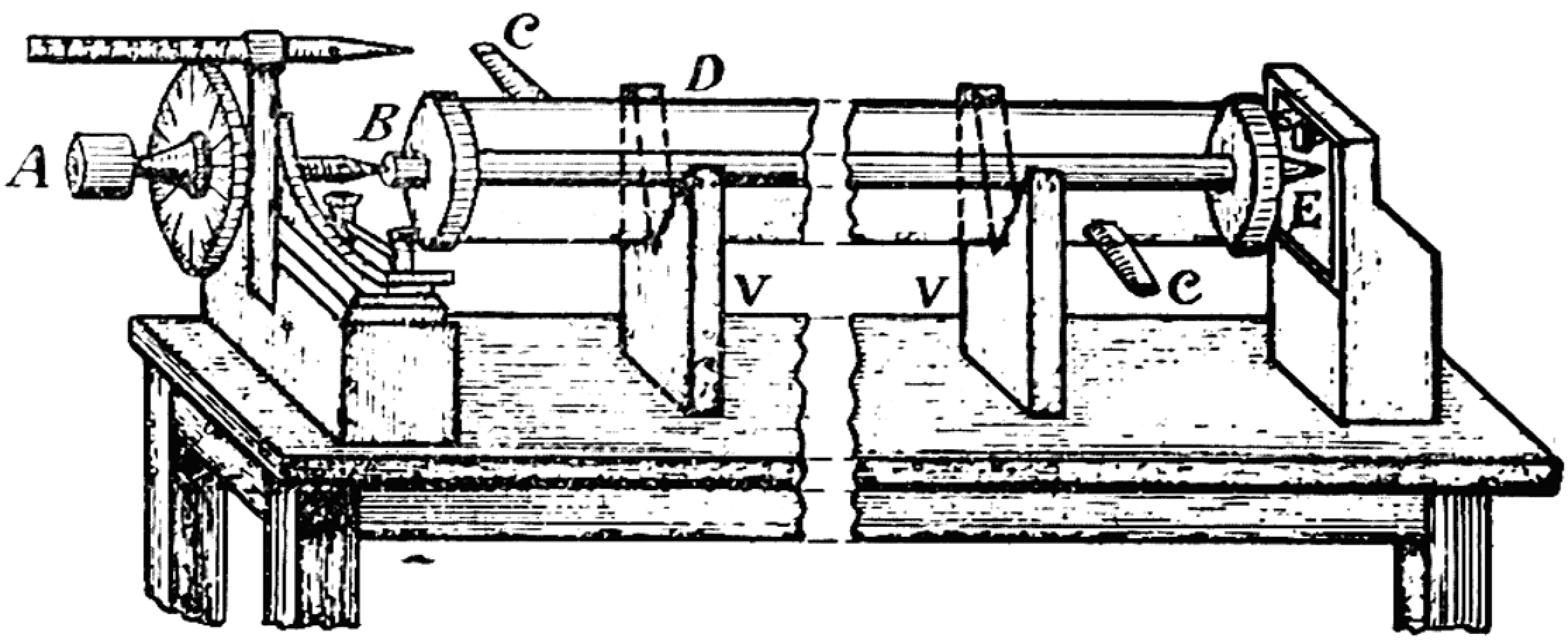


Fig. 8.

A metal rod *B* about 60 cm. long and 0·5 cm. in diameter is enclosed in a tube *D* through which steam can be passed. The ends of the tube are closed by corks through which the ends of the rod *B* project. The tube *D* rests on two V-shaped supports *V*, *V′*; one end of the rod is made pointed and the other is kept flat. The pointed end rests against a *fixed* wooden support, the face *E* of which is covered by a thin metal plate. To the flat end is applied the central leg of the spherometer *A* in such a way that the central leg is in a straight line with the axis of the rod. By connecting one pole of a Leclanche cell to the spherometer, and the other to the metal plate at *E* through a galvanometer, we can find when the central leg *just* touches the rod.

Measure the length of the rod at the room temperature and set it up as explained above. Read the spherometer when the galvanometer needle *just* gives a throw. Turn *back* the head of the spherometer and pass steam till the rod acquires a steady temperature. The rod is thus heated to the temperature of the steam. Turn the head of the spherometer till it makes contact with the rod. The difference between this and the first reading gives the *increase* in length of the rod. With this apparatus the change in length can be measured to about 0·005 mm. We know the original length, the rise in temperature, and the increase in length, hence we can easily find the coefficient of linear expansion.

161. Superficial Expansion.—When a body is heated, it expands in length as well as in breadth. Knowing the increase in length and breadth we can find the increase in area, *i.e.*, superficial expansion of

a body. Like linear expansion, *the superficial expansion also is proportional to the increase in temperature, the original area, and the nature of the material.* Suppose S_0 is the area of the surface at 0°C., S_t the area at t°C., and β the constant depending upon the nature of the material, the increase in area is given by the expression

$$S_t - S_0 = S_0 \beta t \quad \ldots \ldots \ldots \ldots \quad (4)$$

or

$$\beta = \frac{S_t - S_0}{S_0 t} \quad \ldots \ldots \ldots \ldots \quad (5)$$

The constant β is called the **coefficient of superficial expansion.** *It is equal to the increase in unit area produced by 1°C. rise of temperature.*

Relation (4) can also be written as $S_t = S_0(1+\beta t)$. . . . (6)

This relation is very useful in solving numerical examples. There exists a very simple relation between the coefficient of superficial expansion and the coefficient of linear expansion. In order to understand this let us consider a square plate $ABCD$, of side AB (=L). when heated through 1°C., let AB increase to AB' and AD to AD' so that the plate becomes $AB'C'D'$. From what has been said above it is clear that the length of each side is $L(1+\alpha)$ and the area increases from L^2 to $L^2(1+\alpha)^2$.

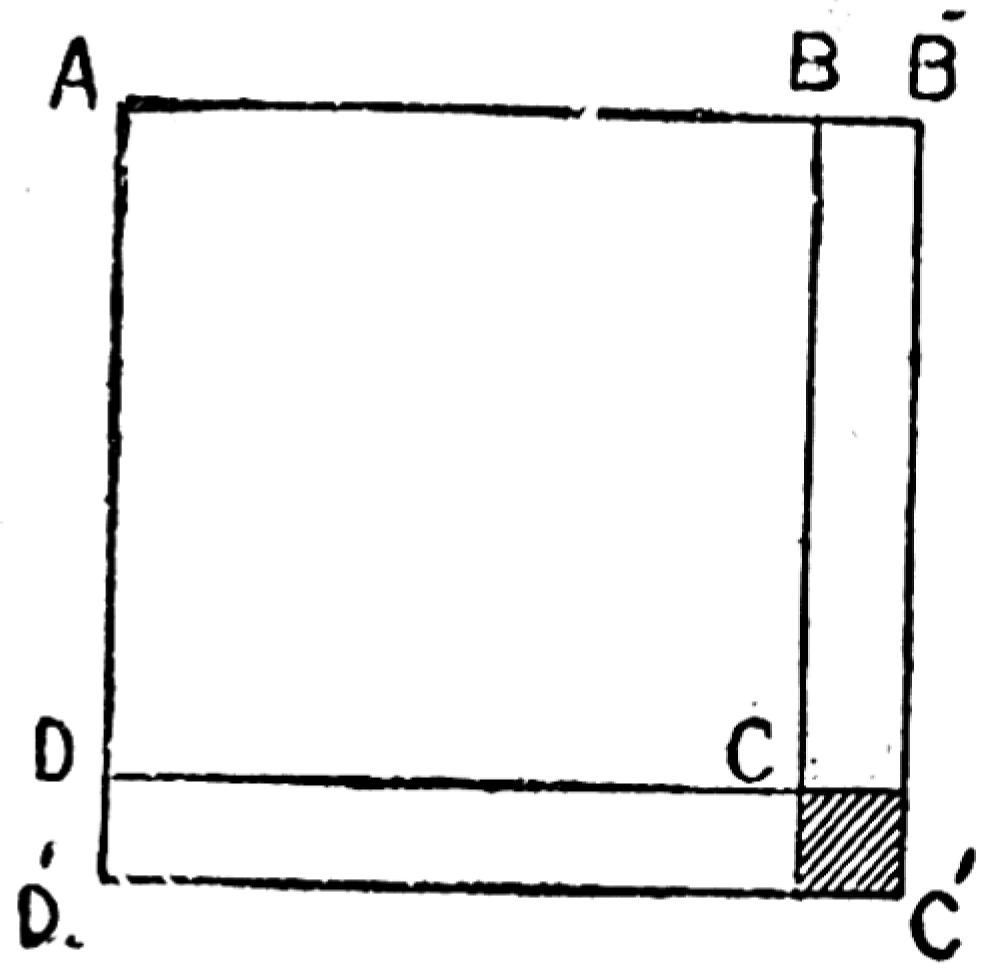


Fig. 9

The increase in area is evidently

$$L^2\,(1+\alpha)^2 - L^2.$$

Using formula (5) we find that the coefficient of superficial expansion,

$$\beta = \frac{L^2(1+\alpha)^2 - L^2}{L^2 \times 1}$$

since t is 1.

Hence $$\beta = (1+\alpha)^2 - 1 = 2\alpha + \alpha^2 = 2\alpha,$$

neglecting α^2, which is too small for our purpose.*

Thus we learn that --

The coefficient of superficial expansion is twice the coefficient of linear expansion.

162. Cubical Expansion.—Like the linear and superficial expansions the cubical expansion also depends upon three factors : (*i*) *the original volume* ; (*ii*) *the rise in temperature* ; and (*iii*) *the constant depending on the nature of the material.* If V_0 be the volume of a body at 0°C and V_t the volume at t°C. and γ the constant depending upon the nature of the material, the increase in volume is given by the relation

$$V_t - V_0 = V_0 \gamma t \quad \ldots \ldots \ldots \ldots \quad (7)$$

or

$$\gamma = \frac{V_t - V_0}{V_0 t} \quad \ldots \ldots \ldots \ldots \quad (8)$$

---

*If a is 0·00002, a^2 is 0·0000000004, which is obviously too small to be taken into account. How small is a^2 in comparison with $2a$ is shown graphically in Fig. 9. Area of square CC', shown shaded, represents a^2 whereas area of rectangle $B'C$ or $D'C$ represents a, if AB or BC be 1 cm.

The constant γ is called the **coefficient of cubical expansion. It is** defined as follows :

The coefficient of cubical expansion is the increase in unit volume produced by 1°C rise of temperature.

To study the relation which exists between the coefficient of cubical expansion and the coefficient of linear expansion, let us consider the expansion of a cube.

Suppose one of its faces is represented by the square $ABCD$ (Fig. 9) at 0°C., the length of each side of which is L. Heat it through one degree and let the face be represented by $AB'C'D'$. Each side will now become equal to $L(1+\alpha)$. The volume will increase from L^3 to $L^3(1+\alpha)^3$.

Using formula (8) we find that the coefficient of cubical expansion

$$\gamma=\frac{L^3(1+\alpha)^3-L^3}{L^3.1} \qquad \text{[since } t=1°.]$$

Hence

$$\begin{aligned}\gamma&=(1+\alpha)^3-1\\&=3\alpha+3\alpha^2+\alpha^3\\&=3\alpha,\end{aligned}$$

neglecting α^2 and α^3, which will be extremely small when compared with α. Thus we see that for a body which expands uniformly in all directions

The coefficient of cubical expansion is three times the coefficient of linear expansion.

163. Change of Density with Rise of Temperature.—We have seen in §122 that density $=\frac{M}{V}$. This shows that if the mass of a body remains constant, the increase in volume will decrease the density of the substance.

Let V_0 be the volume of a body at 0°C., ρ_0 the density at 0°C., V_t the volume at t°C., and ρ_t the density. Since M remains the same $V_0\,\rho_0=V_t\rho_t$,

or

$$\frac{V_t}{V_0}=\frac{\rho_0}{\rho_t}.$$

But $V_t=V_o(1+\gamma t)$,

$$\therefore \qquad \frac{\rho_0}{\rho_t}=\frac{V_o(1+\gamma t)}{V_0}=1+\gamma t,$$

or

$$\frac{\rho_t}{\rho_0}=\frac{1}{1+\gamma t} \qquad \ldots\ldots\ldots\ldots \quad (9)$$

This relation is true for solids, liquids, and gases. But since for solids and liquids γ is small, in their case the formula

$\frac{\rho_t}{\rho_0}=\frac{1}{1+\gamma t}$ is further reduced to $\frac{\rho_t}{\rho_0}=1-\gamma t$.

164. Compensated Pendulum.—We have already said in §60 that t, the time period of a pendulum, is given by

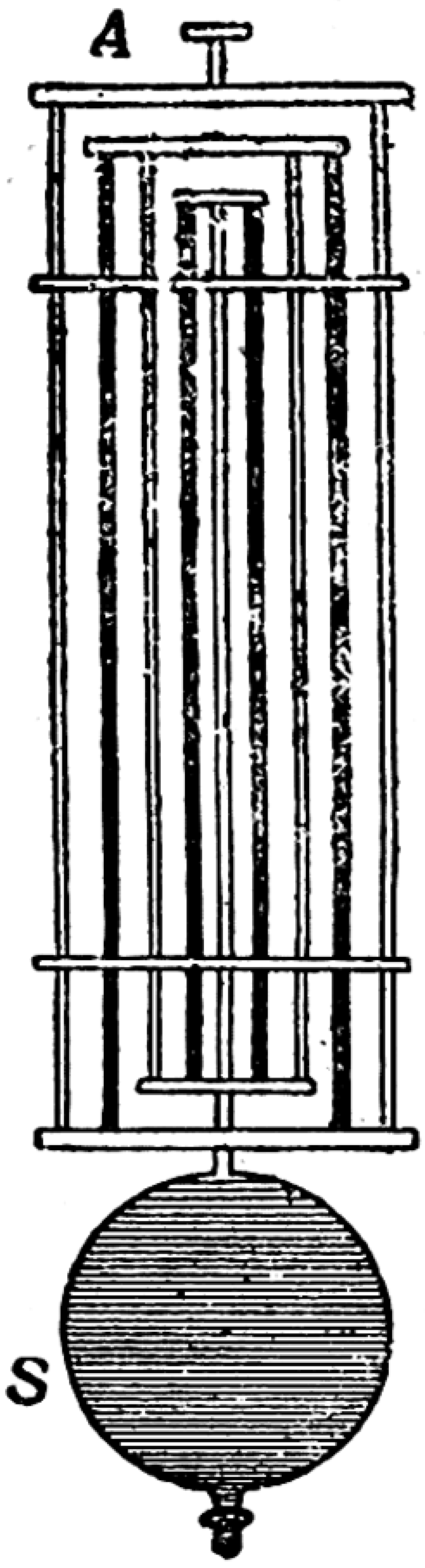
$$t=2\pi\sqrt{\frac{l}{g}}$$

where l is the length of the pendulum, and g the acceleration due to gravity.

It is evident from this relation that if l increases, t becomes greater. Since the length of a pendulum of a clock which is made of some metal, will increase in summer, t will become greater and hence the clock will run slower. In winter on the other hand the length will decrease and hence the clock will run faster. In order to prevent this, the pendulums are made in such a way that the distance between the point of suspension and the centre of gravity* remains the same.

The first person to manufacture a compensated pendulum was John Harrison.* His pendulum is called Harrison's grid-iron pendulum. It consists of 5 steel and 4 brass rods. The steel rods are shown as double lines and brass rods as black lines in Fig. 10. The middle rod which carries the bob passes through holes in the lower horizontal bars and is not attached to them. When such a pendulum is suspended from a fixed support any expansion of the steel rods increases the distance between the point of suspension A and the C.G. of the bob, while that of the brass rods reduces it. The effect of 5 steel rods is the same as of three and of 4 brass rods the same as of two. The total length of the steel rods is about $\frac{3}{2}$ of the total length of the brass rods, and since the coefficient of expansion of brass is about $\frac{3}{2}$ (accurately $\frac{19}{12}$) times that of iron, their expansion is equal but in opposite direction with the result that the effective length AS of the pendulum remains unaltered whatever the temperature.

A



S

Fig. 10.

In some modern clocks the rod of the pendulum is made of invar and the bob of steel or zinc [Fig. 11]. Invar has a very small coefficient of expansion ($\alpha=0{\cdot}0000009$) and hence practically no change in the length of the pendulum takes place with the ordinary fluctuations of temperature. Whatever little change takes place is compensated for by the expansion of the bob in the upward direction. While fixing the bob care is taken to fasten its lower end to the rod so that as temperature increases the bob expands upwards while the rod expands downwards, leaving the effective length *i.e., length* between the point of suspension and $C.G$ of the bob unaltered.

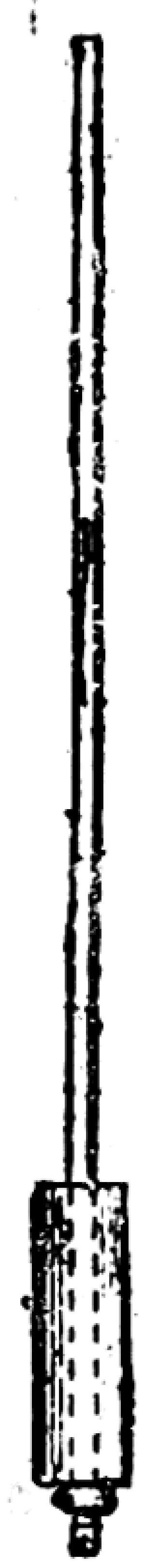


Fig. 11.

164a. Balance Wheel.—We have seen above how with the help of a compensated pendulum a clock is made to keep correct time. In watches and small clocks where a pendulum cannot be used compensated balance wheels are used. A balance wheel, as shown in Fig. 12 is made in two curved segments. The time of the swing

*More accurately the centre of oscillation, which coincides very nearly with the C. G. of a compensated pendulum.

†John Harrison invented in 1726 a spring-driven portable clock and was awarded a prize for this invention. Harrison's grid-iron pendulum is now of historical interest only.

of the balance wheel depends on (*i*) the elasticity of the hair spring and (*ii*) the radius of the balance wheel. An increase in temperature weakens the elasticity of the spring and lengthens the radius of the wheel. Both these changes tend to make the watch lose time. To compensate for this loss in time the segments of the balance wheel are made of two metals, brass on the outside and iron on the inside. When the temperature rises the brass expands more than iron and therefore the segments curve inwards and thereby compensate not only for the increase of radius but also for the weakening of the spring.

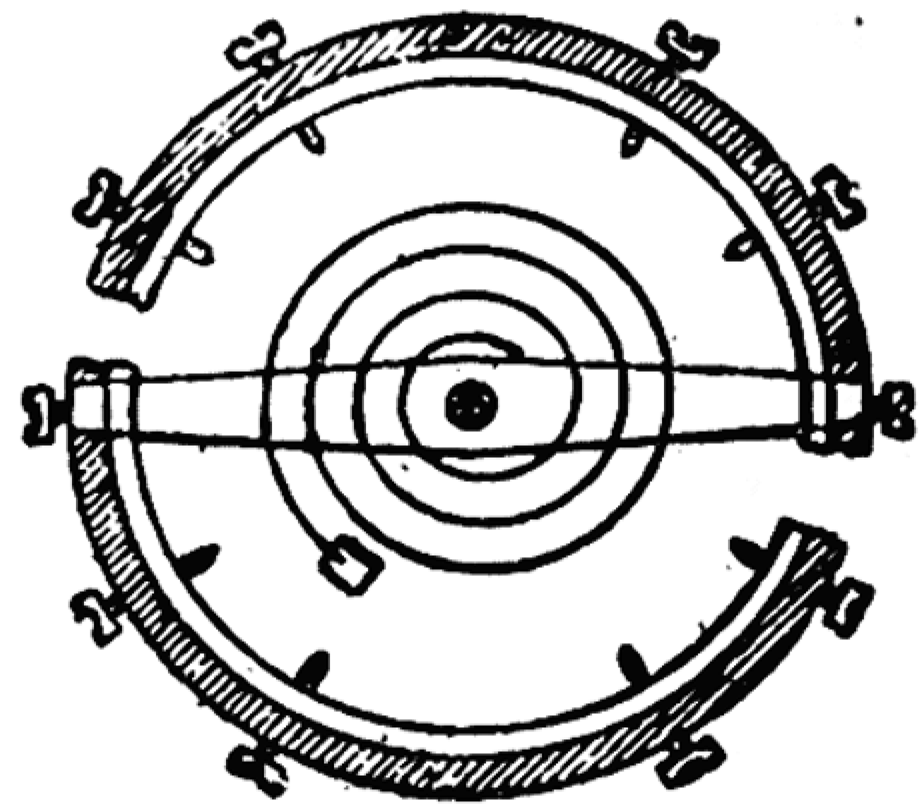


Fig. 12. Balance Wheel.

To make an accurate adjustment small screws are set in the segments. Those near the free ends tend to increase the compensation while those near the fixed ends have the opposite effect.

Nowadays the spring is usually made of a special nickel-steel alloy whose elasticity slightly increases with the temperature. By combining this with an invar balance wheel it is easy to make exact compensation. For the gain in time on account of the slight increase in elasticity with increase of temperature is made up by the loss in time due to the slight increase of diameter of the wheel.

EXERCISES

1. Rails laid on a railway are 60 ft. in length : what space must be left between two rails to provide for alteration in length, assuming that they were laid at 0°C., and that the highest summer temperature is 45°C., the coefficient of linear expansion for steel being 0·000012 ?

Each rail would expand by $L_t - L_0$ or $L_0 \alpha t$,

or $$60 \times 0{\cdot}000012 \times 45.$$

or $$0{\cdot}0324 \text{ ft. or } 0{\cdot}3888 \text{ inch.}$$

Since every rail would expand in both directions, hence one rail would go towards the next by 0·1944 of an inch, the next would also come towards the former by this mount, hence the total distance to be left between two rails is 0·4 inch approximately.

2. Find the increase in volume of a hollow glass cube of 10 cm. side when its temperature is raised from 15° to 25°C. Take the coefficient of linear expansion of glass as 0·0000087.

The increase in volume will be the same whether the cube is hollow or solid. Bearing this in mind let us proceed to find the increase in volume

$$V_{25} = V_{15}\,(1 + \gamma \times 10) = 1000\,(1 + \gamma \times 10)$$

But $$\gamma = 3 \times 0{\cdot}0000087 = 0{\cdot}0000261$$

$$V_{25} = 1000\,(1 + 0{\cdot}000261) = 1000{\cdot}261$$

$\therefore$ Increase in volume $= 0{\cdot}261$ c.c.

3. A lump of sulphur is found to displace 48 c.c. of water at 0°C. What volume of water would it displace at 35°C. if the coefficient of linear expansion of sulphur is 0·000077 ?

Volume at 35°*C*. would be given by the formula

$$V_t = V_0(1+\gamma t), \text{ where } V_0 = 48 \text{ c.c.. } t = 35^\circ.$$
$$\gamma = 3a = 3 \times 0{\cdot}000077$$
$$= 0{\cdot}000231 = 0{\cdot}00023 \text{ (approx.)}$$

Substituting the values we get

$$V_{35} = 48(1 + 0{\cdot}00023 \times 35) = 48(1{\cdot}00805) = 48{\cdot}3864 \text{ c.c.}$$

Since the volume of the lump of sulphur has become 48·3864, evidently it will displace 48·3864 c.c. of water.

4. A brass rod is measured with a wooden scale at 25°C., and is found to be 255 cm. long. Find the length of the rod at 0°C., if the co-efficient of linear expansion of brass is 0·000019. *Ans.* 254·879 cm.

5. What space must be left between two rails 30 ft. long so as to allow for expansion between 0°C and 40°C. assuming that the rails are laid at 12°C., and further, that the coefficient of linear expansion of steel is 0·000012 ? *Ans.* 0·121 inch.

6. A telephone wire is 1 mile long when the temperature is 0°C. What would be its length in summer when the temperature of the atmosphere is 35°C ? Take the coefficient of linear expansion of the material of wire as 0·000012. *Ans.* 1 mile and 2·22 ft.

7. The distance between Allahabad and Delhi is 390 miles. Find the total space that must be left between the rails to allow for a change of temperature from 36° F. in winter to 117°F. in summer. (Take co-efficient of linear expansion as 0·000012.) *Ans.* 0·21 mile.

8. A clock which keeps correct time at 25°C. has a pendulum made of brass whose coefficient of linear expansion is 0·000019. How many seconds a day will it gain if the temperature falls to 0°C. ?
Ans. 20·52 sec.

9. The density of silver at 0°C. is 10·31, and its coefficient of linear expansion is 0·000019. Find its density at 150° C. *Ans.* 10·23.

10. The volumes of two pieces of metal, iron and brass, are 702 and 700 c.c. respectively at 0°C. Find the temperature at which they will have the same volume, the coefficients of linear expansion being 0·000012 and 0·000018 respectively. *Ans.* 159·7°C.

11. Find the increase in the surface of a hollow copper ball 1 foot in radius when it is heated from 32°F to 164°F. Take the coefficient of linear expansion of copper as 0·000017. *Ans.* 0·03 sq. ft.

12. How are modern clocks and watches compensated for variation of temperature ?

13. An iron plate is cut with each side equal to 100 cm. when room temperature is 15°C. Find the increase in area when room temperature is 40°C. What will be the increase if you do not neglect α^2 ? Take $\alpha = 0{\cdot}000012$. *Ans.* (*i*) 6 sq cm. (*ii*) 6·000036 sq. cm.

14. A mechanic employed in a telephone company strings wires between poles on a hot day. Should he leave the wires a little slack or stretch them as tightly as possible and why ?
Ans. Should leave them slack.

CHAPTER III

The Expansion of Liquids

165. Coefficients of Real and Apparent Expansion.—That liquids expand when heated was shown in the first chapter while discussing the effects of heat. If the experiment that was explained there be repeated with different liquids, it will be seen that they expand by different amounts. Since liquids do not possess any shape of their own, but always take up the shape of the vessel containing them, it is absurd to speak of their linear or superficial expansion. We are concerned in their case with the increase in volume only or with the cubical expansion owing to the fact that liquids must be kept in some vessel a complication comes in. In order to heat the liquid, we must heat the vessel which increases in size and before the expansion of the liquid becomes visible the liquid has to fill up this increase in volume. The fact that we see a liquid rising rapidly in level in a vessel shows that liquids expand much more than solids. The *expansion that we observe* is the difference between the real expansion of the liquid and the expansion of the vessel. This shows that in the case of liquids we have to deal with two kinds of expansion, **real** and **apparent.**

Let the volume of the flask A (Fig. 13) at 0°C. be V up to the mark P. Fill it with a liquid at 0°C. up to P and heat it to t°C. For the sake of simplicity suppose that heating takes place in two stages *i.e.*, at first only the flask gets heated and then the liquid. When the flask is heated to t°C., the volume up to P will no longer be V, it will be greater. If the coefficient of cubical expansion of glass be g, the volume up to P will be $V(1+gt)$; *i.e.*, the volume will increase by $V\times gt$. If the liquid does not expand, it must fall down, for its volume is yet V. Suppose it falls to the level Q. Obviously the volume of the portion of the neck between P and Q is Vgt. Suppose now that the liquid is also heated to t°C., and that it comes up to the mark R. If γ_r be the coefficient of real expansion of the liquid, the volume V of the liquid (at 0°C.) will become $V(1+\gamma_r t)$ at t°C., and reach up to R.

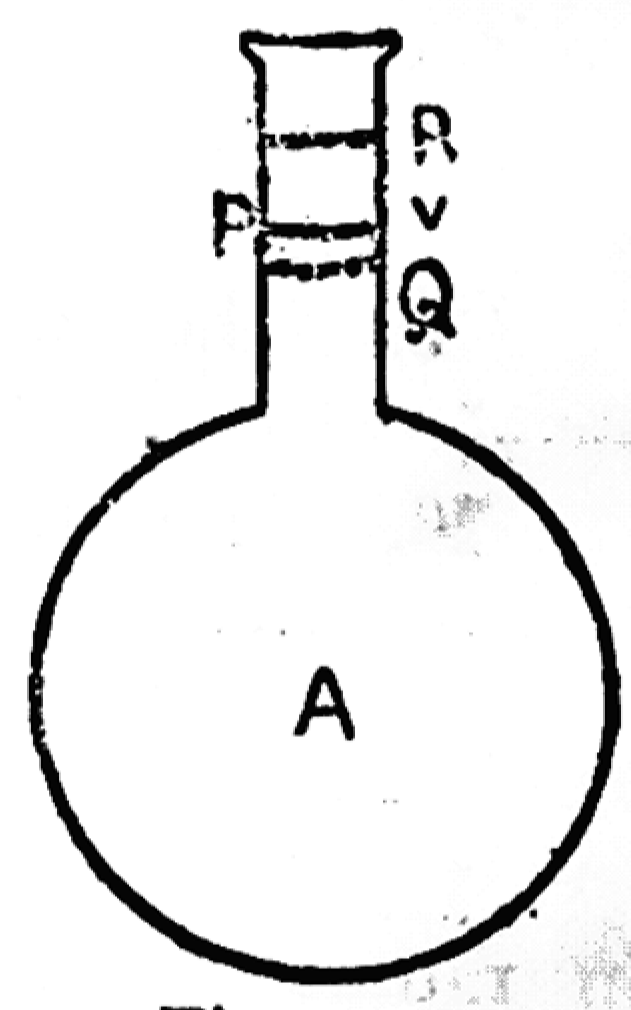


Fig. 13.

Evidently the volume of the portion of the neck between Q and R is $V\gamma_r t$. But if we were not to take into consideration the expansion of the vessel, we would have taken PR, *i.e.*, $(QR-PQ)$ or $V(\gamma_r-g)t$ as the expansion. In other words $V(\gamma_r-g)t$ is the apparent expansion and, the coefficient of apparent expansion

$$\gamma_a=\frac{V(\gamma_r-g)t}{Vt}$$

$$=\gamma_r-g,$$

or

$$\gamma_r=\gamma_a+g.$$

Hence *the coefficient of real expansion of a liquid is equal to the sum of its coefficient of apparent expansion and the coefficient of cubical expansion of the material of the vessel.**

166. Determination of the Coefficient of Apparent expansion of a Liquid by the Weight Thermometer Method.—If we neglect the increase in size of the vessel, the increase that we get in the volume of a liquid is the apparent expansion. Dividing this by the original volume and the rise of temperature we get the coefficient of apparent expansion. To determine it experimentally we generally use the weight thermometer method. Take a weight thermometer made of glass shown in Fig. 14. Weigh it when empty. Let its weight be w. Place below its capillary end a small dish containing the liquid whose apparent expansion is to be determined. By alternately heating and cooling the thermometer, nearly fill it with the liquid. Play a Bunsen flame around the bulb of the thermometer to heat the liquid and expel the air and the moisture, keeping the capillary end all the while below the surface of the liquid in the dish. Next surround the bulb with ice so that its temperature is 0°C. The thermometer is thus filled up with the liquid at 0°C.

Fig. 14.

Place a clean dry dish, say of weight a, below the capillary end, and heat the thermometer to t°C. Collect the liquid which overflows in the dish and let its weight be m. Weigh the thermometer with the remaining liquid, and subtract the weight w of the thermometer, and let the weight of the liquid remaining in the thermometer at t°C. be M.

It is evident that M grams of the liquid completely fill the thermometer at t°C., whereas $M+m$ grams completely fill it at 0°C. This means in other words that M grams of the liquid when heated from 0°C. expand so much that they occupy the volume occupied by $M+m$ grams of the liquid at 0°C, *i.e.*, the increase in volume is equal to the volume occupied by m grams at 0°C. neglecting of course the expansion of the vessel. Since the volumes occupied by M and m grams of the liquid at 0°C., are proportional to their weights, for volumes we can substitute the weights. Thus the apparent increase in volume is proportional to m, the original volume to M, the rise in temperature being t°C. Hence the coefficient of apparent expansion

$$\gamma_a = \frac{m}{M \times t}$$

---

*To be precise γ_r is only *approximately* equal to $\gamma_a + g$, Let us see why ?

The real volume of the liquid at $t°$ is $V_r = V(1+\gamma_r t)$. This is also equal to $V_a(1+gt)$ which is the correct volume of the flask up to V_a graduation at t°C. Hence

$$V(1+\gamma_r t) = V_a(1+gt) \quad \ldots\ldots\ldots\ldots \quad (i)$$

The apparent volume $V_a = V(1+\gamma_a t)$. Substituting this value of V_a in equation (i) we get

$$V(1+\gamma_r t) = V(1+\gamma_a t)(1+9t)$$

or

$$1+\gamma_r t = (1+\gamma_a t)(1+gt) = 1+\gamma_a t + gt + \gamma_a g t^2$$

or

$$\gamma_r = \gamma_a + g + \gamma_a g t.$$

It is only when we neglect $\gamma_a gt$ which is small, being the product of two small quantities, that $\gamma_r = \gamma_a + g$.

or
$$\frac{\text{Mass of liquid expelled}}{\begin{pmatrix}\text{Mass of liquid left behind}\\ \text{at } \textit{higher} \text{ temperature}\end{pmatrix}\times\begin{pmatrix}\text{Rise of}\\ \text{temperature}\end{pmatrix}}$$

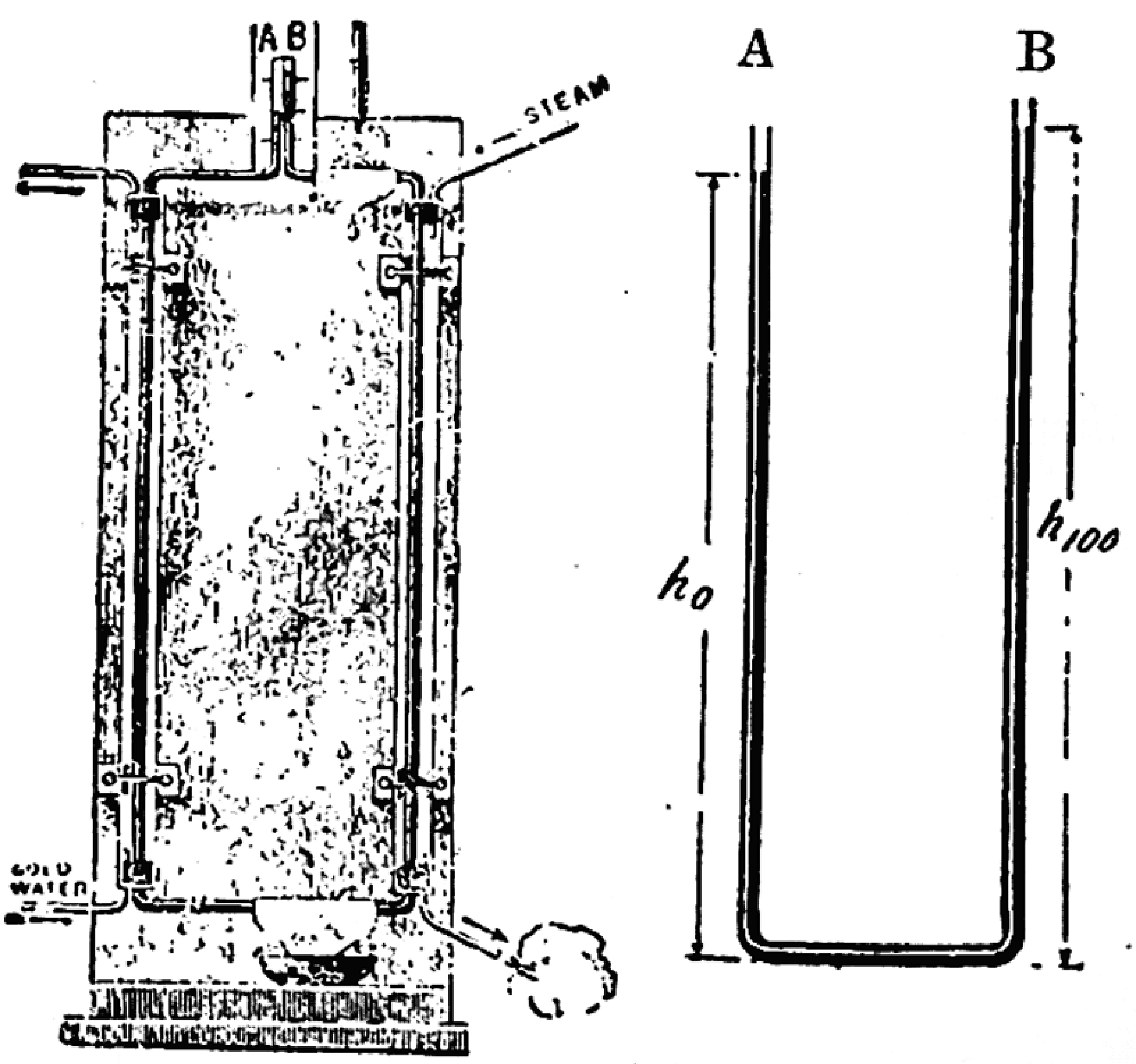
This formula is accurate enough, even if the lower temperature is not 0°C.

It may be remarked here that if we know the coefficient of apparent expansion of a liquid, by finding the weight of the liquid expelled and of the liquid left behind at higher temperature we can find the unknown temperature. That is why this instrument is called a Weight Thermometer.

If it be desired to determine the coefficient of real expansion of a liquid with the help of the weight thermometer, add to γ_a, the coefficient of cubical expansion of the vessel, for $\gamma_r = \gamma_a + g$.

We shall now consider Dulong and Petit's method of determining the coefficient of real expansion of liquids.

167. Determination of the Coefficient of Real Expansion of a Liquid by the Method of Dulong and Petit.—In its essentials the apparatus used by Dulong and Petit consists of a U-tube. The method is based on the principle that if two liquids produce equal pressure (*i.e.*, are in equilibrium), their heights are inversely proportional to their densities. Fig. 15 (*a*) shows the actual apparatus, and Fig. 15 (*b*) a rough sketch of the essentials.



(*a*) (*b*)

Fig. 15.

A and *B* are the two arms of the U-tube. Each arm is surrounded by a wider glass tube ; through the tube surrounding *A* ice-cold water flows whereas through the tube surrounding *B* steam is passed. Both the arms contain the same liquid, but owing to the difference of temperature the density is different on the two sides. Let h_0, ρ_0 be the height and density of the liquid in the arm *A*, and h_{100}, ρ_{100} the

height and density of the liquid in the arm B. Since the liquids are in equilibrium,

$$h_0\rho_0 = h_{100}\rho_{100},$$

or

$$\frac{\rho_0}{\rho_{100}} = \frac{h_{100}}{h_0}.$$

But $\frac{\rho_o}{\rho_{100}} = 1 + \gamma.100$, (see §163), where γ is the coefficient of real cubical expansion of the liquid.

Hence

$$\frac{h_{100}}{h_o} = 1 + \gamma.100,$$

or

$$\gamma = \frac{h_{100} - h_o}{h_o.100}$$

The formula shows that the value of γ obtained by this method is independent of the expansion of the tubes.

Regnault made a series of experiments and found that the coefficient of real expansion of mercury is not exactly the same at all temperatures. His results are as follows :

Coefficient	of real	expansion	between	0°	and	100°C.	=0·0001815.
,,	,,	,,	,,	0°	,,	200°C.	=0·0001841·
,,	,,	,,	,,	0°	,,	300°C.	=0·0001866.

From this we see that the expansion of mercury is not quite uniform with the rise of temperature.

168. Expansion of Water.—We have considered the expansion of liquids in general, but the expansion of water deserves special notice. Generally all liquids go on increasing in volume, *i.e.*, decreasing in density as they are heated ; but water behaves in a strange way. Its volume decreases when it is heated from 0°C to 4°C (and therefore density increases) but above 4°C, it begins to expand and hence its density begins to decrease, *the density of water at 4°C being maximum*. To show experimentally that the density is maximum at 4°C. take a flask of 200 to 300 c.c. capacity fitted with a cork having two holes. Fill one-seventh of the flask with mercury and the rest with coloured water. Since the coefficient of cubical expansion of mercury is 7 times as much as the coefficient of cubical expansion of glass, mercury will expand as much as the flask and hence the space above mercury will remain the same at all temperatures. This method of filling one-seventh of a glass flask with mercury enables us to study the real expansion of all liquids directly. Pass through one hole a glass tube of narrow bore and through the other a thermometer. Fix behind the glass tube a paper scale. Push the tube a little downward so that the coloured water stands 3 or 4 cm. above the level of the cork. Place the flask in a freezing mixture of ice and salt, and observe carefully the level of the coloured water in the tube ; it will be noticed that at first the level falls rapidly, then slowly, and finally becomes steady. The thermometer will be found to register at this stage 4°C. As the cooling continues below 4°C. the level rises again, at first slowly, but more rapidly afterwards. It is clear from this experiment that the volume of water is minimum, and hence **the density maximum at 4°C.**

The following table shows the volume of water at various temperatures between 0° and 100°C., the volume at 4°C. being taken as unity.

0° C.	... 1·00013	50° C.	... 1·01205
3° ,,	... 1·00003	60° ,,	... 1·01698
4° ,,	... 1·00000	80° ,,	... 1·02885
10° ,,	... 1·00027	90° ,,	... 1·03566
30° ,,	... 1·00433	100° ,,	... 1·04315

The mean coefficient of cubical expansion of water between 4°C. and 100°C is 0·000432, of alcohol 0·001 and of petroleum 0·0009.

Fig. 16 shows graphically the volume of 1 gm. of water at various temperatures between 0° and 20°C.

The fact that water possesses maximum density at 4°C. plays a very important part in the economy of Nature. The cooling of water in winter goes on till the ponds and tanks are all at 4°C. Further cooling results in the water at the surface becoming lighter and therefore remaining at the top, leaving the rest of the water at 4°C., and hence unaffected. Further cooling of the top layers may result in their freezing. Since ice is a bad conductor of heat, it does not allow the lower layers to be cooled by the cold outside. This enables the aquatic animals to continue to live even in the most severe winter.

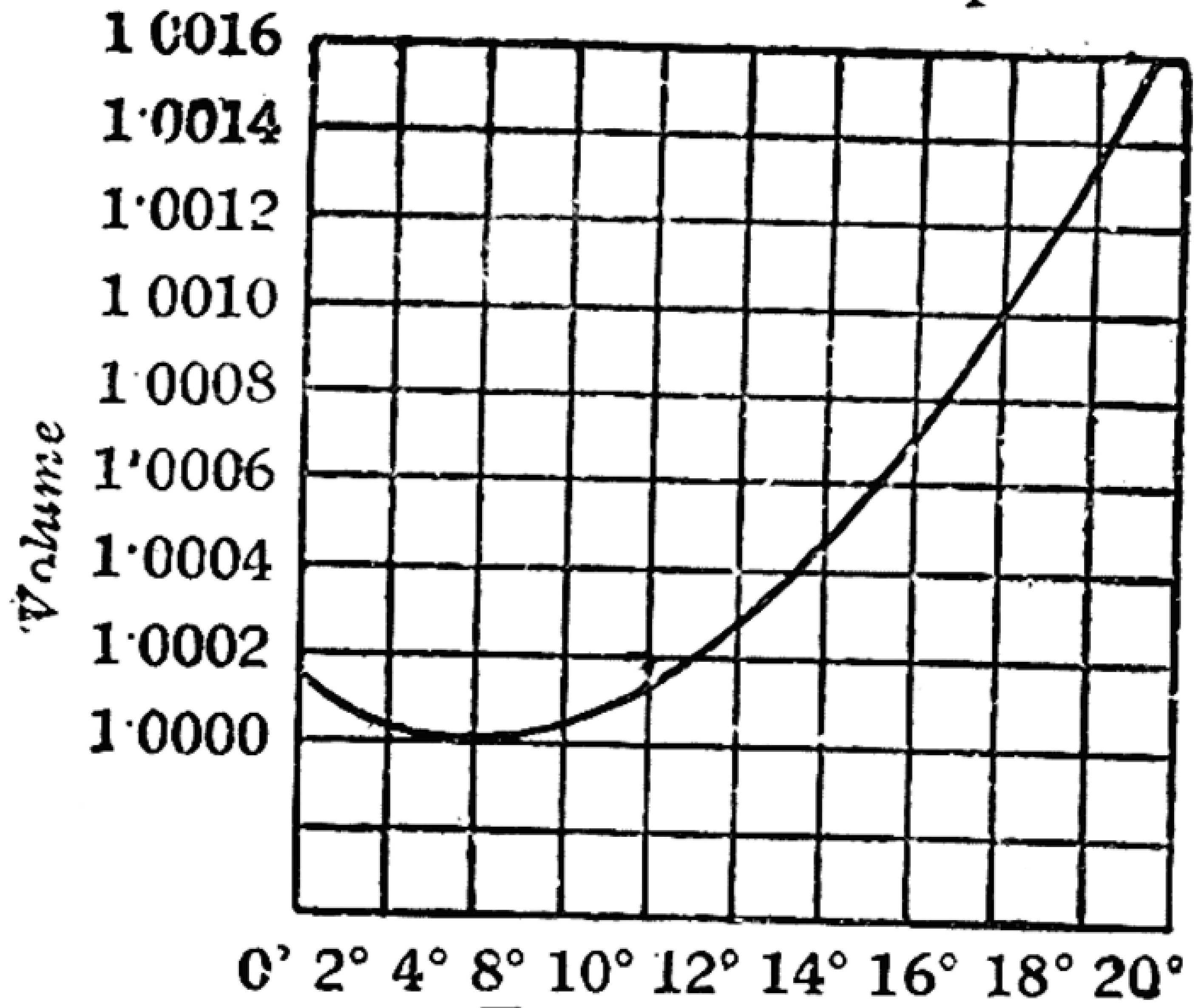


Fig. 16.

EXERCISES

1. A weight thermometer weighs 40 gm. when empty, and 490 gm. when filled with mercury at 0°C.; on heating it to 100°C., 6·85 gm. of mercury escape. Calculate the coefficient of cubical expansion of glass, given that the coefficient of real expansion of mercury is 0·000182.

$$\gamma_a = \frac{m}{Mt} = \frac{6{\cdot}85}{443{\cdot}15 \times 100} = 0{\cdot}0001546.$$

But

$$\gamma_r = \gamma_a + g,$$

$$\therefore\ g = \gamma_r - \gamma_a = 0{\cdot}000182 - 0{\cdot}0001546 = 0{\cdot}0000274.$$

2. A weight thermometer which contains 50 grams of mercury at 15°C. is heated to a certain temperature, and the mercury expelled is found to weigh 0·945 gm. Find the temperature to which the thermometer was heated, coefficient of apparent expansion of mercury being 0·000155.

$$0{\cdot}000155 = \frac{0{\cdot}945}{49{\cdot}055 \times (t-15)},$$

or

$$t - 15 = \frac{0{\cdot}945}{49{\cdot}055 \times 0{\cdot}000155} = 124{\cdot}3,$$

$$\therefore\ t = 139{\cdot}3°C.$$

3. Find what length of a glass tube 750 mm. long must be filled with mercury in order that the volume of the tube unoccupied by mercury may be the same at all temperatures. The coefficient of cubical expansion of mercury is 0·000182, and of glass 0 000026.

Since the coefficient of expansion of mercury is seven times that of glass, if mercury be 1/7th of the volume of the glass tube, it will expand as much as the glass tube. Since the tube has a uniform bore, its volume will be proportional to its length. Hence the mercury must fill $\frac{750}{7}$ mm. of the tube ; or 107·14 mm.

4. How much mercury will overflow when a weight thermometer which contains 40 gm. of mercury at 0°C. is heated to 100°C., given that the apparent expansion of mercury in glass is $\frac{1}{6500}$?

Ans. 6·0606 gm.

5. A glass weight thermometer contains 147·4 gm. of mercury at 10°C., at 95°C., 2·02 gm. of mercury are expelled. Find the coefficient of expansion of mercury in glass. *Ans.* 0·000163.

6. A glass weight thermometer contains 67 gm. of a liquid at 10°C. When the temperature is raised to 100°C., 1 gm. of the liquid overflows. What is the coefficient of absolute expansion of the liquid assuming the coefficient of linear expansion of the glass to be 0·000009 ?

Ans. 0·000195.

7. A weight thermometer which contains 100 gm. of mercury at 0°C. is placed in an oil-bath and the mercury expelled is found to weigh 2 gm. Find the temperature of the bath. Take γ_a for mercury as 0·00016. *Ans.* 127·6°C.

8. In an experiment with Dulong and Petit's apparatus to find the expansion of a liquid the column at 32°F. was 62·3 cm. high and at 212°F. it was 5·67 cm. higher. Find the coefficient of real expansion of the liquid (for 1°C.). *Ans.* 0·00091.

9. The coefficient of real expansion of mercury is 0·00018 and the coefficient of linear expansion of glass is 0·000009. Find what length of a tube of glass 300 mm. long must be filled with mercury in order that the volume unoccupied by mercury may remain the same at all temperatures. *Ans.* 45 mm.

10. Find what length of a glass tube 95 cm. long must be filled with mercury so that the *length* of the tube unoccupied by mercury may be the same at all temperatures. Given that mercury expands 7 times as much as glass does. *Ans.* 5 cm.

11. A long capillary tube of glass contains a thread of mercury at 10°C., the length of which according to a brass scale attached to the tube is 150·56 cm. What will be the length recorded by the scale at 55°C ?

Take coefficient of linear expansion of glass as 0·000009, of brass as 0·000019 and coefficient of real cubical expansion of mercury as 0·00018. *Ans.* 151·53 cm.

12. 100 gallons each of water, of petroleum and of alcohol are measured at 4°C. They are found to be 100·96, 103·6, and 104·4 gallons respectively when the temperature rises to 44°C. Find the mean

coefficient of cubical expansion of water, petroleum and alcohol. Compare the coefficient of expansion of water that you get with the mean coefficient of water between 4 and 100°C. and explain the reason for the difference. *Ans.* (*i*) 0·00024, 0·0009 and 0·0011.

13. Is it correct, "A gallon of vinegar weighs more in winter than in summer"?

Find the coefficient of expansion of vinegar if a tin of it at 28°C. weighs 8·403 lb. and at 5°C. it weighs 8·457 lb. *Ans.* 0·00028.

CHAPTER IV

The Expansion of Gases

169. While discussing the expansion of solids and liquids with rise of temperature we did not take into consideration the pressure to which they were subjected. It was because the change of pressure does not appreciably change their volume. But the case of gases is different. Their volume changes considerably with change of pressure even when temperature remains the same. It is, therefore, that we keep the pressure constant throughout an experiment designed to study the expansion of a gas with rise of temperature. When it is done we are sure that the change in volume is due to heating only.

The fact that we can change the pressure, volume, or temperature of a given mass of gas, is often expressed by saying that gases have three *variables*, p, v, and t. Now either all these quantities may be independent, or some relation may exist between them, which means simply that if we know any two of them we can calculate the third. It will be seen afterwards that such a relation does exist. But before we consider this relation we shall discuss some experiments in which we shall keep one quantity constant, vary the second, and study the change produced in the third. We can do so in three ways :—

(*i*) We can vary p, keep t constant, and study the change produced in v ; or

(*ii*) We can vary t, keep p constant, and study the change produced in v; or

(*iii*) We can vary t, keep v constant, and study the change produced in p.

We shall take up these cases one by one.

170. Relation between Pressure and Volume of a Gas at Constant Temperature.—We shall refer to it very briefly here, for we have already dealt with it in §140. It was explained there that *the volume of a gas varies inversely as the pressure, provided the temperature remains constant*, or that the product of pressure and volume of a gas is constant, provided its temperature remains the same. Expressing it in symbols we say that

$$pv = \text{constant (say } K\text{)}.$$

This is known as **Boyle's Law.**

According to this law the product of p and v should remain constant, whatever the value of p. But experiments tell us that at high pressures pv does not remain the same as at low pressures. At first it slightly decreases in value and then increases with pressure. A gas which obeys Boyle's law is called a *perfect gas.* Dry air, hydrogen, nitrogen, and oxygen, etc. are very nearly perfect gases.

171. Relation between Temperature and Volume of a Gas at Constant Pressure.—This enquiry corresponds to the study of the cubical expansion of solids and liquids with rise of temperature. It is found that gases are not only the most expansible of all bodies, but that their expansion is most regular. To study experimentally the expansion of gases with rise of temperature, Regnault's apparatus shown in Fig. 17 is used. Bulb A of known volume is connected by a thin glass tube to a U-tube $BCDE$ containing mercury, the arm DE being longer than the arm CB. There is a tap T below D through which the mercury in the arm DE can be run out when required. There is a small capillary tube t connected to the thin glass tube with the help of the stopcock G. The stopcock enables us to connect bulb A with U-tube $BCDE$ and cut it off or connect it with the capillary tube and cut it off from the U-tube. To perform the experiment the bulb A is connected to the capillary tube and is exhausted. It is then filled with hot air and re-exhausted. By repeating this process the interior of the bulb is thoroughly dried. The bulb is then filled with the gas whose coefficient of expansion is to be determined ; and is connected to the U-tube. The vessel M which surrounds the bulb is filled with pounded ice, and the pressure of the gas is adjusted so that mercury in the U-tube stands at level f in both the arms. The gas in the bulb A is at 0°C. and is at the atmospheric pressure. The ice is now removed from the vessel M, and steam is allowed to play upon the vessel A. The gas in it gets heated and consequently expands pushing the column of mercury downwards in the arm BC. As a result of it the level of mercury column in the other tube rises. To make the pressure in the bulb atmospheric again, the tap T is opened and mercury is allowed to run out, so that it is at the same level in both the arms as shown at f' and f''. The arm BC is graduated, hence we know the increase in volume. Knowing original volume and the rise of temperature, we can find the **coefficient of expansion of the gas** which is defined as *the increase in volume produced in unit volume at* 0°C., *when it is heated through* 1°C., *the pressure remaining the same.* From the result of such expriments, it is found that the coefficient of expansion of a gas is $\frac{1}{273}$ or 0·00366 of its volume at 0°C. This result is true for almost all gases. It should be noted that the expansion that we get in this experiment is apparent expansion, but since the increase in the volume of the vessel is negligible in comparison with the increase in the volume of the gas, we can suppose with a fair degree of accuracy that this expansion is the real expansion.

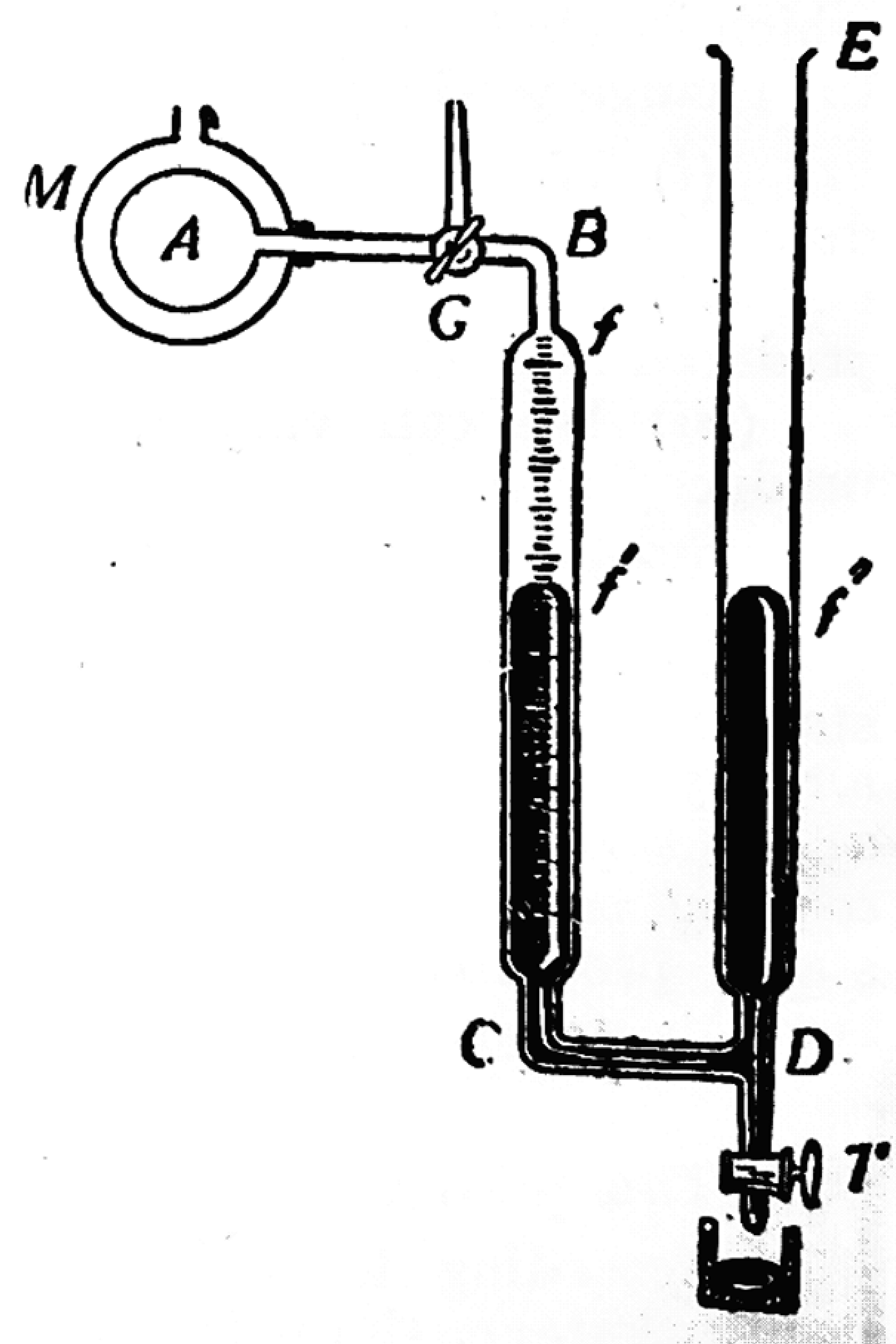


Fig. 17.

To sum up, we have learnt that *if a given volume of a gas be heated through 1°C., it expands by* $\frac{1}{273}$ *of its volume at 0°C., provided the pressure is kept constant.* This law is known as **Charles's Law.**

172. The student should mark the words **volume at** 0°**C**. in the definition of Charles's Law. In the formulæ relating to the expansion of liquids and solids we did not specify any lower temperature because their expansion was so small, that a^2, etc , could be neglected ; if that were not the case, there also we would have used the lower temperature as 0°C.

Let us see why it is important to take the lower temperature as 0°C. in the case of gases and not in the case of solids or liquids. Suppose we take at 0°C., 273 c c. of a gas,

at	1°C.	the volume	becomes	274 c.c. or 273 $(1+\frac{1}{273})$,
,,	2°C.	it	,,	275 c.c. or 273 $(1+\frac{2}{273})$,
,,	100°C.	it	,,	373 c.c. or 273 $(1+\frac{100}{273})$,
,,	101°C.	it	,,	374 c.c. or 273 $(1+\frac{101}{273})$

or in general at t°C., its volume becomes

$$V_t = V_0\left(1+\frac{t}{273}\right)$$

We see from the above example that the increase in volume for one degree rise of temperature is always $\frac{1}{273}$ of its volume at 0°C. In the above example it is 1 c.c. whether the gas is heated from 100° to 101° or from 0° to 1°. If we do not mind the words "of its volume at 0°C." we get wrong result. For instance, 373 c.c. of a gas at 100°C. should become 374·367 c.c. as shown below—

$$\begin{aligned} V_{101} &= 373(1+\tfrac{1}{273}) \\ &= 373+\tfrac{373}{273} \\ &= 373+1{\cdot}367 \\ &= 374{\cdot}367. \end{aligned}$$

This certainly is not the case, for, as said above, the volume of 373 c.c. of a gas at 100°C. is found to be 374 c.c. at 101°C. This shows why we should remember the words '*of its volume* at *0°C.*'

In the case of solids this point does not materially affect the result. Suppose we have a rod of copper which is 100 cm. at 0°C.

at	1°C.	it will	become	100 (1+0·000017)	or 100·0017 cm.
at	100°C. ,,	,,	,,	100 (1+0·0017)	or 100.1700 cm.
at	101°C, ,,	,,	,;	100 (1+0 001717)	or 100·1717 cm.

Calculating directly $L_{101} = L_{100}(1+a)$

$$= 100{\cdot}17(1+0{\cdot}000017) = 100{\cdot}171703.$$

The difference in two calculations comes out to be 0·000003 of a cm., which is too small to be ordinarily taken into account.

Next let us suppose we want to find the volume at temperature t_2 of a gas when its volume at temperature t_1 is given, the pressure remaining the same. From what has been said above, the student

would think that he must know the volume at 0°C. before he can find the volume at t_2°C. This, however, is not necessary, for he can proceed in the following manner.

$$V_{t_1}=V_0\left(1+\frac{t_1}{273}\right)$$

and
$$V_{t_2}=V_0\left(1+\frac{t_2}{273}\right)$$

Dividing one by the other we get

$$\frac{V_{t_2}}{V_{t_1}}=\frac{1+\dfrac{t_2}{273}}{1+\dfrac{t_1}{273}}=\frac{273+t_2}{273+t_1},$$

or
$$V_{t_2}=V_{t_1}\times\frac{273+t_2}{273+t_1}.$$

173. Relation between Pressure and Temperature of a Gas at Constant Volume.—When a gas is heated at constant volume, its pressure increases. *The ratio of the increase of pressure to the pressure at 0°C. when a gas is heated through 1°C. at constant volume, is called the* **coefficient of increase of pressure.** To study the relation between the pressure and temperature at constant volume the apparatus shown in Fig. 17 is used. The procedure is exactly the same as has been explained in §171, with the only difference that instead of opening the tap T to make the level of mercury same in both the arms, we pour mercury in the tube E, so that the level of mercury in BC may come once more to f. When it is so, the difference in the height of mercury columns is noted, and from that the total pressure to which the gas in the vessel is subjected is calculated. It is found that the coefficient of increase of pressure of a gas is $\frac{1}{273}$ of its pressure at 0°C.

To sum up, the relation between pressure and temperature at constant volume is as follows :

If a given volume of a gas be heated through 1°C., its pressure increases by $\frac{1}{273}$ of its pressure at 0°C., provided the volume is kept constant.

Here also the student should note carefully the words *pressure at 0°C.*

We can write this relation as

$$P_t=P_0\left(1+\frac{t}{273}\right).$$

If pressure is to be calculated at a temperature t_2 when it is given at temperature t_1, then either we must first find the pressure at 0°C., or use the following relation,

$$P_{t_2}=P_{t_1}\times\frac{273+t_2}{273+t_1}.$$

The two coefficients *i.e.* the coefficient of expansion and of increase of pressure are equal to each other. We can derive the same result theoretically.

174. Let us start with a volume V_0 of a gas at pressure P_0 and temperature 0°C. Heat it to t°C., keeping pressure constant ; the new volume V_t will be given by the equation

$$V_t = V_0(1+ct).$$

where c is the coefficient of expansion of the gas. Now let us change the pressure (keeping the temperature constant) so that the volume V_t is reduced to V_0 once more ; call this new pressure P_t. By Boyle's law, we have

$$P_0 V_t = P_t V_0$$

$$\therefore \quad P_t = P_0 \frac{V_t}{V_0}$$

but $\dfrac{V_t}{V_0} = 1+ct$; substituting this value in the above equation we get

$$P_t = P_0(1+ct).$$

This shows that in the case of a gas which obeys Boyle's law the two coefficients are equal.

175. Air Thermometer.—We have already said in §154 that an air thermometer is used as standard thermometer with which all other kinds of thermometers are compared. Since air expands 20 times as much as mercury, an air thermometer is far more sensitive than a mercury thermometer; moreover air expands much more uniformly with the rise of temperature than mercury. The most important reason, however, for taking air thermometer as standard thermometer is that its temperature scale agrees very closely with the scale derived from theoretical principles into which we cannot enter, as they are beyond the scope of this book.

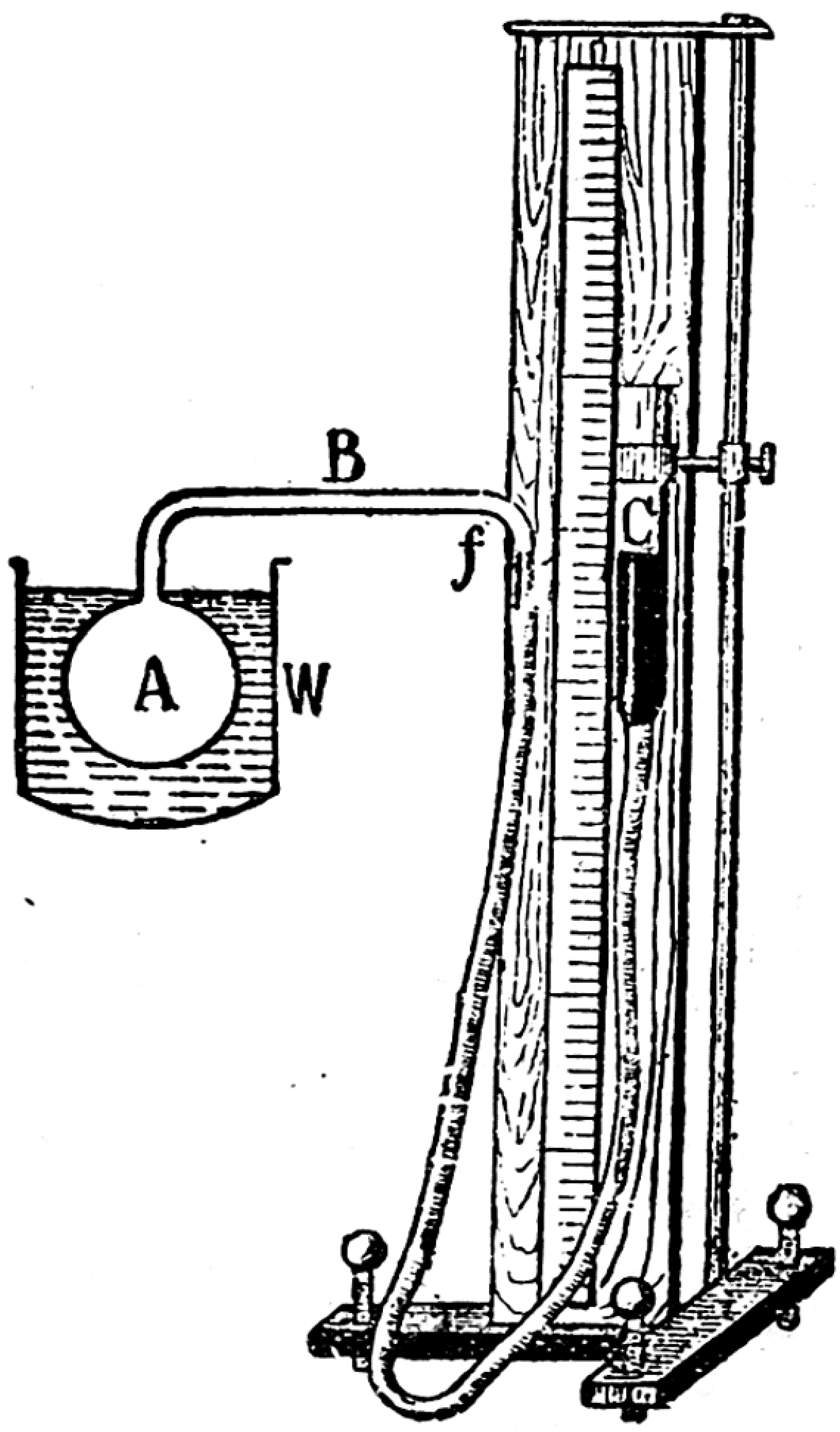


Fig. 18.

The air thermometer may be either of constant pressure type or of constant volume type, according as the pressure or volume is kept constant while measuring temperatures.

Since in the constant pressure form of air thermometer, a considerable portion of the gas is at a lower temperature than the gas in the bulb, correction must be applied, which is by no means a simple affair, hence in practice the constant volume form is used. The actual standard constant volume air thermometer is very cumbersome and unwieldy for

everyday work. Hence in laboratories Joly's form is ordinarily used. It consists of a glass bulb A ending in a capillary tube B bent twice at right angles and connected by a pressure tubing to a glass tube C and is mounted on a vertical board as shown is Fig. 18. The pressure tubing is filled with mercury. Let us see how we can measure temperatures with this apparatus. Surround the bulb A by a can W containing ice and wait for some time to allow the gas in A to come to 0°C. Adjust position of C so that the mercury column in B stands at a point f as near the bend as possible. Read the level of the mercury column in both the tubes and let the difference in reading be h. Now put a burner underneath the can and heat the water to the boiling point. When it has boiled for about 15 minutes adjust the position of C so that the mercury column in B is once more at the point f. Note the difference in level of mercury column in tubes B and C and let the difference be H. Read the barometer and let the atmospheric pressure be P. Find from the tables the boiling point of water. Let us suppose for simplicity that the boiling point is 100°C. Now surround the bulb A by the body whose temperature is to be measured, and when the temperature of the gas in A is same as that of the body, adjust the level of C and bring the mercury column in B at f. Read the difference in level and let it be H'. The pressure acting on the gas in the three cases is $P+h$, $P+H$, and $P+H'$. Let us call these pressures, P_0, P_{100}, and P_t. The value of c, *i.e.*, the coefficient of increase of pressure can be found from P_{100} and P_0, for $P_{100}=P_0(1+100c)$ or $c=\dfrac{P_{100}-P_0}{P_0\times 100}$.

Substituting this value of c in the relation $P_t=P_0(1+ct)$ we can find the value of t as shown below :—

$$t=\frac{P_t-P_0}{P_0\times c}=\frac{(P_t-P_0)P_0}{P_0(P_{100}-P_0)}\times 100$$

$$=\frac{P_t-P_0}{P_{100}-P_0}\times 100.$$

176. Absolute Temperature.—Now let us enquire how a gas will behave if instead of being heated it is cooled at constant pressure. Suppose we take 273 c.c. of a gas in a long tube at 0°C. and at atmospheric pressure. On cooling the tube the gas will contract, decreasing by $\frac{1}{273}$ of its volume at 0°C. (*i.e.*, $\frac{273}{273}$ or 1 c.c.) for every centigrade degree that it is cooled below zero. At -100°C. the volume will be 173 c.c. and at -200°C. the volume will be 73 c.c. only. It is evident that if the gas could be cooled to -273°C. the volume would be zero. The result is, of course, physically impossible, for a few degrees before this temperature is reached the gas is liquefied and hence it no longer obeys Charles's law

If we regard this -273°C. as zero and measure temperatures from it upwards, 0°C. will correspond to 273°, and 100°C. to 373°C. and t°C. to $273°+t$.

This -273°C.* is called the **Absolute Zero**, and the scale of temperatures mentioned above, *i.e.*, $273+t$, is called the **Absolute Scale** of temperatures. In solving problems relating to gases given at the end

*On Fahrenheit scale it is $-459{\cdot}4$°.

of this chapter the use of the absolute scale will be found convenient.

We shall sum up once again the relations which we have proved already,

$$\frac{V_{t_2}}{V_{t_1}}=\frac{273+t_2}{273+t_1}=\frac{T_2}{T_1},$$

where T_2 and T_1 are absolute temperatures. This relation can be expressed more simply by $\frac{V_{t_2}}{T_2}=\frac{V_{t_1}}{T_1}=\frac{V}{T}$ =a constant. Thus we see that Charles's law leads us to the conclusion that

The volume of a given mass of a gas at constant pressure is proportional to its absolute temperature.

Similarly, the law of pressures can be expressed as

$$\frac{P_{t_2}}{P_{t_1}}=\frac{273+t_2}{273+t_1}=\frac{T_2}{T_1}, \quad \text{or} \quad \frac{P_{t_2}}{T_2}=\frac{P_{t_1}}{T_1},$$

or more simply $\frac{P}{T}$ =a constant, *i.e., the pressure of a given mass of a gas at constant volume is proportional to its absolute temperature.*

177. General Relation between Pressure, Volume and Temperature of a Gas.—It frequently happens that when a gas is heated, its volume and pressure both change. To find in such a case the new volume or pressure we have to combine the laws of Charles and Boyle. Let us see how that is done.

Let V denote the volume of a given mass of gas, enclosed in a cylinder [Fig. 19(*a*)], T its absolute temperature and P its pressure.

Change the temperature from T to T' and as a result of it let the pressure change from P to P' and the volume from V to V'. Let us find the relation between the various quantities.

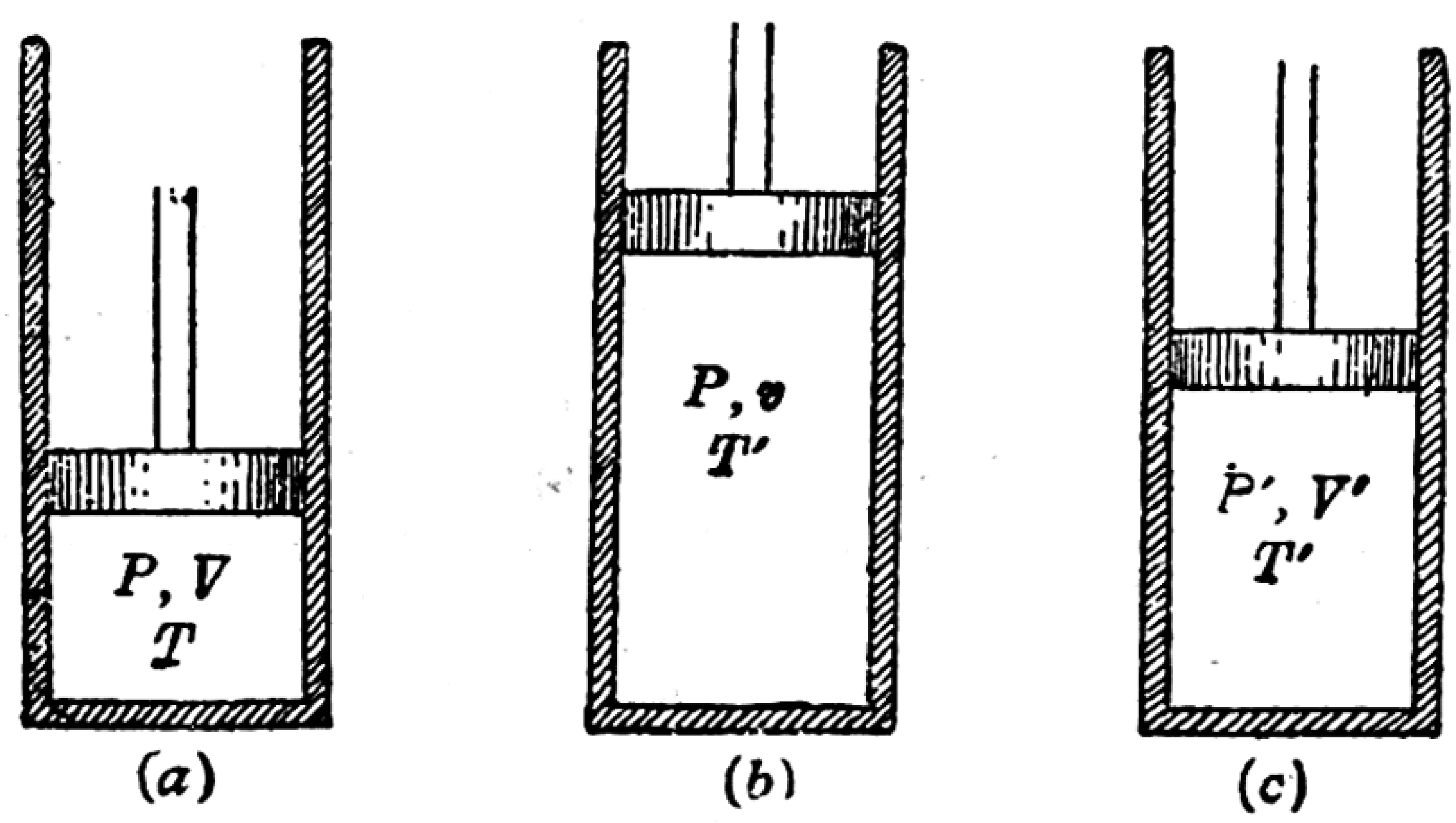


Fig. 19.

For the sake of simplicity suppose that the two changes take place separately. Let first the gas be heated at constant pressure to T' absolute). By Charles's law the new volume v will be related to the initial volume V by the relation $\frac{V}{T}=\frac{v}{T'}$. Therefore $v=\frac{V}{T}T'$ (Fig. 19 (*b*)].

Let the temperature be now kept constant and the pressure be changed from P to P'. By Boyle's law we have

$$P'V'=Pv,$$

where V' is the final volume. Substituting the value of v obtained above we get

$$P'V'=PV\frac{T'}{T},$$

or

$$\frac{P'V'}{T'}=\frac{PV}{T}.$$

In other words, we find that $\frac{PV}{T}$=a constant. If we denote the constant by R, we can write this relation as

$$\frac{PV}{T}=\frac{P'V'}{T'}=R.$$

This equation is generally called the *gas equation*.

The value of R depends upon the mass of gas taken. If the mass be 1 gram, the constant is usually denoted by r. Its value for a gas can be easily calculated from the relation

$$\frac{P_0V_0}{T_0}=r,$$

where V_0 is the volume of 1 gm. of the gas at pressure P_0 and temperature T_0 Absolute.

Let us find the value of the constant for 1 gm. of air; V_0 at 0°C (*i.e.*, 273° A) and at atmospheric pressure is $\frac{1}{0{\cdot}001293}$ c.c.

Substituting values for V_0, P_0 and T_0 we get

$$r=\frac{76\times13{\cdot}56\times981}{0{\cdot}001293\times273}$$

$$=2{\cdot}87\times10^6 \text{ ergs per gram per°C or}(A).$$

For hydrogen $r=4{\cdot}13\times10^7$,, ,, ,, ,, ,,

As shown above the value of 'r' varies with the nature of the gas But if we were to take 1 gram-molecule* of a gas the value of the constant is found to be the same for all gases.

This value is called Universal Gas Constant and is denoted by R. Its value is equal to $8{\cdot}26\times10^7$ ergs per gram-molecule per 0°C(or A)

EXERCISES

1. Compare the volumes of a given mass of air at 33°C. and −30°C., the pressure being the same.

$$\frac{V_{33}}{V_{-30}}=\frac{273+33}{273-30}=\frac{306}{243}=\frac{34}{27}=1{\cdot}259.$$

2. A litre of dry air weighs 1·293 gm. at N.T.P. At what temperature will a litre of air weigh 1 gm., the pressure being 72 cm.?

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*By a gram-molecule is meant a mass in grams equal to the molecular weight of the substance.

At N.T.P. 1 gm. of air would have $\frac{1}{1 \cdot 293} \times 1000$ or 773·4 c.c. as the volume. The problem is reduced to this : at what temperature would 773·4 c.c. at N.T.P. become 1000 c.c. at 72 cm. pressure ?

Applying
$$\frac{PV}{T} = \frac{P'V'}{T'},$$

we get
$$\frac{76 \times 773 \cdot 4}{273} = \frac{72 \times 1000}{T'}$$

or
$$T' = \frac{72 \times 1000 \times 273}{76 \times 773 \cdot 4} = 334 \cdot 4$$

or
$$= 61 \cdot 4°C.$$

3. A steel cylinder placed in melting ice is filled with compressed oxygen at a pressure of 42 atmospheres ; if the cylinder is taken out of the ice and is allowed to stand in a room at 26°C., what will be the pressure of the gas in the cylinder ?

$$P_t = P_0\left(1 + \frac{t}{273}\right) = 42\left(1 + \frac{26}{273}\right)$$

$$= 42 \times \frac{299}{273} = 46 \text{ atmospheres.}$$

4. A gas at 0°C, and 74 cm. pressure is contained in a vessel of 2 litres capacity. What will be the pressure when the temperature rises to 24°C. ? *Ans.* 80·5 cm.

5. Find the temperature at which the column of a given mass of air will be half as much again as it is at 15°C. the pressure being the same. *Ans.* 159°C.

6. The volume of a certain mass of gas is 145 c.c. at 17°C and a pressure of 72·5 cm. of mercury. What will be the volume if the temperature falls to 7°C., pressure remaining the same ? *Ans.* 140 c.c.

7. On heating a certain substance it is found that 380 c.c. of oxygen are given off, the temperature being 23°C. and pressure 74 cm. What would be the volume of oxygen measured at the normal pressure and temperature ? *Ans.* 341·25 c.c.

8. 54·02 c.c. of a gas at 22°C. and 74 cm. pressure on cooling down to 0°C. became 49·3 c.c. at a pressure of 75 cm. ; calculate the coefficient of expansion of the gas. *Ans.* 0·003685.

9. A litre of hydrogen at N. T. P. weighs 0·9 gm. What is the weight of a litre of this gas at 27°C. and 740 mm. pressure ? *Ans.* 0·797 gm.

10. A closed vessel is filled with air at N.T.P. If the vessel can withstand a pressure of 4 atmospheres, find the temperature to which the vessel can be heated before it bursts. *Ans.* 819°C.

11. What is the weight of a litre of air collected at the top of a mountain where the barometer stands at 60 cm. and the temperature is 15°C ? (1 litre of air at N. T. P. weighs 1·29 gm.) *Ans.* 0·9653 gm.

12. Find out the absolute zero and the coefficient of increase of volume of a gas on the Fahrenheit scale. *Ans.* $-459{\cdot}4°$F. and $\frac{1}{491{\cdot}4}$.

13. Prove the gas equation $pv = RT$, and calculate the value of R for 1 gm. of air; given that the density of air at N.T.P. is 0·001293 gm. per c.c. *Ans.* $2{\cdot}872 \times 10^6$.

14. A bicycle tyre is inflated with air to a pressure of 1·4 atmospheres when the temperature is 71°F. What will be the pressure of the air in the tyre when the temperature rises to 121°F., the volume remaining constant? *Ans.* 1·53 atmospheres.

CHAPTER V

Calorimetry

178. In the preceding chapters, we studied the methods of measuring temperature and the changes produced in volume by heat, without caring to know how much heat was utilized in a process. Now we shall consider this part of the subject, *i.e.*, measurement *of the quantities of heat*. This subject is called **calorimetry.** Before we begin to deal with it, let us understand clearly what we mean by the expression *quantity of heat*. Heat a kettle half full of water to 80°C. Take out of it a cup of water. The temperature of water in both the vessels is the same but the quantity of heat is quite different in the two. It is obvious that the water in the kettle will give out much more heat while cooling to the room temperature than the water in the cup. Of course the heat given out by water in the act of cooling is the same as it takes when it is heated through the same range of temperature. This illustration tells us the difference between the temperature and the quantity of heat. Now let us see on what factors does the quantity of heat given out by a body while cooling depend. Scoop out a hollow in a block of ice. Introduce into it 100 gm. of iron nails heated to the temperature of steam. They will melt a certain amount of ice. Take next 200 gm. of iron nails and heat them to the same temperature. When they are introduced into the hollow, they will melt double the amount of ice. It is obvious that the heat which ice requires to be melted comes from the hot nails, and further that the heat given out by the nails in the second case, is twice as much as the quantity given out by the nails in the first case. This shows that the quantity of heat given out by a body in falling through a certain range of temperature depends upon its mass, *the greater the mass the greater the quantity of heat given out.*

Introduce next into the hollow the same mass of nails twice, once after heating them to 100°C. and second time to 200°C. It will be noticed that the ice melted in the second case is double in quantity of the ice melted in the first case. This shows that the amount of heat given out by a body while cooling depends upon the fall of temperature, *the greater the fall of temperature, the greater the quantity of heat given out.*

Introduce now into the hollow in ice same mass of brass nails, heated to the same temperature. It will be found that the amount of heat given out by them is different, showing thereby that the heat given out by a body in falling through a given range of temperature depends upon its *nature.*

We can include all the above-mentioned three conditions in the relation,

$$Q = Sm\theta \qquad \text{. . . . .} \qquad (i)$$

where Q is the quantity of heat given out by a body, S a constant depending upon its nature, m its mass, and θ the fall in temperature.

Formula (*i*) represents also the heat absorbed by the above body when it is heated through θ degrees.

It is clear from what has been said above that heat is a measurable quantity. Now let us see how to measure it. Evidently we must first select a unit.

In the English system the unit of heat is known as the British Thermal Unit (written as B. Th. U.). It is defined as the quantity of heat required to heat 1 pound of water through 1°F.

In the C.G.S. system the unit is known as the **calorie.** *It is the amount of heat required to heat one gram of water through* 1°C. Of course the heat given out by one gram of water in cooling through 1°C is also the same.

One B. Th. U. is equal to 252 calories. There is another unit also called Calorie. It is usually written with capital C. and is 1,000 times bigger than the calorie defined above. It is the amount of heat required to raise the temperature of 1 kilogram of water by 1°C. This unit is used by Physiologists when they talk of the fuel value of various foods.

If we substitute in formula (*i*) 1 gram for m, 1°C. for θ and 1 calorie for Q, we get $S=1$. This means that the constant S is equal to one for a substance which requires 1 calorie to heat 1 gram of it through 1°C. This as we know is the case with water. Hence for water the formula (*i*) reduces to $Q=m\theta$.

179. Specific Heat.—We have already seen that equal masses of different substances in falling through the same range of temperature give out different amounts of heat. We expressed it by saying that the amount of heat given out by a body when falling through a certain range of temperature was proportional to the constant S depending upon the nature of the material. Let us study this constant in greater detail.

Suppose we take m grams of water and m grams of copper, and heat or cool them through θ°C. The heat, Q, absorbed or given out by water will be equal to $m\theta$ whereas heat given out by copper, say Q', will be equal to $m\theta S$. Dividing one by the other, we get

$$S=\frac{Q'}{Q}$$

The constant S is called the specific heat. We define it thus :

The specific heat of a substance is the ratio of the amount of heat required to raise the temperature of a given mass of it through a certain range to the amount of heat required to heat the same mass of water through the same range of temperature.

When heat is measured in calories, the specific heat is equal to the number of calories required to heat 1 gram of the substance through 1°C.

It is clear from formula (*i*) that the amount of heat required by a body to be heated through 1°C. is equal to mS. This quantity is called the **thermal capacity** of the body.

180. Before we begin calorimetry proper, let us understand the basis underlying all experiments in calorimetry. Suppose 100 grams of water at 100°C. are mixed with 100 grams of water at 0°C., it is found that the temperature of mixture is very nearly 50°C. It is not exactly 50°C. because some heat is absorbed by the vessel containing cold water and some is lost on account of radiation etc. If the vessel had not absorbed any heat and there had been no loss due to radiation, the temperature would have been exactly 50°C. In that case the temperature of the hot water would fall through 50°C., therefore it would lose $100 \times 50 = 5000$ calories. The cold water would be heated through 50°C. therefore it would gain $50 \times 100 = 5000$ calories. This shows that the heat lost by one body is gained by the other. If the student remembers the relation **Heat gained = Heat lost,** all problems dealing with calorimetry will become extremely simple.

It should be noted that this principle is true only if there is no chemical action involved. For instance, if hot sulphuric acid is mixed with cold water the resultant temperature is considerably modified by the heat given out during the chemical action, and the calculation cannot be made in the above-said manner.

181. Water Equivalent of a Calorimeter.—The apparatus used in measuring heat is called a *calorimeter*. Generally it consists of a copper vessel C fitted with a lid (not shown in the figure) through which pass a stirrer and a thermometer. To protect the calorimeter from the effect of the atmosphere it is placed inside a second copper vessel. It is either suspended within the outer vessel F with the help of hooks, or is placed on a felt pad. The space between the two vessels is packed with dry cotton. In order to minimise the effect due to radiation, the outer surface of C and the inner surface of F are polished.

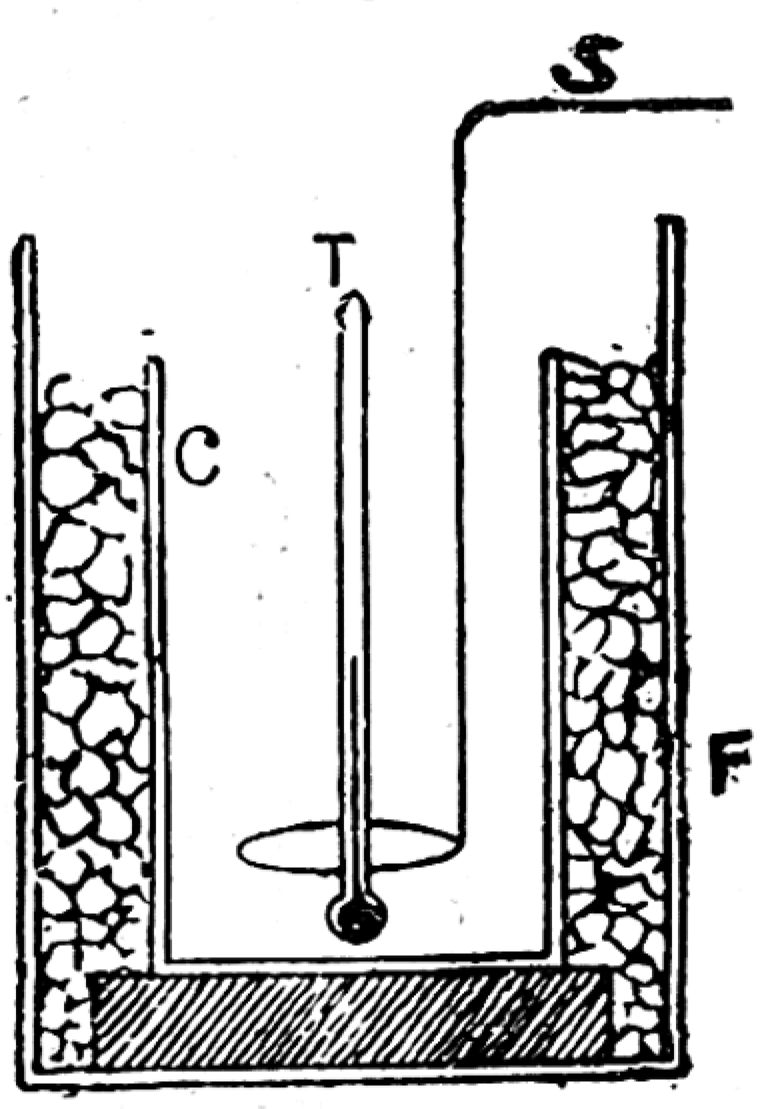

Fig. 20.
Calorimeter.

Whenever we mix a solid and a liquid or two liquids at different temperatures, some heat is necessarily emitted or taken by the vessel containing the mixture *viz.*, the calorimeter, which may be regarded as an extra quantity of water. The amount of water to which a calorimeter is equivalent, or in simple words *the amount of water which absorbs or emits the same quantity of heat as the calorimeter, is called the water equivalent of the calorimeter.* This may be found either by experiment or by calculation.

Let us find experimentally the amount of heat absorbed or emitted by a calorimeter when it is heated through 1°C. Take an empty calorimeter along with a stirrer, and weigh it. Let its weight be m. Fill it about one-third with cold water and reweigh it. Subtracting the first from the second weight, get the weight of the cold water. Let it be m_1. Note its temperature, and let it be t°C. Add hot water at temperature T, enough to fill calorimeter two-thirds. Stir the water and note the final temperature with a *half degree* thermometer; let it be θ. Weigh the calorimeter again, and find from it the weight of hot water added. Let it be m_2 grams.

Now hot water has lost heat, whereas cold water and the calorimeter both have gained heat.

Of course some heat is lost while hot water is being poured and the final temperature is being read. If we neglect this loss and suppose that the water equivalent of the calorimeter is w we can say that

the heat lost by hot water $=m_2(T-\theta)$,

the heat gained by the cold water $=m_1(\theta-t)$.

the heat gained by the calorimeter

$$=w(\theta-t).$$

Since heat gained = heat lost

$$m_1(\theta-t)+w(\theta-t)=m_2(T-\theta).$$

or

$$w=\frac{m_2(T-\theta)-m_1(\theta-t)}{(\theta-t)}$$

$$=\frac{m_2(T-\theta)}{(\theta-t)}-m_1.$$

If s be the specific heat of the material of which the calorimeter is made, the heat gained by the calorimeter would be $ms(\theta-t)$, whereas in the above calculation we have taken $w(\theta-t)$ as the heat absorbed by the calorimeter. Comparing the two expressions we find $w=ms$ *i.e.*, the water equivalent is numerically equal to the thermal capacity of the calorimeter.

182. Determination of the Specific Heat of a Solid.—Any one of the following methods may be used to determine the specific heat of a solid body :

(1) The given body is heated to a high temperature and is immersed in a known mass of water at room temperature (or if the body be soluble in water, in some other liquid of known specific heat). The rise in temperature of the water (or of the liquid) is noted, and therefrom the specific heat of the solid is calculated. This method is called the **Method of Mixtures.**

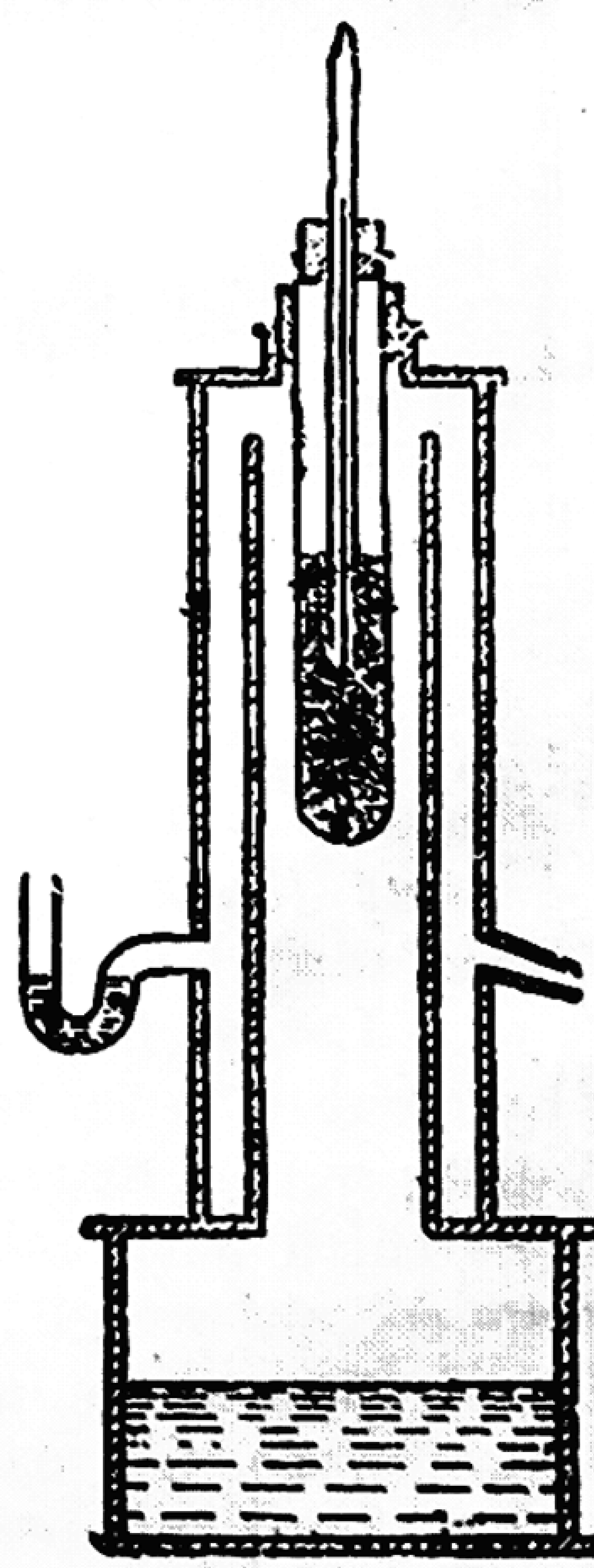

Fig. 21.

(2) The given body is heated to a certain temperature and is made to melt ice. From the amount of ice melted, the heat given out is calculated, and therefrom the specific heat of the substance is determined. This method is called the **Method of Fusion of Ice.**

(3) The given body is heated to the temperature of steam by allowing steam to condense on it. From the amount of steam condensed, the heat taken up by the body is calculated and therefrom the specific heat of the substance is determined. The method is called the **Steam Calorimeter Method.**

We shall consider at this stage the method of mixtures only, leaving the other methods for a later stage.

Method of Mixtures.—Suppose we wish to find the specific heat of brass nails. The first step is to heat the nails to a known temperature. The apparatus generally used for heating iron or brass nails or lead shot in the laboratories is shown in Fig. 21. It is an ordinary hypsometer. Its mouth is fitted with a cork through which an iron or copper tube passes. Drop the nails into the tube so as to surround the bulb and part of the stem of the thermometer, which is placed in the middle of the tube. Put some water in the hypsometer and place the tube in position. On heating the water the temperature of the nails inside the tube will rise, and if heating is continued long enough, the temperature of the nails will become constant. This will happen when the temperature of the nails is the same as that of steam. When the temperature has been constant, say at $T°$ for about ten minutes, the nails are ready to be transferred. In the meantime a calorimeter, whose water equivalent is w, is weighed and is filled two-thirds with cold water. Suppose the weight of cold water is m gm. and its temperature t°C. The nails are quickly transferred from the tube to the water in the calorimeter and the water is stirred ; the final temperature is read with a *half-degree* thermometer. Suppose it is θ°C. To find the weight of the nails which have been dropped into the calorimeter, it is weighed once again. Let the weight of nails be M gm.

Let us suppose that the specific heat of the nails is S.

The heat lost by nails is $MS(T-\theta)$. The heat absorbed by water and calorimeter is $(m+w)\ (\theta-t)$.

But heat lost=heat gained.

Hence $$MS(T-\theta)=(m+w)(\theta-t).$$

or $$S=\frac{(m+w)(\theta-t)}{M(T-\theta)}.$$

Since a hot substance always loses heat, *i.e.*, radiates heat and a cold substance gains heat from its surroundings, to get correct result we must take into account the heat lost by the calorimeter and its contents. or heat gained from the surroundings. A simple method of doing this is *to start with the temperature of the calorimeter as much below the temperature of the atmosphere as the final temperature is expected to be above it.* When this is done the heat lost by the calorimeter when its temperature is higher is compensated by the heat that it gains during the period its temperature is lower than that of the atmosphere.

183. **Determination of the Specific Heat of a Liquid.**—We can either use (1) the method of mixtures, or (2) the method of cooling.

(1) In the method of mixtures the procedure is exactly the same as in the case of solids. The calculations also are made in exactly the same manner. The only difference is that in this case we know the specific heat of the solid in place of the liquid. While selecting a solid, it should be remembered that it should have no chemical action on the liquid.

(2) The given liquid is heated to a certain temperature and is allowed to cool. The rate at which it cools is observed, and is com-

pared with the rate at which water cools under similar conditions. From the rates of cooling the specific heat of the liquid is determined. This method is called the **Method of Cooling.** Before we explain the details of the method we shall briefly explain the law of cooling. If a hot liquid, say water, is taken in a calorimeter and its temperature is recorded after every 30 seconds, on plotting a graph showing the relation between the temperature and the time, we get a curve of the form shown in Fig. 19.

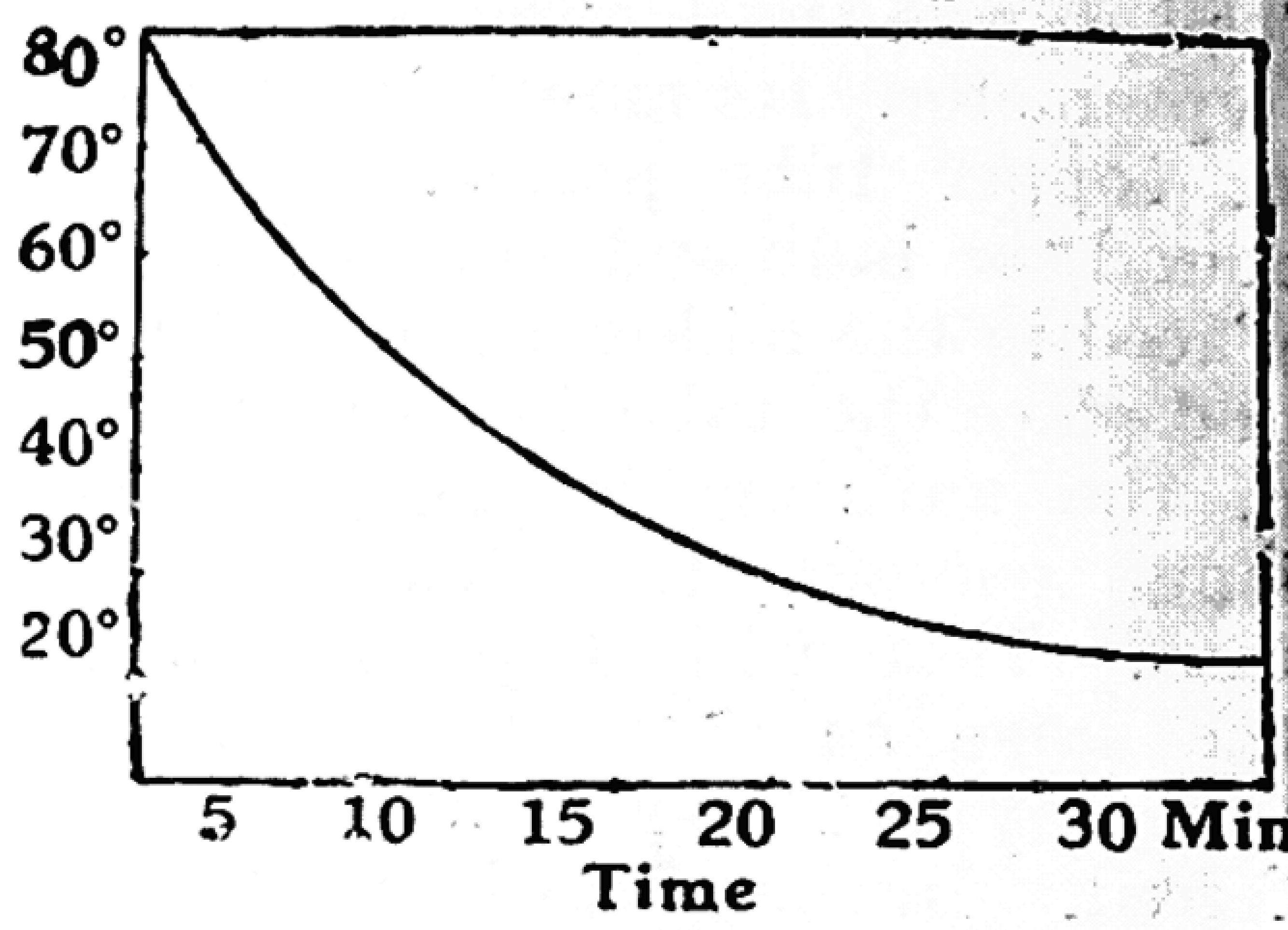

Fig. 22.

It is clear from the curve that the greater the difference between the temperature of the liquid and that of the atmosphere the greater the fall of temperature and hence the greater the amount of heat lost during one second. This fact was first expressed in the form of a law by Newton, and hence is called after him **Newton's law of cooling**. It may be stated as :

The rate at which a body loses heat is proportional to the difference between the temperature of the body and that of its surroundings.

If hot water be allowed to cool in two or three calorimeters it will be seen that *the amount of heat lost per second depends upon the extent and nature of the radiating surface.*

Now if the temperature difference and the extent and nature of the radiating surface be the same in any two cases, it is found that the rate of loss of heat is the same *i.e.*, the rate of cooling is independent of the nature of the liquid. This fact is made use of in finding the specific heat of a liquid.* Both the given liquid and water are heated approximately to the same temperature. Equal volumes of the two are taken and placed in similar vessels. They are allowed to cool under similar conditions and time is noted which each takes to fall through the same range of temperature, say for instance, from 65° to 55°C. Let us suppose the mass of the liquid is M, and of the same volume of water is m ; and further, that the time taken by the liquid is T and by the water is t. Let the specific heat of the liquid be S and the water equivalent of the calorimeter be w.

Heat lost in T seconds by the liquid is $M \times S \times 10$, and
Heat lost by the calorimeter is $w \times 10$.
The rate of cooling of the liquid is

$$\frac{M \times S \times 10 + w \times 10}{T} \text{ or } \frac{(MS+w)10}{T}$$

Similarly, the rate of cooling of water is $\dfrac{(m+w)10}{t}$

Since the rates are equal,

$$\frac{(MS+w)10}{T} = \frac{(m+w)10}{t},$$

*It is difficult to get accurate results by this method.

or $$(MS+w)=(m+w)\frac{T}{t},$$

or $$S=\frac{(m+w)\dfrac{T}{t}-w}{M}.$$

Caution.—*The rates of cooling are equal and not the rates of fall of temperature.*

184. Specific Heat of a Gas.—The determination of the specific heat of a gas is much more difficult than that of a solid or a liquid because when a gas is heated a part of the heat supplied is spent in doing external work on account of increase in volume. The expansion and hence work done is maximum when a gas is heated at constant pressure. In the case of solids and liquids the expansion is so small that the external work is negligible. On the other hand if a gas is heated at constant volume no external work is done and hence the whole of the heat supplied is used in raising its temperature. Corresponding to these two methods of heating there are two specific heats. One is called the *specific heat at constant pressure*, and the other the *specific heat at constant volume.* The actual experimental details are rather complicated to be explained here, but it may be remarked that the determination of the specific heat of a gas at constant pressure was carried out by Regnault in 1850, and the specific heat of a gas at constant volume by Joly in 1888. The principle of Joly's method will be explained in §209. Here we shall very briefly explain the principle of Regnault's method which is simply a modification of the method of mixtures. The gas whose specific heat is to be determined is compressed in reservoir A (Fig. 23). From there it is allowed to flow at constant pressure registered by the manometer M through a big copper coil im-

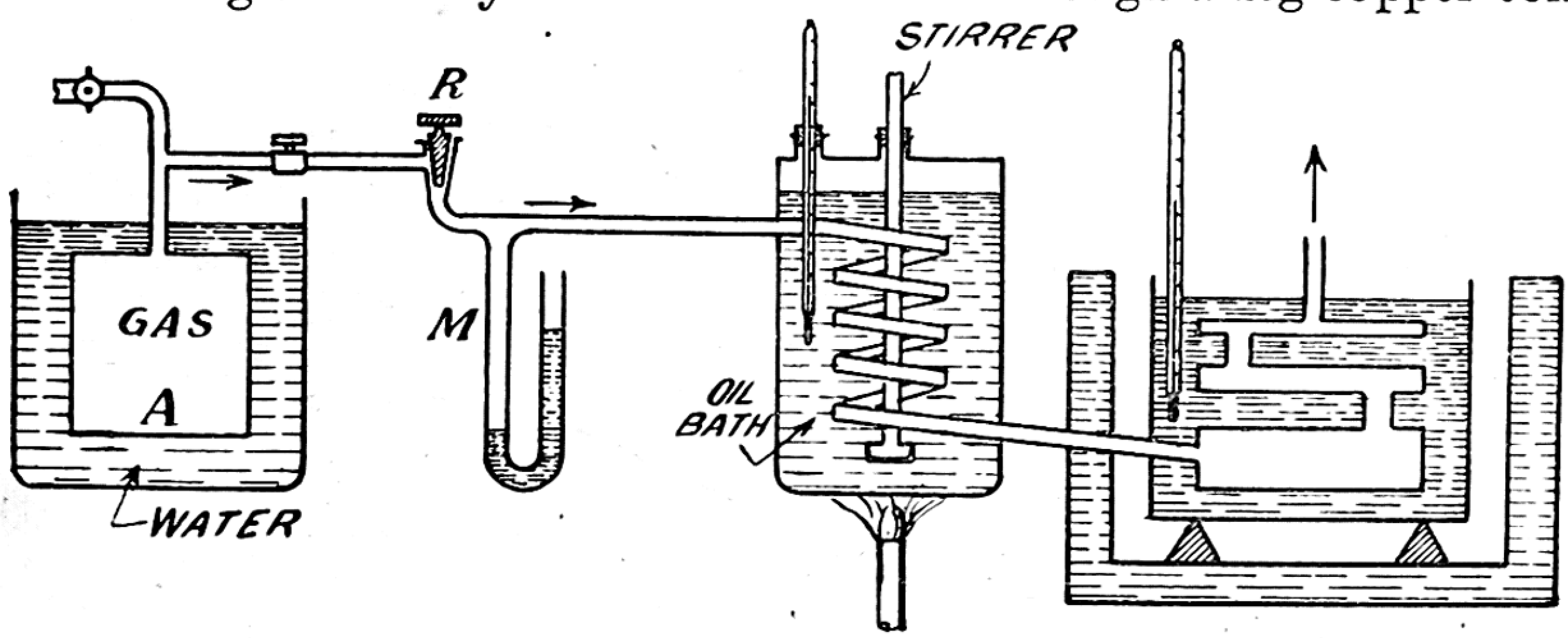

Fig. 23.

mersed in hot oil bath and then through a spiral of copper tube immersed in a known quantity of water in a calorimeter. To keep the pressure constant, regulating valve R is opened more and more as the pressure in the reservoir decreases. The gas is heated by the oil bath and is passed into the calorimeter where it is cooled by the water. After a certain time the rise in temperature of water is noted. The mass of the gas passed through the spiral is calculated from the change of pressure in the reservoir.

Let P_1 and P_2 be the initial and final values of pressure.

Mass of the gas passed $m=(\rho_1-\rho_2)V$, were ρ_1 and ρ_2 are the

initial and final densities, V is the volume of A and $T°$ is the temperature in Absolute degrees of the gas in A.

Since
$$\frac{\rho_1}{\rho_2}=\frac{P_1}{P_2}$$

we get
$$\frac{\rho_1-\rho_2}{\rho_2}=\frac{P_1-P_2}{P_2}$$

$\therefore$
$$(\rho_1-\rho_2)V=\left(\frac{P_1-P_2}{P_2}\right)V\rho_2$$

$e_1 - e_2$

$\frac{P_1 e_0 273}{T 67} - \frac{P_2 e_0}{67 T}$

But we know that $\frac{P_2}{\rho_2 T}=\frac{P_0}{\rho_0 T_0}$, where P_0 is normal pressure and ρ_0 is the density at N. T. P., hence $\frac{P_2}{\rho_2 T}=\frac{76}{\rho_0 273}$

$\frac{e_0 273 (P_1-P_2)}{67 T}$

and so
$$\rho_2=\frac{P_2\rho_0 273}{76T}$$

$P_1 - \frac{P_1 e_0 273}{T \times 76}$

Hence
$$m=\frac{\rho_0 V(P_1-P_2)273}{76T}$$

Knowing the mass m of the gas, the fall of temperature from t to $\frac{t_1+t_2}{2}$, where t_1 is the initial temperature of the water in the calorimeter, and t_2 the final temperature, and w the weight of water including the water equivalent of calorimeter and its contents and t the temperature of oil bath, we can write for heat lost

$$ms_p\left(t-\frac{t_1+t_2}{2}\right)$$

and for heat gained
$$w(t_2-t_1)$$

$\therefore$
$$ms_p\left(t-\frac{t_1+t_2}{2}\right)=w(t_2-t_1)$$

or
$$s_p=\frac{w(t_2-t_1)}{m\left(t-\frac{t_1+t_2}{2}\right)}.$$

The specific heat at constant pressure as said above, is greater than the specific heat at constant volume. The specific heat of air at constant pressure at 20°C. is, for example, 0·242 whereas the specific heat at constant volume at the same temperature is 0·172.

184a. The Relation between the Specific Heats of a Gas.—Let us take 1 gram of a gas in a cylinder shown in (Fig. 24) fitted with a weightless and friction-less piston. Let the pressure acting on the gas be P and the area of the piston be A. Heat the gas from $T_1°$ to $T_2°$ Absolute, first at constant volume and then at constant pressure. Let us find the amount of heat necessary in each case.

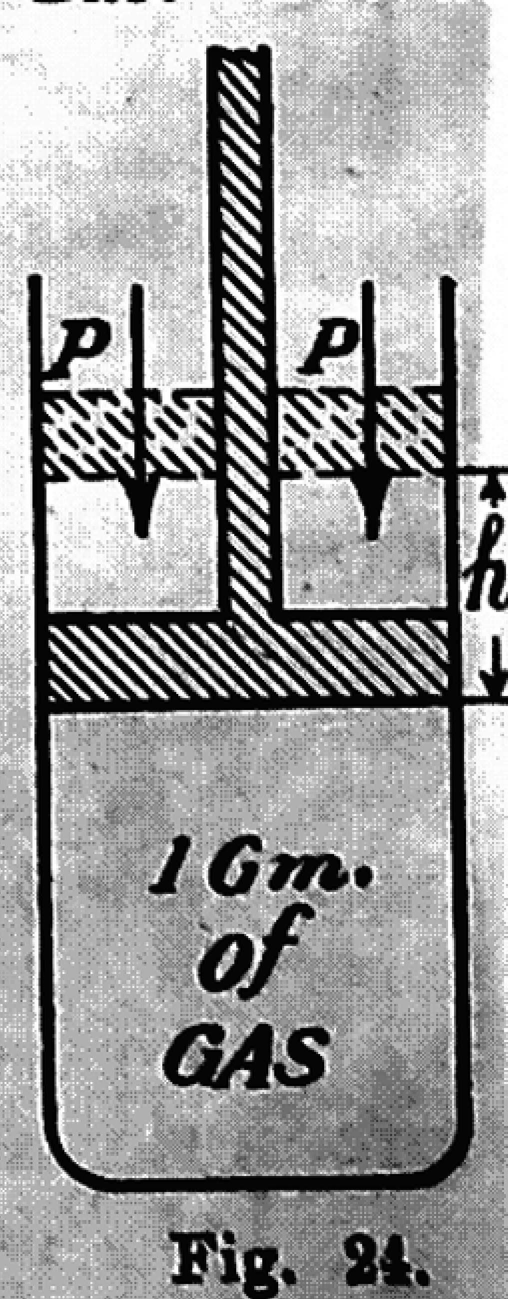

Fig. 24.

1st Case. When the gas is heated at constant volume from $T_1°$ to $T_2°$, the pressure will increase from P_1 to P_2. To keep the volume constant we shall have to increase the load on the piston. The amount of heat required will be by definition

$$s_v(T_2-T_1) \text{ calories.}$$

This heat is utilised in increasing the internal energy of the gas.

2nd Case. Now release the piston and let it rise

through height h. As the gas expands it cools because of the external work done. Supply *some more* heat to bring the gas to T_2°.

The external work done $=PAh=P(V_2-V_1)$, where V_2 is the final volume and V_1 the initial volume.

The total heat supplied in this case

$$=s_v(T_2-T_1)+\frac{P(V_2-V_1)}{J}$$

where J is the mechanical equivalent of heat.

We shall see in §239 that $J=4{\cdot}2\times10^7$ ergs per calorie.

The total heat is also equal to

$$s_p(T_2-T_1).$$

Equating the two expressions we get

$$s_p(T_2-T_1)=s_v(T_2-T_1)+\frac{P(V_2-V_1)}{J}$$

or
$$s_p(T_2-T_1)-s_v(T_2-T_1)=\frac{P(V_2-V_1)}{J}$$

or
$$(s_p-s_v)\ (T_2-T_1)=\frac{P(V_2-V_1)}{J}$$

or
$$s_p-s_v=\frac{P(V_2-V_1)}{J(T_2-T_1)}.$$

From gas equation $\qquad \dfrac{PV}{T}=r,$

we get
$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}=r$$

Since pressure remains constant we can write this relation as

$$\frac{PV_1}{T_1}=\frac{PV_2}{T_2}=r$$

or
$$PV_2=rT_2.$$

and
$$PV_1=rT_1$$

or
$$P(V_2-V_1)=r(T_2-T_1)$$

or
$$\frac{P(V_2-V_1)}{T_2-T_1}=r.$$

Substituting r for $\dfrac{P(V_2-V_1)}{T_2-T_1}$ in relation for s_p-s_v we have

$$s_p-s_v=\frac{r}{J}.$$

For air we know that $r=2{\cdot}87\times10^6$ ergs

$$\therefore \qquad s_p-s_v=\frac{2{\cdot}87\times10^6}{4{\cdot}2\times10^7}=0{\cdot}068$$

Compare this difference with the difference in the experimental values given above.

Table of Specific Heats.

Alcohol	0·61	Lead	0·031
Aluminium	0·21	Mercury (liquid)	0·033
Copper	0·095	Sand	0·19
Iron	0·109	Silver	0·056
Ice	0·502	Water	1·000
Kerosene oil	0·51	Turpentine	0·428

185. From the table of the specific heats it is clear that of all the common substances water has the highest specific heat. On account of this fact water is used in hot-water bottles or for heating rooms etc. In Nature also this fact plays a very important part. The oceans do not get heated so soon in day-time as the earth which has about one-fifth as much specific heat as water, and hence send a cool breeze towards the land, called the sea-breeze. After sunset, however, the land cools more rapidly than the oceans and hence the direction of the breeze is reversed giving the land breeze. It is on account of high specific heat of water that the extremes of climate are absent on an island.

You will notice from the table that the specific heat of mercury is $\frac{1}{30}$th part of the specific heat of water. This is one reason why mercury is used in thermometers in preference to water. A mercury thermometer absorbs only $\frac{1}{30}$th as much heat as is absorbed by a water thermometer of the same dimensions and hence responds more quickly and is more accurate.*

185a. Fuels.—The science of calorimetry is very important to the industrialist as it enables him to grade the fuels by determining their heating or calorific value. By fuel is meant a substance which combines with atmospheric oxygen in the process of combustion. It may be found in nature as such or may be prepared artificially. To the first type belong wood, coal, crude oil, etc. and to the second charcoal, coke, alcohol, petrol, coal gas, etc. The principal combustible constituents of a fuel are carbon and hydrogen or their compounds. When a fuel burns the carbon in it combines with oxygen and produces CO_2 while the hydrogen forms H_2O. If the supply of air is sufficient these are the only products and the combustion is complete and the amount of heat produced is maximum. If the supply of air or oxygen is insufficient the fuel burns incompletely with the result that a good deal of its heating value is lost. If carbon for instance burns incompletely a part of it may change into smoke and a part into carbon monoxide and the rest into carbon dioxide. For each pound of carbon burnt incompletely we may lose as much as two-thirds of its heating value. This is why so much attention is paid to the design of furnaces or grates. In the following table are given the calorific values in B. Th. Units when 1 lb. of a fuel is *completely* burnt.

Calorific Values of Fuels.

Coal { Anthracite	15,000	Wood	6,000
Coal { Bituminous	14,000	Charcoal	12,700
Coal { Lignite	12,000	Petroleum	18,500
		Alcohol	13,000
Coke	13,000	Petrol	20,000

*It takes less heat from the body and hence causes less lowering of temperature than a water thermometer.

185b. Experimental Determination of the Calorific Value of a Fuel.—To determine the heat produced when a sample of fuel is burnt completely a special type of calorimeter is required. There are several types in use. We shall describe what is called *Bomb Calorimeter* [Fig. 25]. It is used whenever accurate results are required. It consists of a strong steel or gun metal vessel B, called bomb, fitted with a gas-tight cover C. A small quantity of the fuel, say one gm. of coal, powdered and dried, is placed in a platinum crucible E held in position inside the bomb by means of a loop of wire D. A piece of fine steel or platinum wire w, w' is covered by coal in the crucible. The bomb is now closed and oxygen under a pressure of 20 to 25 atmospheres is passed into it through a pipe F. The stopcock G is then closed and the bomb is placed inside the calorimeter H containing a known quantity of water, the temperature of which is measured by a sensitive thermometer T. To avoid the loss of heat due to convection etc., the vessel H is surrounded by a second vessel K which itself is surrounded by another vessel L. The space between K and L is filled with water.

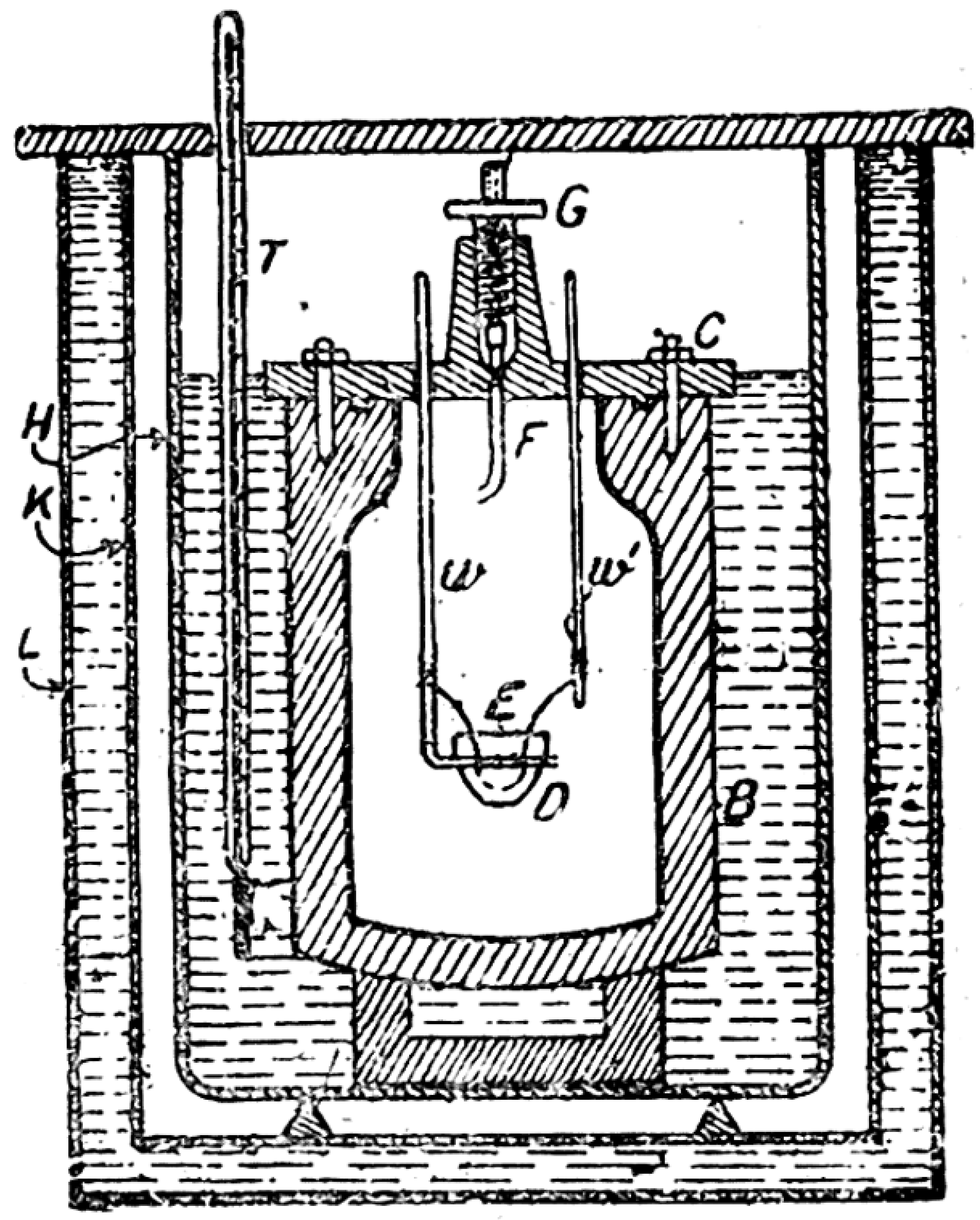

Fig. 25.
Bomb Calorimeter.

On connecting wires w, w' to a battery an electric current strong enough to heat wire D to incandescence is passed. This ignites the coal which burns completely on account of the ample supply of oxygen. The heat produced raises the temperature of water which is constantly stirred by stirrers (not shown in figure) until the temperature of water ceases to rise. Let

M be the mass of water in lb.
W ,, ,, water equivalent in lb.
m ,, ,, coal burnt
$t°F$,, ,, initial temperature of water
$T°F$,, ,, final ,, ,, ,, ,,
and Q ,, ,, calorific value of coal in B. Th. Units.

Then $$Q=\frac{(M+W)(T-t)}{m}.$$

Example.—In a test with a bomb calorimeter containing 4·4 lb. of water, 0·035 oz. of coal was burnt. The rise in temperature was 5·2°F. The water equivalent of the apparatus was 1·6 lb. Find the calorific value of coal per lb. in B. Th. Units.

$$Q=\frac{(4{\cdot}4+1{\cdot}6)5{\cdot}2}{0{\cdot}035\times\frac{1}{16}}=\frac{6\times83{\cdot}2}{0{\cdot}035}=14263 \text{ B. Th. units.}$$

185c. Calorific Value of Foods.—Do you know that when you move your leg, raise your arm, or shut your eyes or when you

breathe or your heart beats you are doing work ? Do you know that your body which is normally at 98·4°F. is almost always radiating heat and is thus losing energy ? Has it ever occurred to you that you cannot do work or lose energy unless you get energy from somewhere ? Just as machines, like a steam engine, take their energy from fuel you do so from the food you eat. In other words food is fuel for you just as petrol is fuel for a motor car engine. Of course each man requires a different quantity of food to meet his requirements. A blacksmith or a carpenter requires about 4000* Calories of heat per day to keep himself fit whereas a clerk or a teacher requires only 2500 Calories. A college student on an average requires about 3000 Calories a day. In the following table are given the calorific values per pound of the various foodstuffs. Calculate and see if you are getting the right amount of Calories per day.

Table of Calorific Value of Foods.

Wheat flour	1,572	Milk (cow)	318	Cabbage	114
Butter	3,442	Potatoes	450	Tomato	100
Bacon	2,730	Eggs	736	Apples	245
Sugar	1,820	Fish	230	Honey	1290
Peas (dry)	1,636	Banana	460	Orange	220

It may be added that you should not only get required number of Calories per day but also the proper quantities of proteins, fats, carbohydrates, mineral salts and vitamins.

EXERCISES

1. Twenty grams of common salt at 91°C., are immersed in 250 gm. of turpentine oil at 13°C., whose specific heat is 0·428. The final temperature is found to be 16°C. Find the specific heat of common salt.

Let the specific heat of common salt be S.

Heat given out by the common salt $=20\times S\times 75$.

Heat gained by turpentine$=250\times 0{\cdot}428\times 3$

$$\therefore \qquad 20\times S\times 75=250\times 0{\cdot}428\times 3,$$

or

$$S=\frac{250\times 0{\cdot}428\times 3}{20\times 75}=0{\cdot}214.$$

2. A ball of iron of specific heat 0·11 and mass 100 gm. is removed from a furnace and is immersed in 250 gm. of water at 0°C. The temperature of the mixture is 33°C. Find the temperature at the furnace. The heat absorbed by calorimeter is to be neglected.

Let the temperature be T°C.

Heat given out by iron ball$=100(T-33)\times 0{\cdot}11$.

Heat taken by water$=250(33-0)=250\times 33$.

Heat gained$=$heat lost,

$$\therefore \qquad 250\times 33=100(T-33)\times 0{\cdot}11$$

or

$$T-33=\frac{250\times 33}{11}.$$

or

$$T=783°\text{C}$$

*Kilogram calories and not gram calories.

3. A piece of silver weighing 25·2 gm. is heated to 101·4°C. and dropped into 136·4 gm. of turpentine at 11·2°C. The temperature of the mixture is 13°C. The water equivalent of the calorimeter is 6·2 gm. The specific heat of silver is 0·056. Find the specific heat of turpentine.

Let the sp. heat of turpentine be S.

Heat given out by silver $= 25{\cdot}2 \times 0{\cdot}056 \times 88{\cdot}4$.

Heat taken by turpentine and calorimeter

$$= 136{\cdot}4 \times S \times 1{\cdot}8 + 6{\cdot}2 \times 1{\cdot}8.$$

But heat gained $=$ heat lost

$$\therefore \quad 136{\cdot}4 \times S \times 1{\cdot}8 + 6{\cdot}2 \times 1{\cdot}8 = 25{\cdot}2 \times 0{\cdot}056 \times 88{\cdot}4,$$

or

$$S = \frac{25{\cdot}2 \times 0{\cdot}056 \times 88{\cdot}4 - 6{\cdot}2 \times 1{\cdot}8}{136{\cdot}4 \times 1{\cdot}8}$$

$$= 0{\cdot}46$$

4. 154 grams of a certain substance at 212°F. are dropped in a vessel containing 182 gm. of water at 15°C. The final temperature is 24°C. Find out the specific heat of the body. *Ans.* 0·14.

5. A steam boiler is made of steel and weighs 10 tons. The specific heat of the material is 0·12. The boiler contains 8 tons of water. Find the quantity of heat required to raise the temperature of the whole from 15°C. to 100°C., assuming no waste.

Ans. 1,751, 680 lb. deg.

6. A calorimeter whose water equivalent is 10 gm. is filled with 100 gm. of water at 80°C. and the time taken for the temperature to fall to 75°C. is 4 min. When filled with another liquid, the weight being 80 gm. the time for the same fall of temperature is 130 seconds. Find the specific heat of the liquid. *Ans.* 0·62.

7. A piece of metal weighing 200 gm. is immersed in 120 gm. of water at 15°C. and the temperature rises to 37°C. If the specific heat of the metal is 0·12, find its original temperature. *Ans.* 147°C.

8. 125 grams of turpentine are contained in a calorimeter (of water equivalent 4·5 gm.) at 13°C. 111·7 gm. of water at 80°C, are mixed with turpentine, and the final temperature is 57°C. Find the specific heat of turpentine. *Ans.* 0·43

9. The specific heats of ether and alcohol are respectively 0·53 and 0·61. If 144 grams of ether at 10°C. are mixed with 180 gm. of alcohol at 31°C., find the resultant temperature. *Ans.* 26·5°C.

10. A glass vessel of weight 6 oz. contains 2 lb. of water, the glass and water being at 20°C. ; 8 lb. of mercury at 45°C. are added. What is the final temperature ? The specific heats of mercury and glass are given to be 0·033 and 0·2 respectively. *Ans* 22·8°C.

11. A tea pot made of silver weighs 300 gm. Tea is made in it from 30 gm of leaves and 800 gm. of water at 100°C. When the temperature of room is 20°C., what difference will it make to the temperature of tea if the pot is heated to 80°C. by washing it with boiling water before making tea ? Take specific heat of silver as 0·056 and of tea leaves as 0·5. *Ans.* 1·2°C.

CHAPTER VI

Change of State

Solid to Liquid

186. While explaining the general effects of heat in §153 it was shown that if a body be heated for a sufficiently long time it changes its state, *i.e.*, if it be a solid it changes into a liquid, and if a liquid it changes into a gas. Changes in the reverse direction take place, if instead of heating a body, we cool it.

The change from the solid to the liquid state is called *fusion* and from the liquid to the gaseous state is called *vaporization.*

In addition to the above two changes a third change of state, *i.e.*, the direct passage of a solid to the gaseous state is also possible. Well-known examples of such substances are camphor, iodine, etc. This change of state is called *sublimation.* The discussion of sublimation is beyond the scope of this book. We shall confine ourselves only to the other two changes of state, *i.e.*, fusion and vaporisation.

187. Fusion.—Fusion or melting is the passage of a solid body to the liquid state. This phenomenon is produced when on account of an increase in the molecular motion the force of cohesion between the molecules which binds them together is overcome. Since this force is different in different substances, evidently for every substance there is a different temperature at which its cohesive force becomes weak, and hence a different melting point. Some substances melt easily ; while others require very high temperatures. There are some which are incapable of fusion on account of want of sufficiently high temperature, such as carbon. Most of the substances, especially crystalline, possess well-defined melting points, and as soon as that temperature is reached the fusion begins to take place abruptly ; while there are some, such as glass and wrought iron, which pass through a plastic stage ; *i.e.*, first become soft, and then grow softer and softer as they are heated till they change into the liquid state. This type of fusion is called *vitreous fusion.*

Almost all amorphous bodies pass more or less through a plastic stage and hence do not possess sharp melting points.

Before we discuss the determination of the melting point of a substance, let us study the phenomenon of fusion.

Take ice from a refrigerator where its temperature is very low say $-10°C$) and place it in a beaker and heat it. At first its temperature will rise up to 0°C. and then remain stationary till all the ice has melted. This stationary temperature, at which ice changes into water, is called the *melting point* of ice. If the burner be removed from below and the water obtained be stirred, it will be seen to solidify at the same temperature at which it melted, showing thereby that solidification is just the reverse process of melting.

If water that is obtained on melting ice be further heated, the temperature will be seen to rise up to 100°C. and then remain stationary till the water is boiled off, *i.e.*, is changed into the gaseous state.

The heat that is used up in raising the temperature of a body is called sensible heat. That heat is not always sensible is clear from the fact that whenever a change of state takes place, temperature does not rise in spite of the fact that the substance receives heat just as before. The *heat which is used up in changing the state of a body without raising its temperature is called* **latent heat**. The word latent means "lying hidden". It is so called because its addition cannot be detected with the help of a thermometer.

188. Change of Volume on Melting.—As a rule substances contract on solidifying and expand on melting. For example, if we pour molten paraffin wax into a beaker and let it solidify, we find a depression at the centre showing that wax has contracted. Most metals and alloys behave in a similar manner. It is for this reason that gold and silver coins are stamped with dies and not cast. But there are some substances which instead of contracting, expand on solidifying. Water is one of them. To see experimentally that water expands on changing into ice take a small cast-iron bottle, fitted at the mouth with a screw stopper to close it, and with sides $\frac{1}{8}$th of an inch in thickness. Fill it completely with water and close it tightly with the stopper. Place it in a mixture of ice and common salt. After some time the bottle will be seen to burst. This will happen when the water inside is changed into ice. Since the volume of ice is greater than that of the same amount of water, it exerts so great a force on the sides of the bottle that they yield to the pressure.

Cast-iron, bismuth, and antimony expand on solidifying. It is for this reason that sharp castings can be made of these metals.

189. Regelation.—A pure substance always begins to melt at the same temperature ; so constant is this temperature that it is used by the chemists to identify substances. It was Lord Kelvin who showed for the first time that the melting point is changed by a change in pressure. He found that as the pressure upon ice is increased, its melting point is lowered by 0·0072°C. for an increase of one atmosphere in pressure. It means, in other words, that under a pressure of 1,000 atmospheres water will not freeze above the temperature of −7·2°C. (approximately). That melting point of ice is lowered by increase of pressure can be shown easily by a simple experiment.

Take a slab of ice and let it rest on two supports. Let a copper wire carrying two equal weights, say 14 lb. each at its two ends be laid on the slab. The wire will be seen to slowly cut its way into the slab and after some time pass completely through it without leaving any permanent mark behind, except that some air bubbles will be seen. The slab will be found to be a single piece as before, although the wire has cut through it. This phenomenon is called *Regelation* (literally refreezing). Let us see how it can be explained.

The pressure of the wire on the ice lowers the melting point, with the result that the ice melts. But before melting it requires some heat. This comes from the neighbouring parts as well as from the

copper wire, with the result that the temperature falls below zero. The wire cuts through the water which goes above it and freezes again, being no more under pressure and with temperature below zero. It gives out heat which it absorbed while melting. This heat goes down to melt the ice immediately below the wire, and so on. This process continues till the wire, passes completely through the slab.

The melting point is not always lowered under increased pressure. In the case of substances like wax or sulphur it is raised. Bunsen, for instance, found that a specimen of paraffin wax which melted at 46·3°C. under a pressure of one atmosphere melted at 49·9°C. under a pressure of 100 atmospheres. To know whether the melting point will be lowered or raised with increase of pressure, remember *that the melting point of a substance is lowered by increase of pressure if it expands on solidifying*; *whereas it is raised if the substance contracts on solidifying.*

190. Laws of Fusion.—The experiments given above lead us to the following laws of fusion :—

I. *Every substance begins to melt at a certain fixed temperature, depending upon the pressure.*

II. *From the moment the fusion commences temperature remains constant until the whole of the substance is melted*

III. *Unit mass of each substance requires a definite amount of heat to change it from the solid to the liquid state without change of temperature.*

IV. *Every substance undergoes a change of volume on melting.*

V. *The melting point of a substance which increases in volume on freezing is lowered by increase of pressure ; whereas it is raised in the case of a substance which decreases in volume on solidification.*

191. Determination of the Melting Point.—For substances which have low melting points, such as paraffin wax, naphthalene, etc., use the following method. Take the substance and melt it in a dish. Take a fine capillary tube. Dip its one end in the melted substance. The liquid will rise in the tube on account of capillary action. Seal its lower end, and attach it to a thermometer with rubber bands near its bulb. With the help of a stand fix the thermometer in such a position that the bulb along with the lower part of the stem and the capillary tube dips in a beaker of water which is heated from below. The water is stirred so that the temperature may remain uniform. At a certain stage the substance will melt and the opacity will disappear. Note this temperature roughly. Remove the burner from below, and let the water cool till the substance solidifies again. Begin to heat the water once more, and when the approximate melting point is near, turn the flame low, so that the temperature of the water rises very slowly. Stir the water and note the temperature at which the substance melts. Remove the burner and let the water cool. Note the temperature when the substance solidifies.* Repeat this once again. Take the mean of four readings as the melting point.

* In general the temperature at which a liquid freezes is the same as the temperature at which the solid melts. But in the case of certain fats like butter, melting point is not the same as the freezing point. For example, butter melts between 28° and 32°C. and solidifies between 20° and 23°C.

A very convenient method, however, is the method of cooling. Take the substance in a beaker and heat it to a temperature much adove its melting point. Pour the melted substance into a calorimeter, anb allow it to cool. Note the temperature at first after every half minute, and then, when the fall of temperature is not rapid, after every minute till the substance is not only solidified, but is also much below the freezing point.

Plot a curve showing the relation between the temperature and the time (Fig. 26.) The flat portion gives the melting point or the freezing point. This part corresponds to constant temperature at which the change of state takes place. The substances which pass more or less through a plastic stage do not give well-defined flat parts. This method is often used in the investigation of the melting point of metallic alloys.

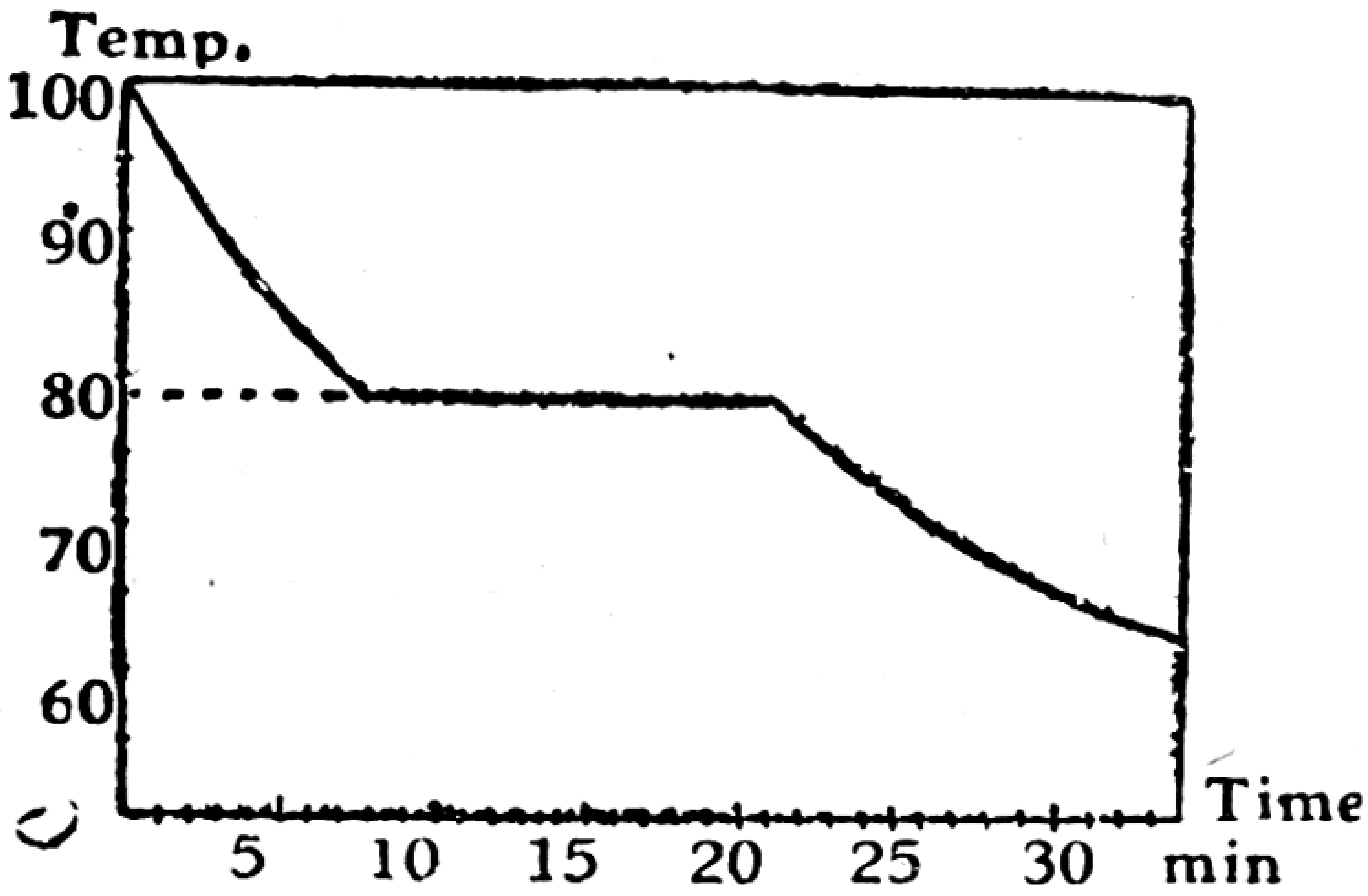

Fig. 26.

Table of Melting Points.

Iridium	... 2230°C.	Naphthalene	... 80°C.
Platinum	... 1755°,,	White wax	... 60°,,
Silver	... 951°,,	Paraffin wax	... 52°,,
Tin	... 232°,,	Butter	... 33°,,
Cane sugar	... 160°,,	Ice	... 0°,,

192. Latent Heat.—We have said above that whenever change of state takes place the temperature does not rise in spite of the fact that the substance is receiving heat. This heat, as we have already said is called *latent heat.* It is important to understand that latent heat is not heat at all, for it exists in the body, not as heat energy, but as molecular potential energy. Since there are two changes of state which a substance can undergo evidently every substance will have two latent heats, one the latent heat of fusion, and the other the latent heat of vaporization.

It is found that a given substance absorbs or gives out per unit mass a definite amount of heat while undergoing a change of state. The amount is different for each substance and for each change of state.

The **latent heat of fusion** *of a substance is defined as the amount of heat required to melt one gram of the substance without change in temperature.* If L stands for the latent heat of fusion of a substance, then $L=$the number of calories required to melt one gram of the substance without change of temperature. If M grams of the substance be just melted without change of temperature the amount of heat used up will be $M \times L$ calories. The **latent heat of vaporization** of a substance is defined as *the amount of heat used up to convert one gram of the substance from liquid to the gaseous state without change in temperature.*

193. To find the Latent Heat of Fusion of Ice.—Take a calori-

meter and weigh it along with the stirrer. Fill it half with water* heated to 5° or 6°C. above the room temperature and weigh again. The difference in the two weights gives the weight of water. Wash some ice-bits with distilled water and dry them with a filter paper. Add them one by one and stir the water. Stop when the fall of temperature is about 10°C. Weigh the calorimeter again. The difference in this weight and the weight of warm water and calorimeter gives the weight of ice added.

Let m be the weight of calorimeter and stirrer,
m_1 ,, ,, weight of warm water,
m_2 ,, ,, weight of ice added,
T ,, ,, temperature of warm water,
θ ,, ,, the final temperature of water,
S ,, ,, sp. heat of the material of the calorimeter,
and L ,, ,, the latent heat of ice.

Heat has been given out by warm water and calorimeter, and has been taken up by ice, partly for melting and partly for raising the temperature of water obtained on melting from 0° to θ°C.

$$\text{Heat given out } = m_1(T-\theta)+mS(T-\theta)$$
$$= (m_1+mS)(T-\theta).$$
$$\text{Heat taken up } = m_2L+m_2\theta.$$

But *heat gained = heat lost.*

$$\therefore \quad (m_1+mS)(T-\theta)=m_2L+m_2\theta.$$

or
$$L=\frac{(m_1+mS)(T-\theta)-m_2\theta}{m_2}$$
$$=\frac{(m_1+mS)(T-\theta)}{m_2}-\theta.$$

The latent heat of ice is found to be 79·6 calories though it is usual to consider it as 80 calories in calculations.

Latent Heat of Fusion

Ice	... 80 calories	Tin	... 14 calories
Paraffin wax	... 35 ,,	Sulphur	... 9·4 ,,
Zinc	... 23 ,,	Lead	... 5·9 ,,
Silver	... 21 ,,	Mercury	... 2·8 ,,

It is seen from the above table that the latent heat of fusion of ice, which is also called the latent heat of water, is very high, and it is well that it is so. If the latent heat of water were low, ice would melt very soon and disastrous floods would result. Moreover the ponds and lakes would freeze very much sooner than they do at present, destroying thereby the aquatic life.

194. Freezing Mixtures.—Before a solid changes into a liquid, it must be supplied with its latent heat of fusion. If the change of state is brought about by heating the substance the necessary heat is supplied from outside. But what happens when a solid changes its

*It is usual to take water 5° or 6° above the room temperature so that the final temperature is 5° or 6° below it. This arrangement enables us to make the loss of heat negligible. For in the first part of the experiment the heat is lost to and during the second part heat is gained from the air.

state without any application of heat, as for instance, when a salt like sodium sulphate* is dissolved in water ? Since no heat comes from outside it must come from the materials themselves which consequently get cooled.† As low a temperature as $-15°C$ can be produced by dissolving sodium sulphate crystals in water. If we mix ice and common salt in proper proportions (*i e.*, one part by weight of salt and three parts of ice), a temperature of $-22°C$ may be produced. If three parts (by weight) of crystalline calcium chloride are mixed with two parts of ice, as low a temperature as $-55°C$ may be obtained. All such mixtures by means of which we produce low temperature are called freezing mixtures.

195. Ice Calorimeters.—The fact that one gram of ice at 0°C requires 80 calories of heat to change it into water at the same temperature is made use of in finding the specific heat of bodies. There are several forms of ice calorimeters with the help of which we can determine the specific heat, but the principle in every case is the following : The body is heated to a high temperature, say T°C., and is dropped into the ice calorimeter, where it cools to the temperature of ice. The heat given out by it is spent in melting ice. The water obtained on melting is collected and its weight determined. Suppose the weight of the body is M gm. and that of the ice melted is m gm. The ice takes $80 \times m$ calories of heat, which must have come from the hot body. As the heat given out by the body is MST, we can write

$$MST = 80m.$$

or

$$S = \frac{80m}{MT}.$$

We shall describe only two ice calorimeters here.

196. Lavoisier and Laplace's Ice Calorimeter.—It consists of an inner metallic vessel which is entirely surrounded by a chamber, packed with broken ice. In order that the ice in this chamber may not melt by the external heat, another chamber, which also is packed with ice is provided. There are two taps through which water can be drained off. One of them is connected with the inner chamber and the other with the outer, as shown in Fig. 27.

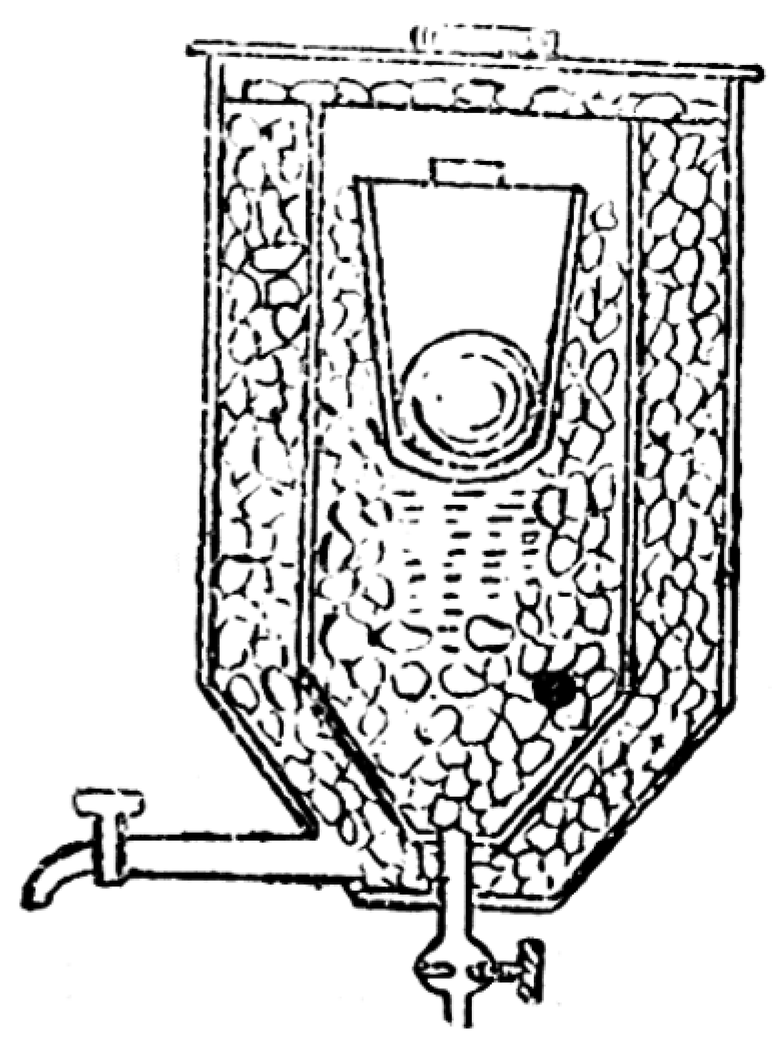

Fig. 27.

The hot body is dropped into the inner vessel ; the weight of the water obtained from the inner chamber gives the weight of the ice melted ; knowing the weight of the body which is dropped and its initial temperature we can find its specific heat.

This form of the ice calorimeter does not yield accurate results. It is interesting only from the historical point of view.

*We are talking of those substances only which have no chemical action with water.

†It should be noted that the cooling is temporary only. It does not last long.

197. Bunsen's Ice Calorimeter.—Bunsen's ice calorimeter yields accurate results and can be used even when only a small quantity of a substance is available.

It consists of a thin glass test-tube A fused into a bigger glass tube B. At its lower end the tube B is connected to a U-tube, the free end of which is connected to a graduated capillary tubing TT, as shown in Fig. 28.

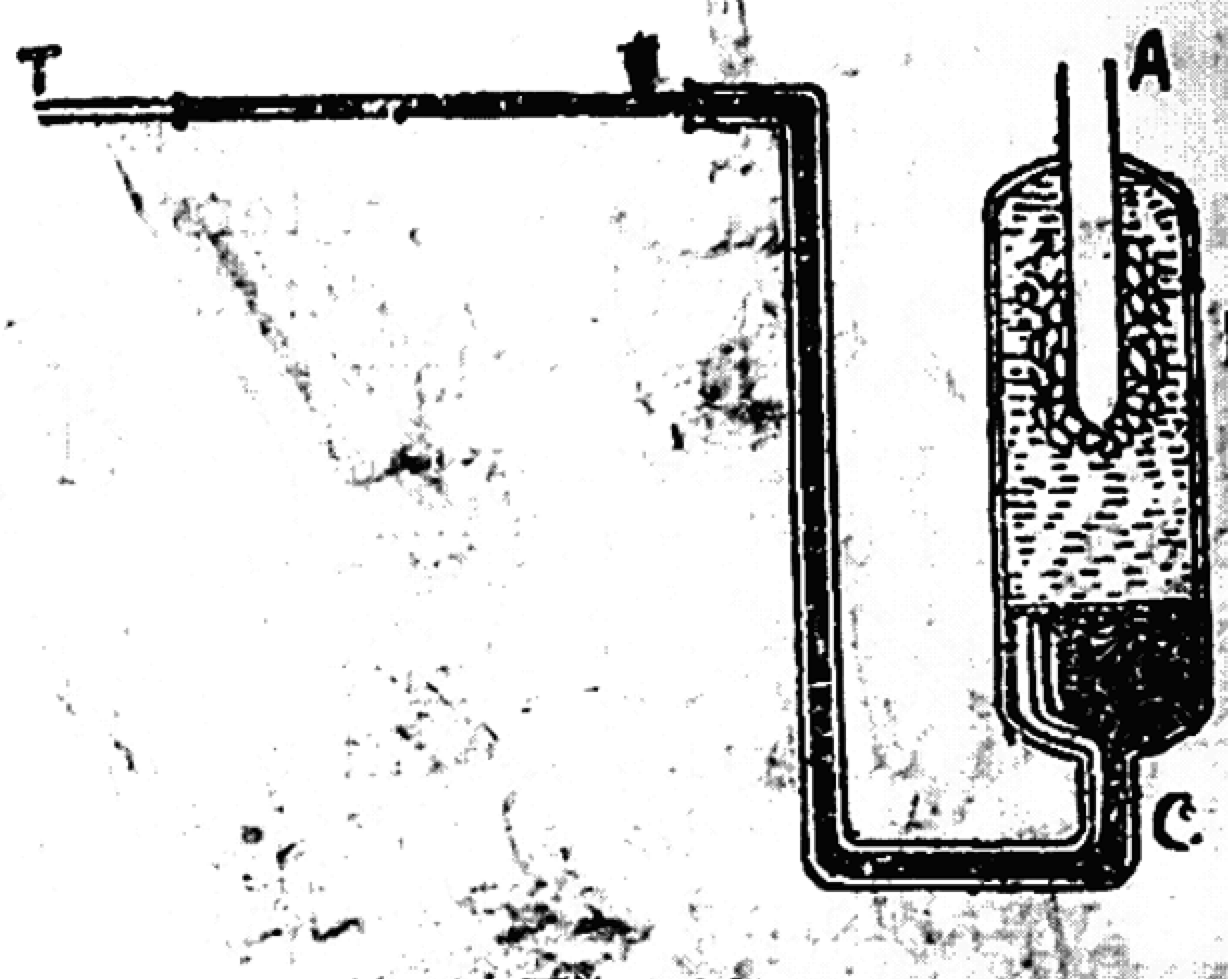

Fig. 28.

To use the apparatus the upper part of the tube B is filled with pure distilled water and the lower part with mercury. The tube TT is filled with a coloured liquid. The whole apparatus is placed in a vessel containing pure ice so that its temperature may remain 0°C. throughout an experiment. Some of the water in the tube B is made to freeze by evaporating ether in the test-tube A. The column of the liquid in the capillary tube is noted. The hot body whose specific heat is to be determined is dropped into the test-tube, which is then corked up to protect it from losing heat to the surroundings. The heat from the body melts some of the ice. And since ice contracts in volume on melting, the column of coloured liquid in the capillary tube moves inwards. Since the tube is graduated, we can know the change in volume. Knowing that 1 gram of ice when melted decreases in volume by 0·09 c.c.,* we can determine how much ice has melted. Applying the relation $S=\frac{80m}{MT}$, the specific heat of the body can be calculated.

EXERCISES

1. Twenty grams of ice at -10°C. are mixed with 240 grams of water at 100°C. Find the final temperature of the mixture. Specific heat of ice may be taken as 0·5 and the latent heat of water as 80.

Let the final temperature be θ.

Heat given out by the hot water $=240\times(100-\theta)$.

Heat taken up by ice is utilized, partly in heating it from -10°C. to 0°C. partly in melting it to water at 0°C., and partly to heat this water from 0° to θ°C.

Therefore heat taken up by ice $=20\times10\times0{\cdot}5+20L+20\times\theta$.

But heat gained $=$ heat lost.

Hence $$20\times10\times0{\cdot}5+20\times80+20\times\theta=240(100-\theta).$$

or $$5+80+\theta=12(100-\theta).$$

or $$13\theta=1200-85=1115.$$

or $$\theta=\frac{1115}{13}=85{\cdot}8\text{°C. (approximately)}.$$

*We know that 1,090 c.c. of ice at 0°C. yield 1,000 c.c. (or 1,000 gm.) of water at 0°C. This means that 1,000 grams of ice on melting decrease by 90 c.c. or 1 gram of ice on melting decreases in volume by 0·09 c.c.

2. Nine grams of ice at 0°C. are added to 121 grams of water contained in a brass calorimeter (specific heat 0·09) weighing 35 grams. The initial temperature of water was 24°C. and the final temperature 17°C. Find the latent heat of water.

$$\text{Heat given out by water and calorimeter} = 121 \times 7 + 35 \times 0{\cdot}09 \times 7 = 847 + 22{\cdot}05.$$

$$\text{Heat taken up by ice} = 9 \times L + 9 \times 17 = 9L + 153,$$

Therefore $$9L + 153 = 847 + 22{\cdot}05.$$

or $$L = \frac{716{\cdot}05}{9} = 79{\cdot}56.$$

3. Ten grams of mercury at 100°C. were introduced into the inner tube of a Bunsen's ice calorimeter. The column of the coloured liquid in the capillary tube of 1 sq. mm. cross-section moved through 37 mm. If 1,000 c.c. of water become 1090 c.c. of ice, find the specific heat of mercury.

First of all let us see how much decrease in volume takes place on the melting of ice. It is obviously equal to $3{\cdot}7 \times \frac{1}{100}$ c.c. or 0·037 c.c.

Now since a decrease of 90 c c. would occur in 1,000 gm., a decrease of 0·037 c.c. corresponds to the melting of

$$\frac{1000}{90} \times 0{\cdot}037 = 0{\cdot}411 \text{ gm.}$$

The heat taken up by ice to melt $= 0{\cdot}411 \times 80 = 32{\cdot}88$ calories,

Heat given out by mercury $= 10 \times 100 \times S$,

Then $$10 \times 100 \times S = 32{\cdot}88,$$

or $$S = 0{\cdot}033 \text{ (approx.)}.$$

4. The specific heat of ice is 0·5. How much heat will be required to change 20·4 gm. of ice at −6·7°C. to water at 15°C. ?

Ans. 2006·34 calories.

5. How many ounces of ice (sp. heat=0·5) would melt if 3 lb. of water at 25°C. are mixed with 1·5 lb. of ice at −10°C. ? *Ans.* 13·5 oz.

6. Would a change in the thermometric scale affect the numerical value of the latent heat of a substance ? The specific heat of ice is 0·5 and its latent heat of fusion is 80 units when the Centigrade scale is used. What are the corresponding numbers on the Fahrenheit scale ?

Ans. Yes : sp. heat same : latent heat 144 units.

7. 143 grams of water were obtained on introducing a kilogram of iron (specific heat 0·114) at 100° into an ice calorimeter. Find the latent heat of ice. *Ans.* 79·7 units.

8. The density of ice is 0·93 gm. per c.c. at 0°C., the latent heat being 80. A piece of metal weighing 100 gram. is heated to 75°C. and is placed in a Bunsen's ice calorimeter. The volume decreases by 0·938 c.c. Find the specific heat of the metal. *Ans.* 0·133.

9. Describe and explain the use of a Bunsen's ice calorimeter. What change of volume in the liquid would be produced in a Bunsen's ice calorimeter if we place in it 2·5 gm. of a substance of sp. heat 0·076 at a temperature of 100°C. ?
Ans. 0·0214 c.c.

10. A glass tumbler of weight 200 gm. contains 300 gm. of water at 46°C. on a hot day. If the temperature of the water is to be lowered to 10°C. how much ice should be added ? Take specific heat of glass as 0·2.
Ans. 136 gm.

CHAPTER VII

Change of State (*Continued*)

Liquid to Vapour

198. Vaporization.—The change from the liquid to the gaseous state is known as *vaporization*. We have already learnt that a liquid on being heated begins to boil and change its state. But we know that boiling is not the only process by which a liquid changes into the gaseous state. There is another process in which this change goes on at all temperatures. Who has not observed that a wet cloth on being exposed to the air becomes dry after some time, or water left in an open dish gradually disappears ? This type of change from the liquid to the gaseous state is known as *Evaporation*.

199. Vapour.—When a liquid changes into the gaseous state we say that it has changed into vapour. Before we proceed any further let us understand the distinction between gas and vapour. Strictly speaking, *a substance in the gaseous condition which can be liquefied by pressure alone is called vapour*. In other words, a gas below its critical* temperature is vapour, and vapour above the critical temperature is gas. According to this definition carbon dioxide at room temperature should be called vapour, for its critical temperature is 31°C. and so also sulphur dioxide, for its critical temperature is 157°C. But we generally call both these substances gases. This shows that the distinction between gas and vapour is not strictly observed in actual practice. In a general way it may be said that *by vapour is meant the gaseous condition of those substances which are liquid at ordinary temperatures, such as alcohol, ether, water*, etc. It should be remembered, however, that the laws of gases are obeyed imperfectly by vapours when their temperature is close to the boiling point of their liquids. At temperatures far removed from the boiling points the vapours obey the gas laws like permanent gases.

200. Vapour Pressure.—Since vapour is gaseous in character, it must exert pressure like a gas. Let us see experimentally if it is true.

Experiment.—Take two clean and dry barometer tubes. Fill them with clean, dry mercury and invert them in a dish containing mercury. Support both the tubes side by side as shown in Fig. 29. The mercury will be seen to fall a little and stand at the same level in both the tubes. With the help of a bent pipette introduce into one of the tubes two or three drops of water. The drops will be found to pass up into the Torricellian vacuum and vaporize there. The column of mercury will be seen to fall showing thereby that the vapour

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*Critical temperature is that temperature above which a gas cannot be liquefied to however great a pressure it may be subjected.

is exerting pressure. Introduce more water drop by drop. It will be seen that the column of mercury falls up to a certain limit, after which no further change in level takes place. If any drop is passed up at this stage it lies at the top merely as a thin film. The vapour at this stage is said to be **saturated,** and the pressure which it exerts is said to be **maximum vapour pressure,** or simply **vapour pressure.** We define (maximum) vapour pressure as follows :

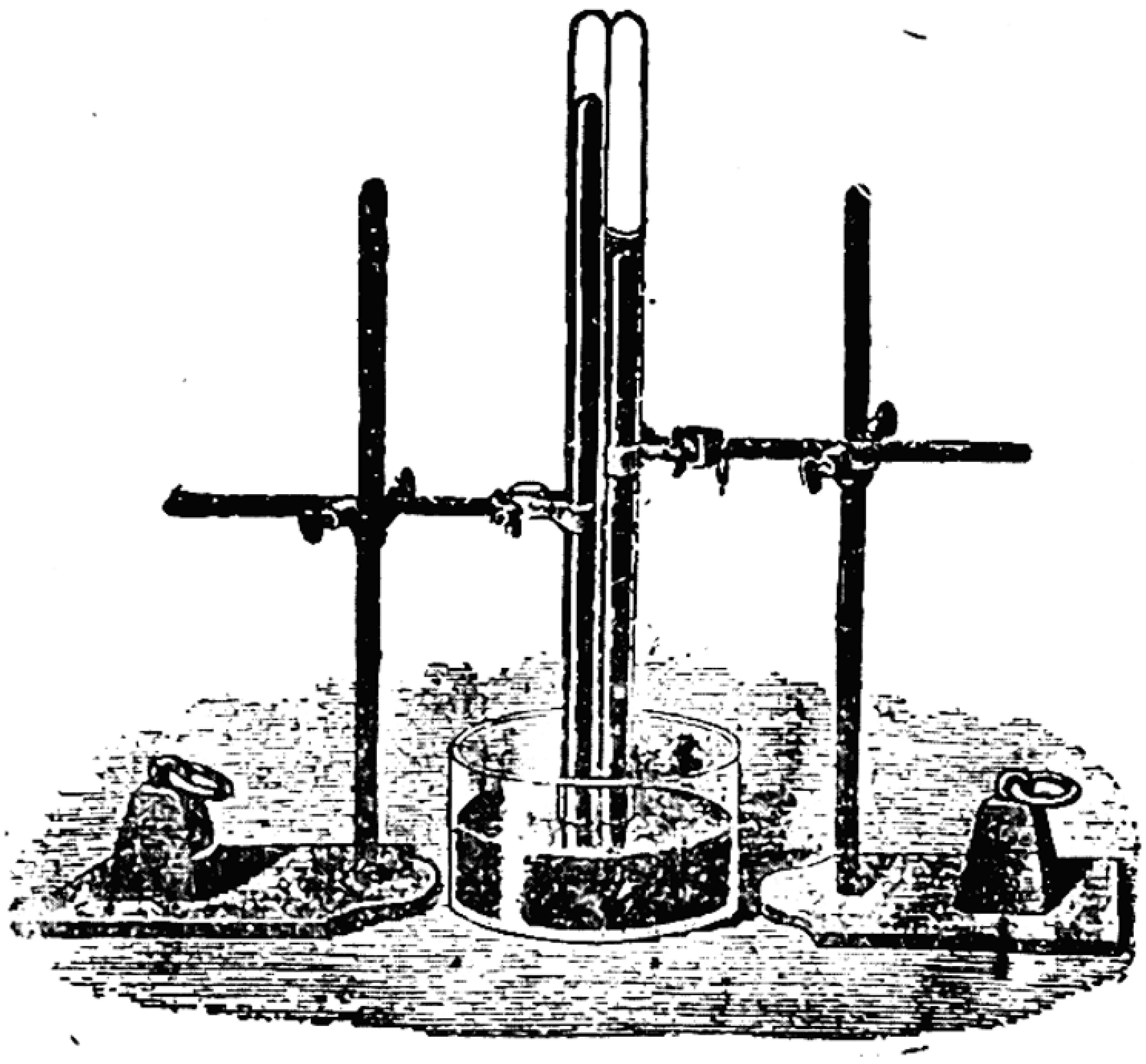


Fig. 29.

The vapour pressure is the pressure exerted in a closed space by vapour in contact with its own liquid.

Now introduce ether drop by drop into the second tube, till the space above the mercury column becomes saturated with vapour of ether. It will be found that the vapour pressure is widely different in the two cases. If, for instance, the temperature of the room be 20°C., the maximum vapour pressure of water will be seen to be 17·4 mm. and of ether 400 mm. This shows that *the vapour pressure of a substance depends upon its nature.*

Let us see now if the volume of vacuum and the temperature of vapour affect the pressure. Take two barometer tubes of different lengths and diameters, fill them with mercury and invert them in a dish of mercury. It will be seen that in each case the height of mercury column is the same. Introduce enough alcohol into the tubes to make the space above the mercury column in each case saturated with vapour. It will be seen that the height of the mercury column is the same in both the tubes, although the amount of alcohol required is widely different. Thus we see that *the vapour pressure is independent of the volume of vacuum.*

Now pass a Bunsen flame up and down one of these tubes which contains saturated vapour in the Torricellian vacuum. It will be noticed that the column of mercury falls down, showing that the vapour pressure increases. If enough liquid be lying at the top of the column of the mercury, the vapour will remain saturated even on heating otherwise it will become unsaturated. As the tube is allowed to cool the level of the mercury column will be seen to rise up gradually, till it comes to the original level once again. This shows that *the vapour pressure depends upon the temperature ; the higher the temperature the greater the pressure.*

To give an idea as to how the vapour pressure of a liquid increases with temperature the pressure of water vapour at various temperatures is given below :—

Pressure of Water Vapour

Temperature.	Pressure.	Temperature.	Pressure.
0° C.	4·60 mm.	95° C.	633·7 mm.
10° ,,	9·16 ,,	99° ,,	733·2 ,,
20° ,,	17·39 ,,	100° ,,	760 0 mm. = 1 atmosphere.
30° ,,	31·51 ,,		
40° ,,	54·87 ,,	101° ,,	787·7 mm.
50° ,,	91·98 ,,	102° ,,	816·0 ...
60° ,,	148·9 ,,	120·6°C.	2 atmospheres.
70° ,,	233·3 ,,	134° ,,	3 ,,
80° ,,	354·9 ,,	144° ,,	4 ,,
90° ,,	525·5 ,,		

If we plot a graph showing the relation between vapour pressure of a liquid and temperature we get a curve of the type shown in Fig. 30. It will be seen that the curve is concave upwards which shows that the vapour pressure increases rapidly at high temperatures.

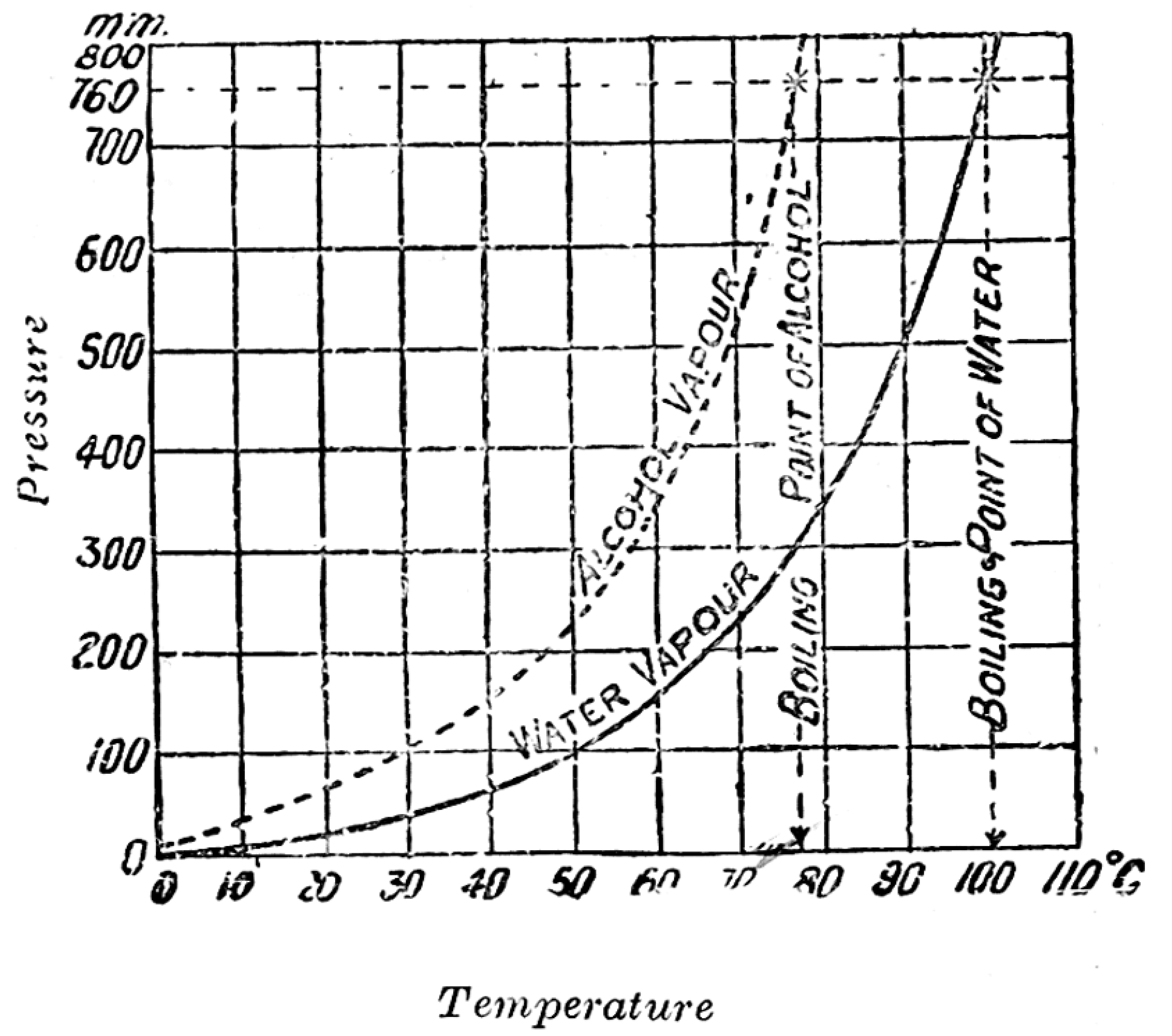


Fig. 30. Graph showing the relation between maximum pressure and temperature of water and of alcohol.

201. In the experiment described in the preceding article we introduced a liquid into vacuum ; we shall now study the case when the liquid is introduced into the space which already contains a vapour or a gas. Take a barometer tube and fill it with clean dry mercury. Invert the tube in a dish containing mercury and introduce air or some other vapour in the Torricellian vacuum. Note the height of mercury column. Now pass a few drops of the given liquid to the top. The liquid will be seen to vaporize more slowly than in vacuum. When the space can no longer hold any more liquid in the form of vapour, it is found that the pressure exerted by the vapour is the same as in the previous experiment. This shows that the pressure of the vapour of a liquid is independent of the presence of any other vapour in the space. This is, of course, true only if there is no chemical action between the two vapours. The total pressure exerted by the mixture is equal to the sum of the pressures which each constituent vapour would exert if it alone were present in the space at the same temperature. Dalton was the first to arrive at these results, and consequently they are called after him **Dalton's Laws.** They are as follows :

(1) *The maximum pressure exerted by a particular vapour in a closed space at the given temperature is independent of the presence of other vapours or gases having no chemical affinity for it.*

(2) *The total pressure exerted in a given space by a mixture of gases and vapours having no chemical action on one another is the sum of the pressures which each constituent would exert when enclosed alone in the same space at the same temperature.*

The first law is applicable to saturated vapours, and the second to both saturated and unsaturated vapours.

202. **Saturated and Unsaturated Vapours.**—We have said in §200 that when a vapour is in contact with its own liquid it is saturated, and exerts maximum pressure, the value of which depends on the temperature of the vapour.

When the vapour exerts pressure less than the maximum pressure corresponding to that temperature, it is called **unsaturated vapour.**

Let us see how to distinguish saturated from unsaturated vapour. The simplest test is to see if the vapour is present in contact with its own liquid. If the liquid is present, the vapour is saturated, for the presence of the liquid is a sure proof that no more liquid can be vaporized. If it is not present the vapour may or may not be saturated. To decide this point, proceed as follows.

Lower the tube containing the vapour thereby decreasing the space above the mercury column. This will compress the vapour and hence increase its density. Now, since the saturated vapour is already exerting maximum pressure and possesses maximum density any attempt to increase the pressure or density must fail. So that if the vapour is saturated the decrease of volume causes condensation. If, on the other hand, the vapour is unsaturated, the decrease in volume causes increase in density or increase of pressure corresponding to Boyle's law. This will go on till the unsaturated vapour becomes saturated; when any further attempt to increase the pressure will fail, and simply result in condensation.

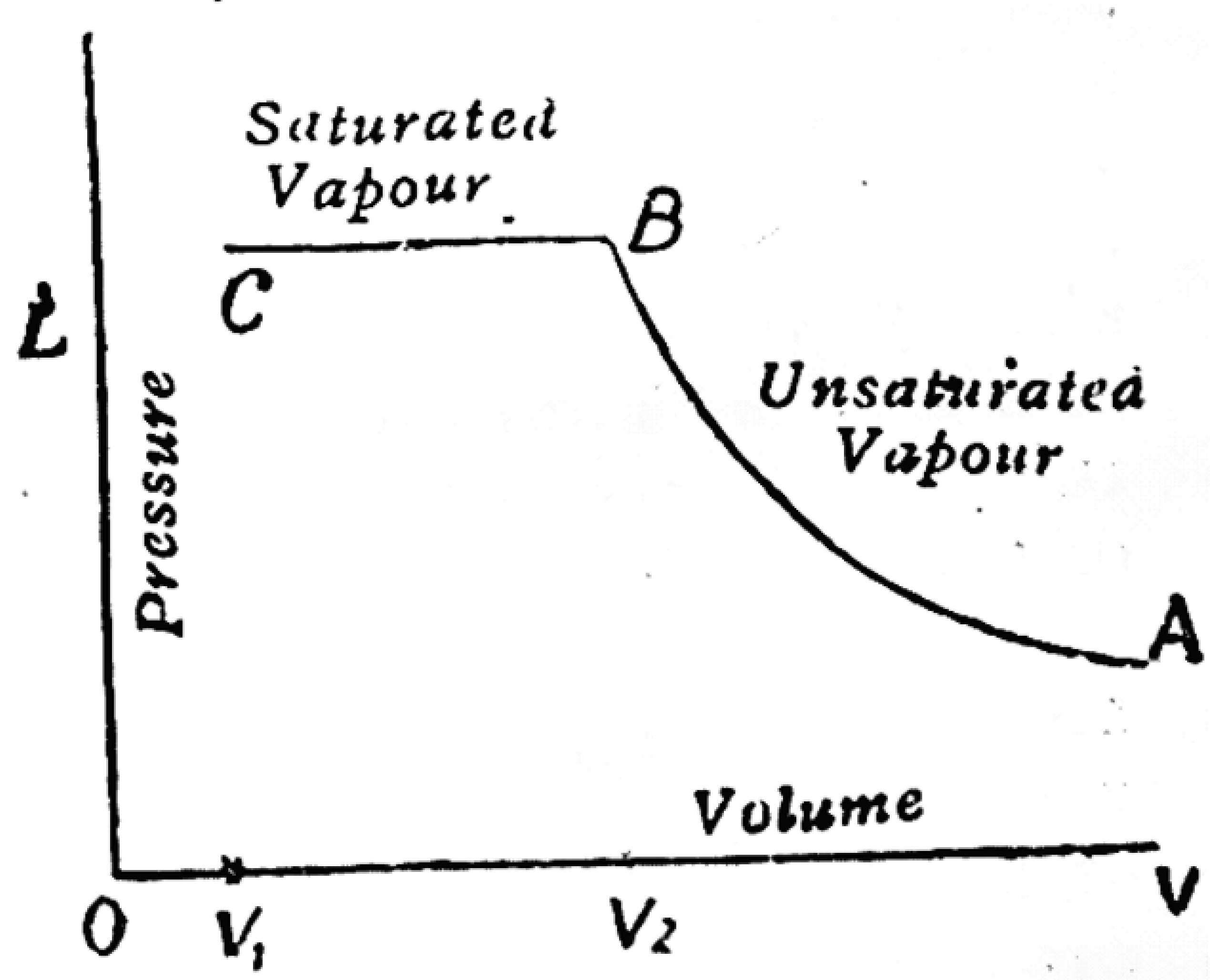


Fig. 31. Graph showing the relation between Pressure and Volume of a Vapour at constant temperature.

The curve in Fig. 31 represents in a striking manner the behaviour of a vapour, when its volume is changed at constant temperature. Let us start with unsaturated vapour having pressure and volume corresponding to point *A* and decrease its volume. The pressure will increase in accordance with Boyle's law and the relation between the two will be represented by the curve *AB*. This shows that unsaturated vapour behaves like a gas.

When the volume is decreased to V_2 corresponding to point *B* on the curve, the pressure corresponds to the maximum pressure. Any further decrease in volume simply condenses the vapour, leaving the pressure unaffected. This result is represented by the part *BC* of the curve which shows that the volume can be changed considerably without changing the pressure.

Now, let us study the behaviour of vapour when its temperature is changed keeping its volume constant. Enclose the tube in a tube of bigger bore and fill the space between the two with cold water. If the vapour be saturated the decrease in temperature will cause condensation, for at a lower temperature less vapour is required to saturate the same space than the amount actually present, and hence the surplus amount must condense.

If, on the other hand, it is an unsaturated vapour the decrease in temperature at constant volume will cause a decrease in pressure. Suppose we start with unsaturated vapour having pressure and temperature corresponding to point A in Fig. 32. On decreasing temperature the pressure will decrease being proportional to the absolute temperature. This is represented graphically by the part AB of the curve. At temperature T_1 the vapour becomes saturated and the pressure becomes maximum corresponding to T_1. Further cooling causes condensation and the vapour pressure begins to fall more rapidly with fall in temperature than before. The behaviour of vapour after T_1 is shown by the part BC of the curve. This part is just like the vapour pressure curve in Fig. 30.

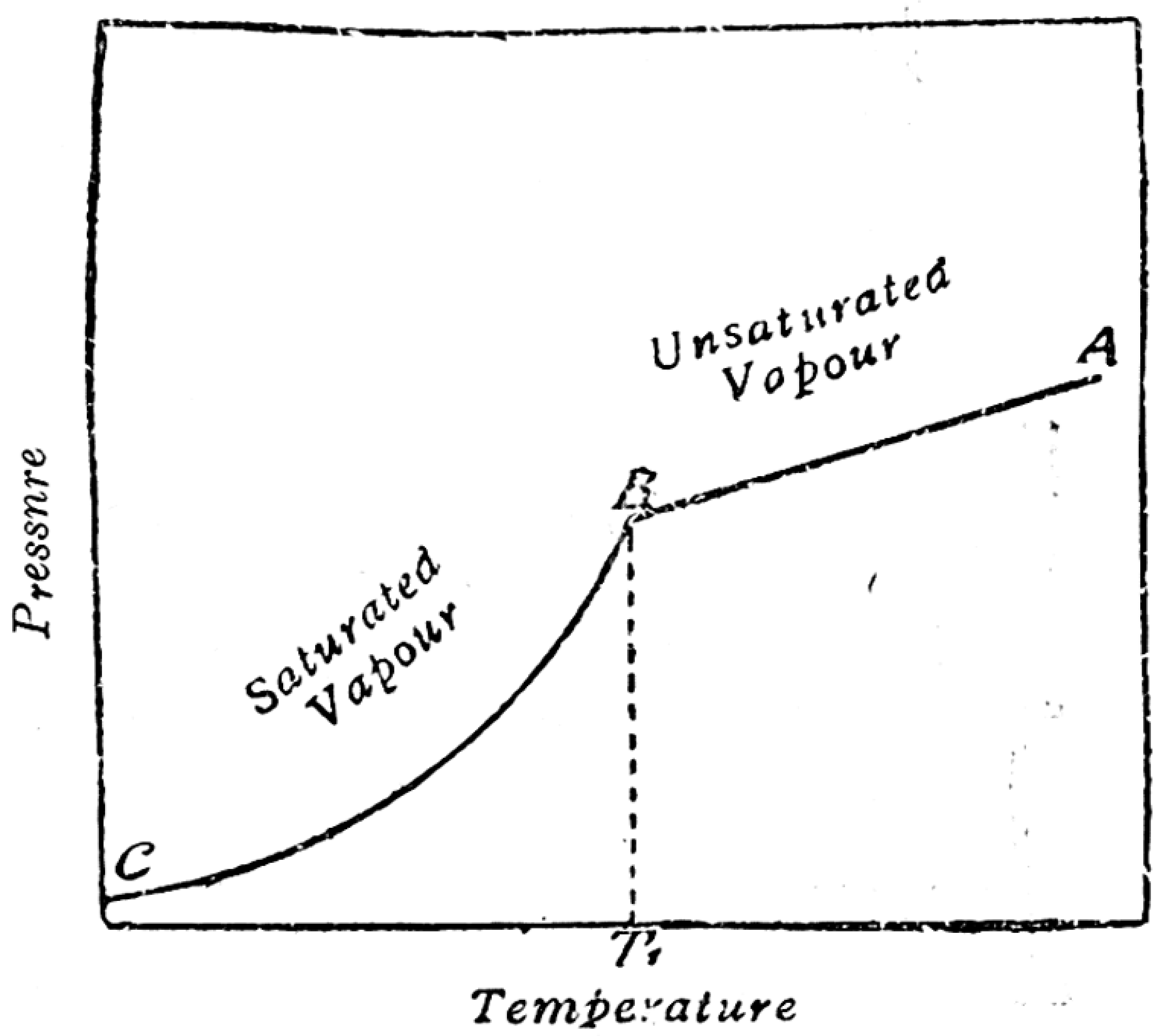


Fig. 32. Graph showing the relation between Pressure and Temperature of a Vapour at constant volume.

203. Ebullition or Boiling.—When a flask containing water is heated from below, the following changes will be observed :

(1) Air dissolved in water will form bubbles on the bottom and the sides of the flask. The bubbles will be seen to grow bigger by degrees and as soon as the temperature of water is 50° or 60°C., they begin to shoot to the surface.

(2) At 70° or 80°C. small bubbles of steam form at the bottom and begin to rise to the surface. Since the upper layers are colder, the bubbles of steam on coming in contact with them condense with sharp clicks producing the sound—called the "singing" of the kettle.

(3) After some time the bubbles of steam cease to condense. On the other hand, they rise to the surface and escape from there into the air. At this stage the temperature of water becomes uniform and the singing of the kettle ceases and the water begins to boil.

(4) However long the boiling is continued, the temperature is found to remain steady. This constant temperature at which a liquid boils is called the *boiling point*.

204. Determination of the Boiling Point.—To determine experimentally the boiling point of a liquid, take a hypsometer and fill

it partly with the liquid. Push a thermometer through the hole in the cork closing the mouth of the hypsometer and see that the thermometer does not dip in the liquid. Heat the liquid. The vapour given off will play upon the bulb and the stem of the thermometer, which will indicate a rise in temperature. Note the highest temperature which remains constant for about five minutes. This constant temperature is the boiling point of the liquid at the atmospheric pressure.

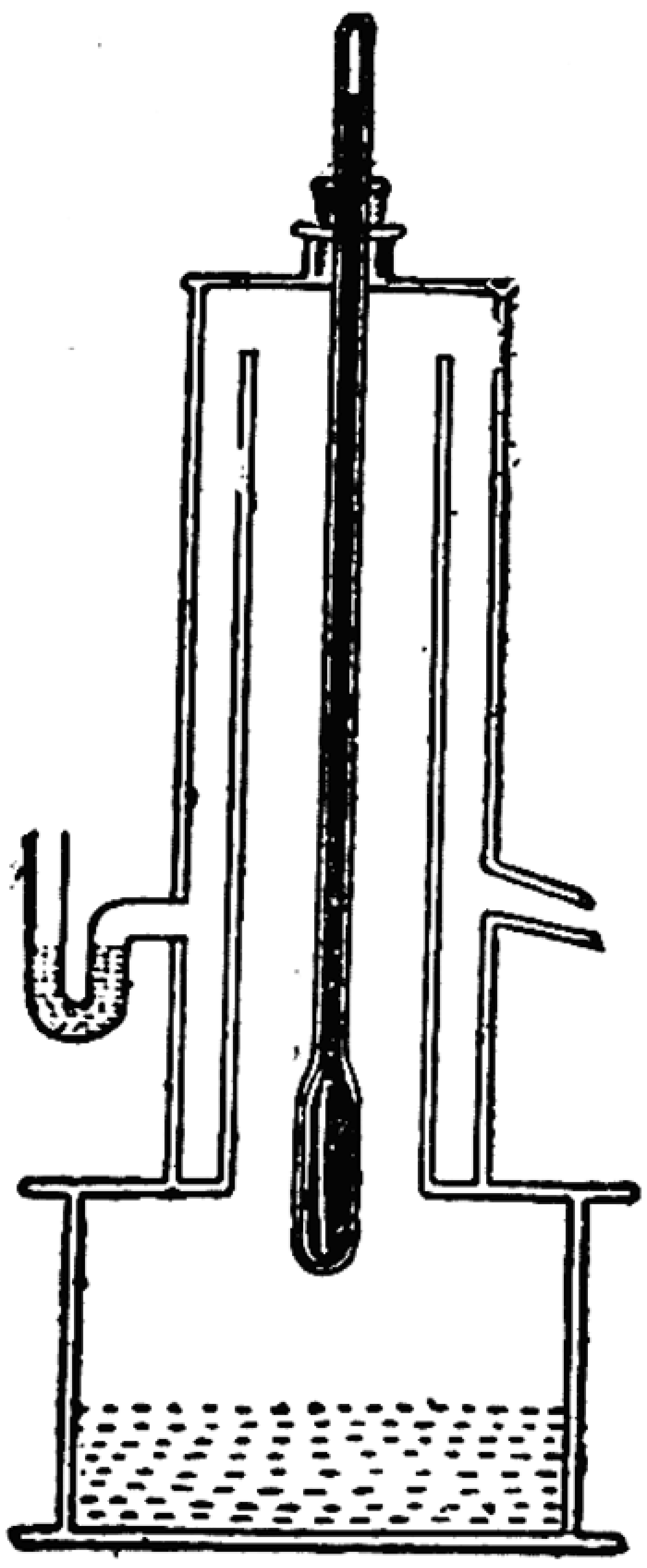


Fig. 33.

Caution.—The thermometer should not dip in the liquid because the temperature of the liquid depends on the state of its purity, whereas the temperature of the vapour depends only on the nature of the liquid and the pressure acting on its surface.

While determining the boiling point of solutions the thermometer should of course dip in the solution, because otherwise it will give the boiling point of the pure liquid.

205. Influence of Pressure on the Boiling Point.—Take a round-bottomed flask of about 300 c.c. capacity. Fill it half with water. Boil the water vigorously for a few minutes, so as to drive out the air and fill the upper part of the flask with steam. Remove the burner and close the mouth with a cork just when the ebullition is about to cease. Turn the flask upside down and allow it to cool in this position until boiling entirely ceases. Then pour some cold water over it and notice that the water inside begins to boil again, and the more you cool the flask the longer theboiling continues. Let us see why it happens so. At the moment the mouth of the flask was closed, the pressure of the steam inside was equal to the pressure of the atmosphere. When water is poured over the bottom, some of the steam condenses and thereby the pressure inside is reduced. Since water begins to boil again, this shows that under reduced pressure water boils at a lower temperature. The more we cool the flask, the greater is the reduction in pressure, and the lower is the boiling point.

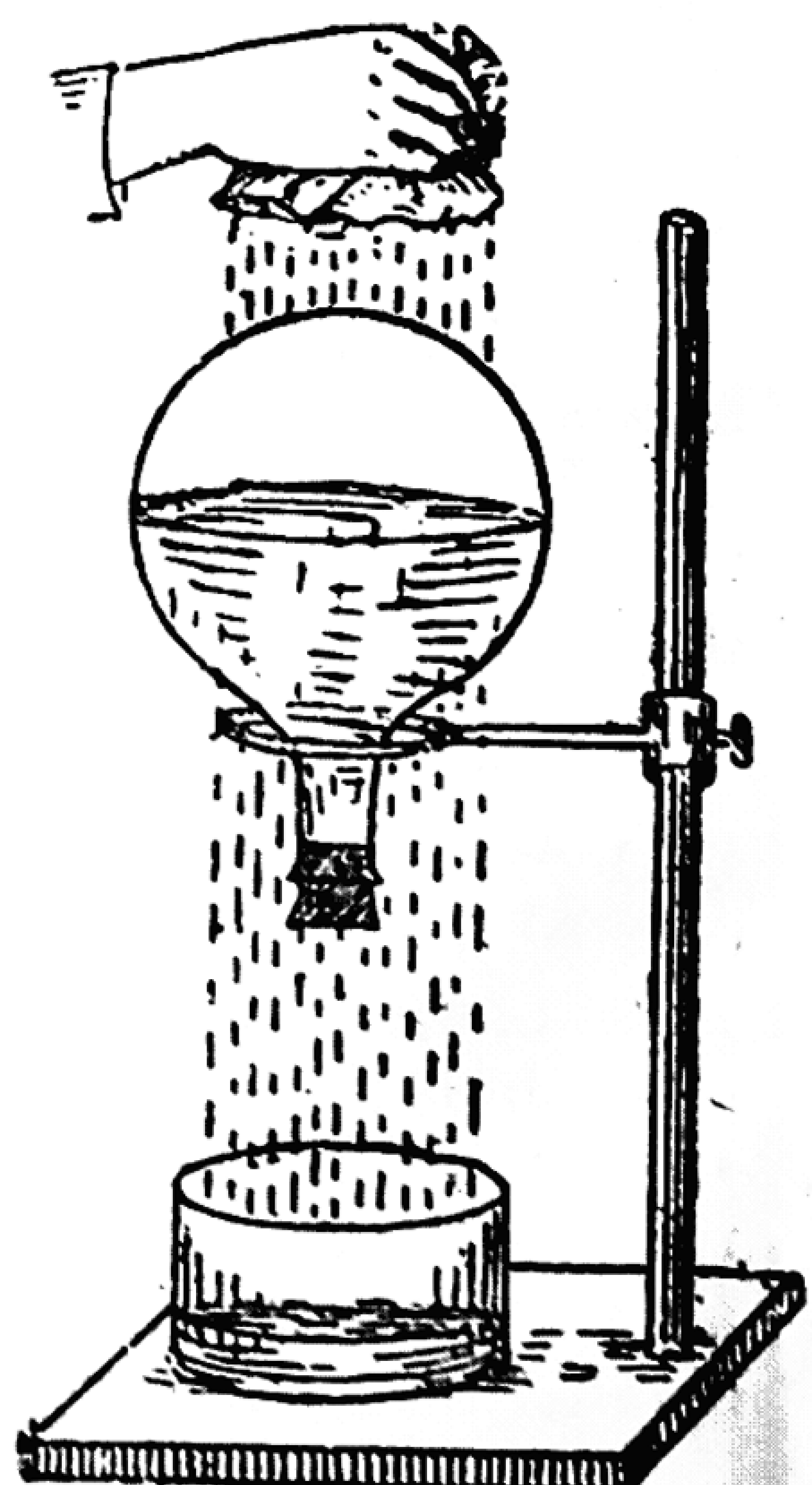
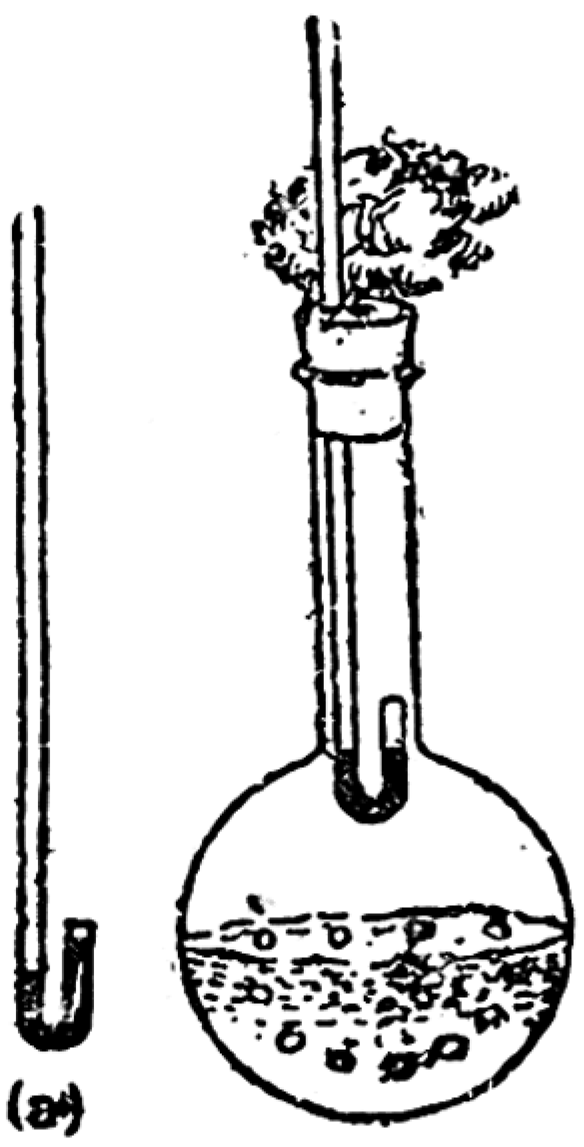


Fig. 34.

On the other hand, the effect of increasing the pressure is to raise the boiling point. For instance, at a pressure of 81·6 cm., water boils at 102°C., and at 107·5 cm., it boils at 110°C. On high mountains where water boils below 100°C., cooking takes a long time and in certain cases cannot be done properly, hence people there cook their articles under pressure in special cookers. In one of the

cookers, called a **digester**, the top is closed with a lid which is held in position by a screw. A valve in the lid is closed by means of a lever carrying a weight. To get different pressures, we have only to change the position of the weight along the lever. With such an apparatus the boiling point of water may be raised to as high a temperature as 200°C.

206. Now let us see whether there is any relation between the external pressure and the maximum vapour pressure at the boiling point. Take a J-tube, whose short limb is closed. Fill the tube with mercury to about 1 cm. from the open end and fill the remaining portion with water. Both the limbs, the shorter as well as the longer, are now filled and there is no air left in the tube. Close the open end with the thumb and invert the tube. Water being lighter will pass into the shorter limb. Now take mercury out of the longer limb so that it stands just above the bend in the longer arm as shown in Fig. 35 (*a*). Introduce the tube into a flask in which water is boiling.


(a) (b)
Fig. 35.

It will be seen that *mercury stands at the same level in both the limbs*, thus showing that the pressure on the two sides is the same. Now since the pressure in the shorter limb is due to the water vapour at the boiling point and that in the longer limb is due to the atmosphere, we learn that at the boiling point the maximum vapour pressure is equal to the external pressure. Keeping this in view, we define the *boiling point as the temperature at which the maximum vapour pressure of a liquid is equal to the atmospheric pressure.*

Table of Boiling Points.

(at pressure of 76 cm. of mercury)

Hydrogen	...	−253° C.	Oil of turpentine	...	159° C.
Sulphur dioxide	...	− 10° ,,	Naphthalene	...	220° ,,
Ether	...	35° ,,	Sulphuric acid	...	318° ,,
Alcohol	...	78° ,,	Mercury	...	357° ,,
Water	...	100° ,,	Sulphur	...	440° ,,

The change in volume that takes place when a liquid changes to the gaseous state is enormous. The most familiar example is that of water. 1 c.c. of it when turned into steam occupies about 1,700 c.c. at the same temperature and pressure.

207. Laws of Ebullition.—We shall now sum up the laws of ebullition. They are as follows :—

I. *Every liquid begins to boil at a certain definite temperature different for each substance depending upon the pressure.*

II. *From the moment boiling commences, the temperature remains constant until the whole of the liquid is boiled off.*

III. *Unit mass of each substance requires a definite amount of heat to change it from liquid to gaseous state without change of temperature.*

IV. *The boiling point of a liquid is raised by increase of pressure and lowered by decrease of pressure.*

V. *There is an enormous increase in the volume accompanying the change from the liquid to the gaseous state.*

207a. Distillation.—If water contains dissolved impurities it can be purified by boiling and condensing its vapour. Not only water but other liquids also are often distilled to purify them. The process of vaporising a liquid and condensing its vapour is called *Distillation.* In case a liquid is a mixture of two or three liquids, as for instance crude petroleum, its constituents can be separated by distillation. When such a mixture is heated in a large steel still the constituents which have low boiling points pass off first as vapour. In the case of crude petroleum, for instance, petrol passes off first. It is followed by kerosene oil. Then comes the gas oil. The oil left behind in the still is the fuel oil.*

The above process of separating a mixture of liquids into various constituents is called *Fractional Distillation.* It is often used in separating alcohol from water.

208. Latent Heat of Steam.—The latent heat of vaporization of water is called the latent heat of steam. To determine it experimentally proceed as follows: Fit a heater with a cork and pass through it a delivery tube bent upwards as shown in Fig. 36. Wrap cotton wool round the tube so that the steam passing through it may not be cooled by the atmosphere. Pass the longer limb of the tube into a steam trap and let a tube from the trap go into the calorimeter fitted with a stirrer and a cover. It is of extremely great importance to remember that we must pass into the calorimeter *dry* steam *i.e.*, steam free from water, for the condensed part is merely water at the boiling point. It is to check the condensed part from going into the calorimeter that the tube is made to slope upwards, and a trap is used. Place a screen to shield the calorimeter from the direct heat of the flame.

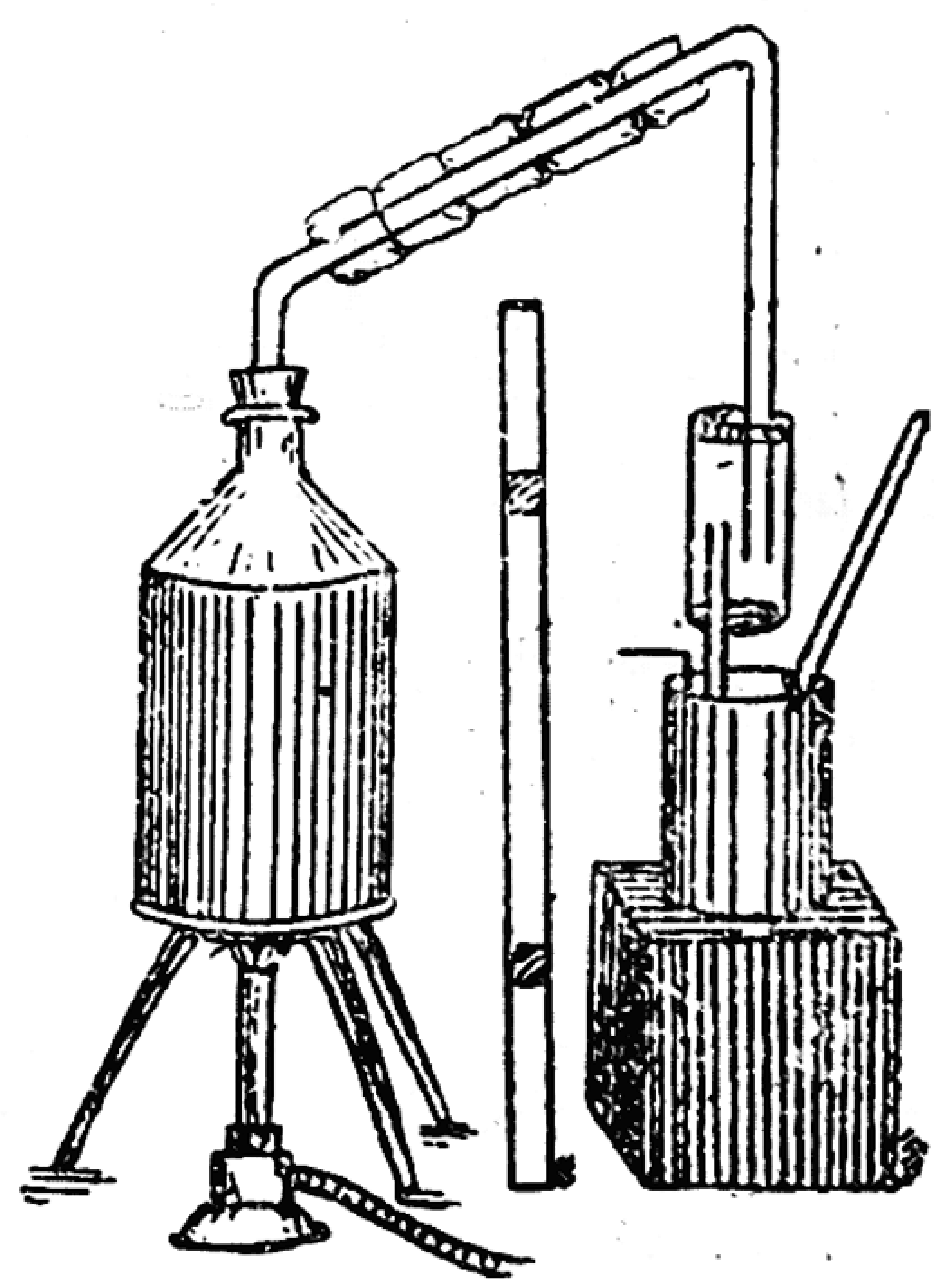


Fig. 36.

Fill half the heater with water and place it over a Bunsen burner. During the time the water boils weigh the calorimeter and the stirrer and let their weight be m, the specific heat of their material being s. Fill the calorimeter half with water, and lower its temperature a little by adding ice. Let the lowered temperature after cooling be t°C. and the weight of water in the calorimeter be m_1 gm. When water is boiling briskly pass so much steam into the water in the calorimeter as may raise its temperature by about 20°C. Stir the water and note the temperature; let it be θ°C. Read the barometer and from the table of vapour pressures find the temperature of the steam at the pressure noted; let it be T°C; it will be very nearly equal to 100°C.; let us, for simplicity, suppose that $T° = 100°$. Weigh the calorimeter and find how much steam has been passed into it. Let its weight be m_2 grams.

We can easily find the latent heat of steam from the above data. Let it be L.

*The fuel oil is often further separated into lubricating oil, paraffin wax and asphalt.

Heat given out by steam in changing its state to water at the boiling point is m_2L, and in cooling from the boiling point to the final temperature θ, $m_2(100-\theta)$ calories.

This heat has been used to raise the temperature of water and calorimeter. The heat taken up by them is

$$m_1(\theta-t)+ms(\theta-t),$$

$$\therefore \qquad m_2L+m_2(100-\theta)=(m_1+ms)(\theta-t),$$

or

$$L=\frac{(m_1+ms)(\theta-t)}{m_2}-(100-\theta).$$

The latent heat of steam at 100°C. is found to be 537* calories. This shows that it takes more than five times as much heat to change a gram of water at 100°C. into steam at the same temperature as it takes to heat the same amount of water from 0° to 100°C.

To prevent condensed steam from passing into the calorimeter, Berthelot used the apparatus represented in Fig. 37. It consists of flask A closed at the top and fitted with a tube B passing through its bottom. The tube ends in a conical ground glass joint D, which fits into a spiral condenser immersed in water in calorimeter E. The calorimeter is placed in a double walled vessel, the space between the walls of which is filled with water to provide a constant temperature enclosure.

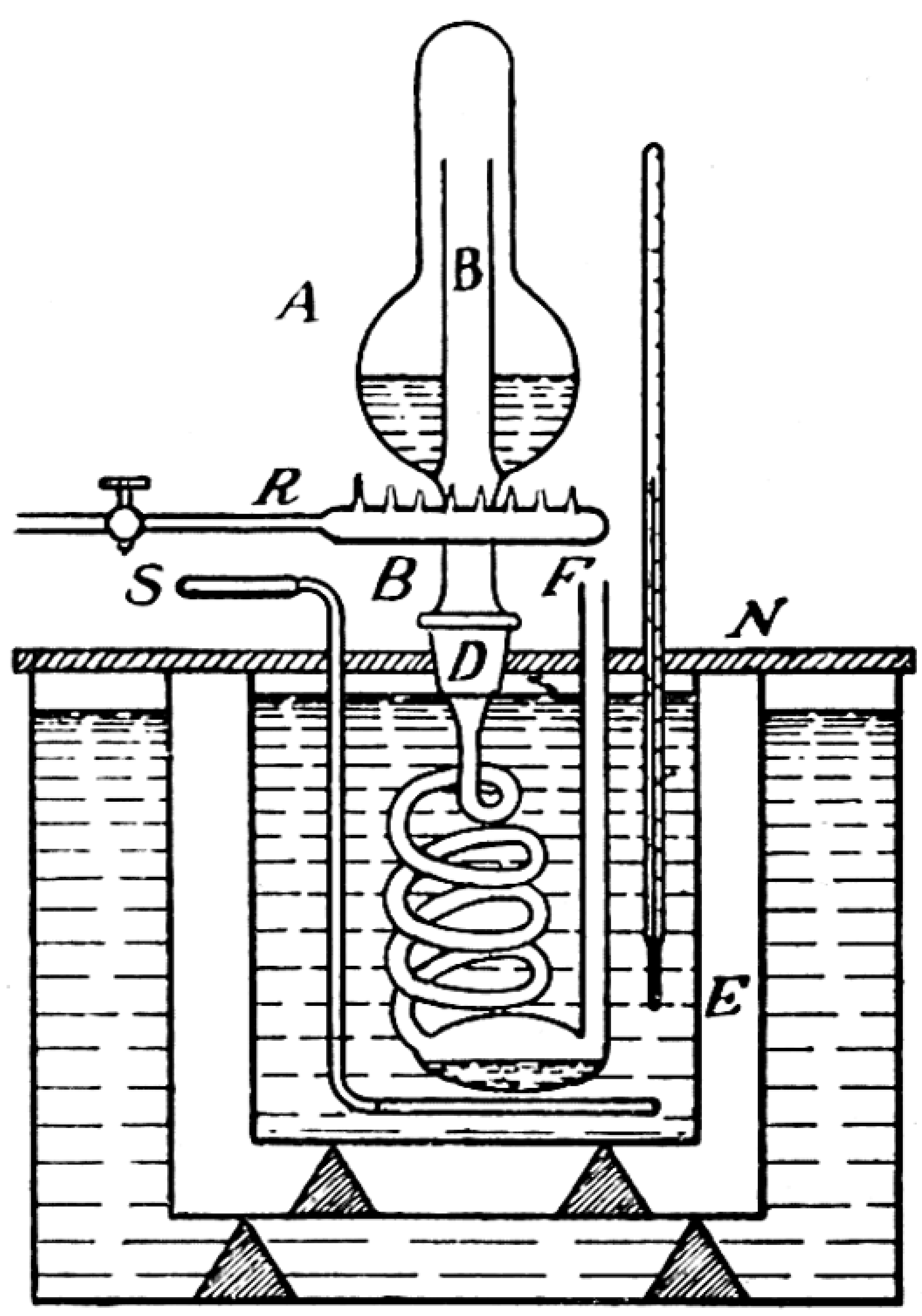


Fig. 37.

The water or the liquid of which the latent heat is required is boiled in the flask A heated by means of a ring burner R. As the spiral condenser is open to the atmosphere at F, the liquid boils at the atmospheric pressure. The calorimeter is shielded from the heat of the burner by a screen of wood N covered with metal.

The temperature of the water in the calorimeter E is measured and steam or vapour, if any other liquid is used, is passed into the spiral condenser, until the temperature of water in E rises by about 15° or 20°C. The steam or vapour supply is now cut off and the highest temperature of the water is read and the condenser is removed and weighed to get the weight of steam or vapour condensed. Let

m be the mass of water in the calorimeter,
m_1 ,, ,, ,, ,, calorimeter and stirrer,
s_1 ,, ,, specific heat of their material,
m_2 ,, ,, mass of condenser,
s_2 ,, ,, specific heat of its material,
m_3 ,, ,, mass of steam or vapour condensed,

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*It should be noted that the latent heat of steam is different at different temperatures. Roughly it may be said that the lower the temperature at which vaporisation takes place the greater the latent heat, *e.g.*, at 0°C. the latent heat of vaporisation of water is 606·5 calories.

t ,, ,, initial temperature of water in calorimeter.
T ,, ,, temperature of steam or vapour.
θ ,, ,, final temperature of water in calorimeter.
L ,, ,, latent heat of vaporisation.

Equating the heat lost to the heat gained we get

$$m_3L+m_3(T-\theta)=(m_1s_1+m_2s_2+m)(\theta-t)$$

or

$$L=\frac{(m_1s_1+m_2s_2+m)(\theta-t)}{m_3}-(T-\theta).$$

This method has the following advantages over the ordinary condensation method described earlier :—

(*a*) The amount of *condensed steam* that goes to the condenser is reduced to a minimum by making it pass through a tube surrounded by boiling water.

(*b*) The mass of the steam condensed is measured more accurately than it is possible when the steam is allowed to condense in a calorimeter containing 150 to 200 grams of water. For if the weight of the condenser is 25 grams and of the steam condensed 10 grams, the final weight of the condenser is 35 grams, which is easier to measure accurately than 200 or 210 grams.

Table of Latent Heats

(at boiling points under pressure of 1 atmosphere)

Water	... 537 Calories	Carbon disulphide	... 87 Calories
Alcohol	... 202 ,,	Oil of turpentine	... 74 ,,
Ether	... 90 ,,	Iodine	.. 24 ,,

209. Joly's Steam Calorimeter.—Joly devised a very simple and accurate method of determining specific heats based on the fact that if a body is immersed in steam at 100°C. steam condenses on it until its temperature is raised to 100°C. The heat required by the body to attain this temperature is supplied by the latent heat given out by the steam in condensing. This method is particularly useful in the determination of the specific heat of a gas at constant volume. A simple form of Joly's Steam Calorimeter is shown in Fig. 38. It consists of a metal enclosure called steam chamber in which hangs from the arm of a delicate balance a copper globe about 7 cm. in diameter. Steam is admitted into the chamber at the upper end through a wide tube and escapes through an outlet at the bottom. The globe is exhausted and is weighed. The gas whose specific heat is to be found is now filled in the globe at a pressure of

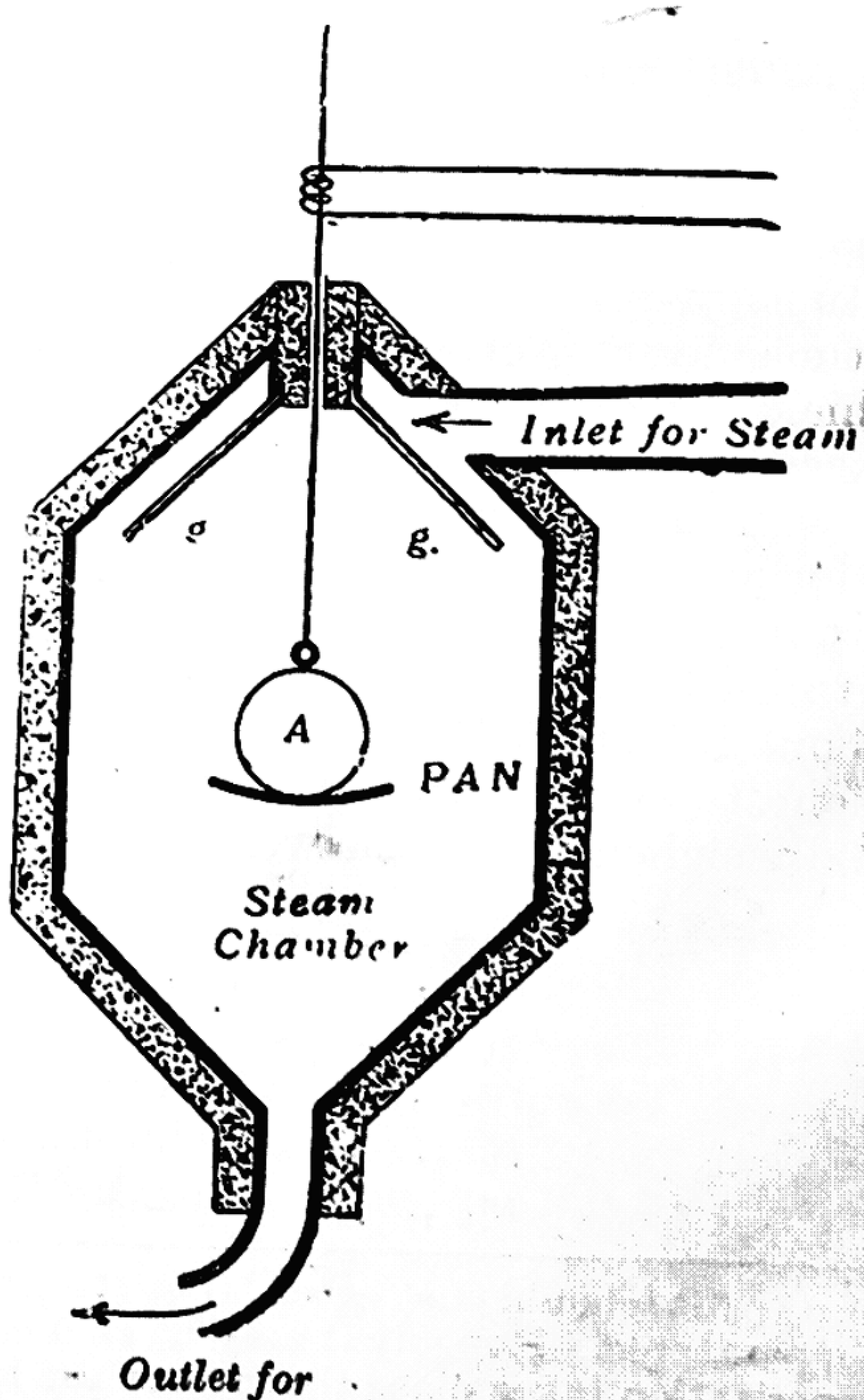

Fig. 38, Joly's Steam Calorimeter.

10 to 20 atmospheres and the mass of the gas determined. After waiting for some time so as to allow the gas to attain the temperature of the chamber, steam is admitted freely through the upper tube. As soon as the steam comes into contact with the globe containing the gas it condenses on the globe until the latter has the same temperature as the steam. The amount of the steam condensed is determined by adding weights to the other pan of the balance to restore equilibrium. The drops which form on the roof of the chamber are prevented from falling on the pan by a thin copper shield *gg*.

Let the weight of the steam condensed be W : part of it has been utilized to heat the copper globe and the pan underneath it and part to heat the gas. If m be the mass of the copper globe and pan and s the specific heat of their material, M the mass of the gas and S its specific heat, then the heat gained by the globe and gas$=(MS+ms)\ (100-t)$ where t is the initial temperature of the gas and globe and 100°C. is the final temperature.

Heat given out by steam in condensing$=WL$.

But $$\text{heat gained}=\text{heat lost}$$

$$\therefore \quad WL=(MS+ms)(100-t)$$

or $$WL-ms(100-t)=MS(100-t)$$

or $$S=\frac{WL}{M(100-t)}-\frac{ms}{M}.$$

Usually a preliminary experiment is first performed in which the steam is allowed to condense on the exhausted globe. This enables us to find the weight of the steam necessary to raise it and the pan from the initial temperature to 100°C. This weight is subtracted from the weight of steam condensed in the actual experiment and the difference gives the weight of steam used in heating the gas from $t°$ to 100°C.

In practice it is found that the condensation on the suspending wire causes serious errors. To avoid this a small platinum coil is wound round it and a current is passed through the coil just sufficient to maintain the suspending wire at 100°C., thus preventing steam from condensing on it.

In an actual experiment it was found that the mass of air enclosed in the copper globe at 20°C. and a pressure of 20 atmospheres was 4·33 gm. and the amount of steam condensed was 0·11 gm.

Let us calculate the specific heat of air at constant volume taking $L=540$.

$$0{\cdot}11\times 540=4{\cdot}33\times(100-20)\times S$$

$$\therefore \quad S=\frac{0{\cdot}11\times 540}{4{\cdot}33(100-20)}=0{\cdot}172$$

Thus we learn that the specific heat of air at constant volume is 0·172.

210. Evaporation.—Boiling differs from evaporation in that it takes place only at a fixed temperature depending on the nature of the liquid and the pressure acting on it, whereas evaporation takes place at all temperatures. Boiling takes place throughout the mass of the liquid, whereas evaporation takes place only at the free exposed surface. Whether a vessel is shallow or deep, ebullition takes place with

equal rapidity, but evaporation is considerably affected by it, the shallower the vessel, the greater the evaporation.

If a liquid is enclosed in a space, the evaporation goes on till the space above the liquid becomes saturated with vapour; after that no more evaporation takes place. But if, on the other hand, the liquid is exposed to open air, the atmosphere cannot be saturated, and hence evaporation goes on constantly.

The rate of evaporation is found to depend on the following factors:

(1) *The extent of the free surface of the liquid.* Take equal amounts of water in a dish and in a cup and expose them to the sun. You will find that the water in the dish disappears very much sooner than the water in the cup. This shows that the greater the free surface of the liquid, the more rapid the evaporation.

(2) *The rapidity of the renewal of air in contact with the surface of the liquid.* The layers lying above the liquid become almost saturated with the vapour after some time, and therefore hardly absorb any more vapour which makes the evaporation very slow. If the layer in contact with the liquid be rapidly removed, as happens when wind blows, the evaporation becomes rapid.

(3) *The temperature of the liquid and of the air.* The higher the temperature of the atmosphere and of the liquid, the greater the rate of evaporation. It is on account of this reason that wet clothes dry quickly near a stove or that water in a dish or from the surface of a pond evaporates more quickly in sunshine than in shade, in summer than in winter.

(4) *The dryness of the air.* The drier the air the more vapour it will require for saturation. It is for this reason that wet clothes dry more quickly on a dry than on a rainy day, although the temperature on both days may be the same. Hence *the drier the air, the more rapid the evaporation.*

(5) *The pressure on the surface of the liquid.* The lower the pressure, the lower the boiling point, and hence the quicker the passing of the liquid into the gaseous state. In vacuum it becomes very rapid.

(6) *The nature of the liquid.* The more volatile a liquid, the faster it evaporates; for instance, alcohol evaporates faster than water, chloroform faster than alcohol and ether faster than chloroform.

211. Cooling caused by Evaporation.—We have seen that whenever a change of state takes place heat is absorbed. We found the same thing in the case of a freezing mixture where the substance changed from the solid to the liquid state. Corresponding to that we have here cooling caused by evaporation. Whenever evaporation takes place, heat required for the change of state is taken from the liquid or from the surrounding objects if not supplied from outside. To see that it is so, pour ether over the hand. It will disappear in a very short time leaving the hand cold. The cold produced by evaporation may be made so great as to freeze the liquid itself.

To show it experimentally take a **cryophorus**, which consists of two glass bulbs connected by a tube as shown in Fig. 39. One of these bulbs is filled with a volatile liquid (like water or ether) and the rest of the space contains nothing except its vapour. Transfer the whole of the liquid to one of the bulbs (say *B*) and surround the other by a freezing mixture; the vapour in the latter *i.e.*, *A* will condense, thereby decreasing the pressure in *B*, where rapid evaporation will take place. Heat required for change of state is taken from the remaining liquid in *B*, which is thereby cooled. If the experiment is continued for some time the liquid in *B* will be cooled to the freezing point and eventually freeze.

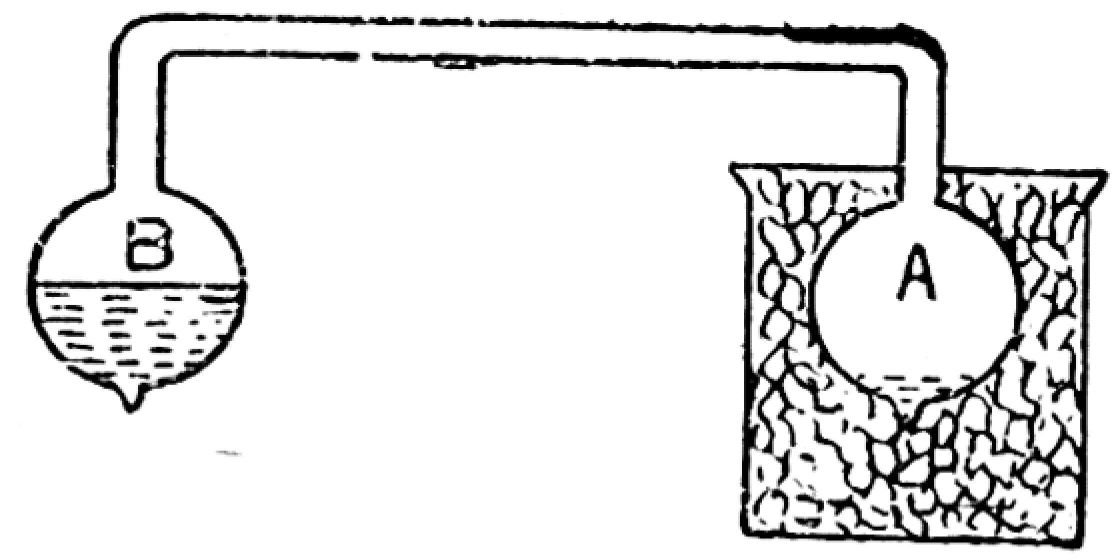

Fig. 39.

212. Applications of Cooling caused by Evaporation.—Every day we come across the applications of this principle. The refreshing effect produced on a hot day by watering a road is due to the cooling produced by evaporation of water. The pores in the earthen pots allow the water to come out to the outer surface where evaporation takes place, and thereby cold is produced and hence water remains cool. Refrigeration, *i.e.* the production of low temperatures is due to the rapid evaporation of a liquid like ammonia. The machines used for the production of artificial cooling are called **Refrigerators.** They are used either to maintain a chamber at low temperature for the storage of perishable articles like fruits etc., or to keep tanks of brine well below 0°C, for the manufacture of ice. In order to make clear the working of the refrigerators we shall briefly explain the action of ammonia ice plant.

The plant consists of the following parts : (*i*) A compressor worked by an engine, (*ii*) two sets of coils, condenser (*I*) and evaporator (*J*), (*iii*) ice-tanks, (*iv*) regulating valve, (*v*) high pressure and low pressure gauges.

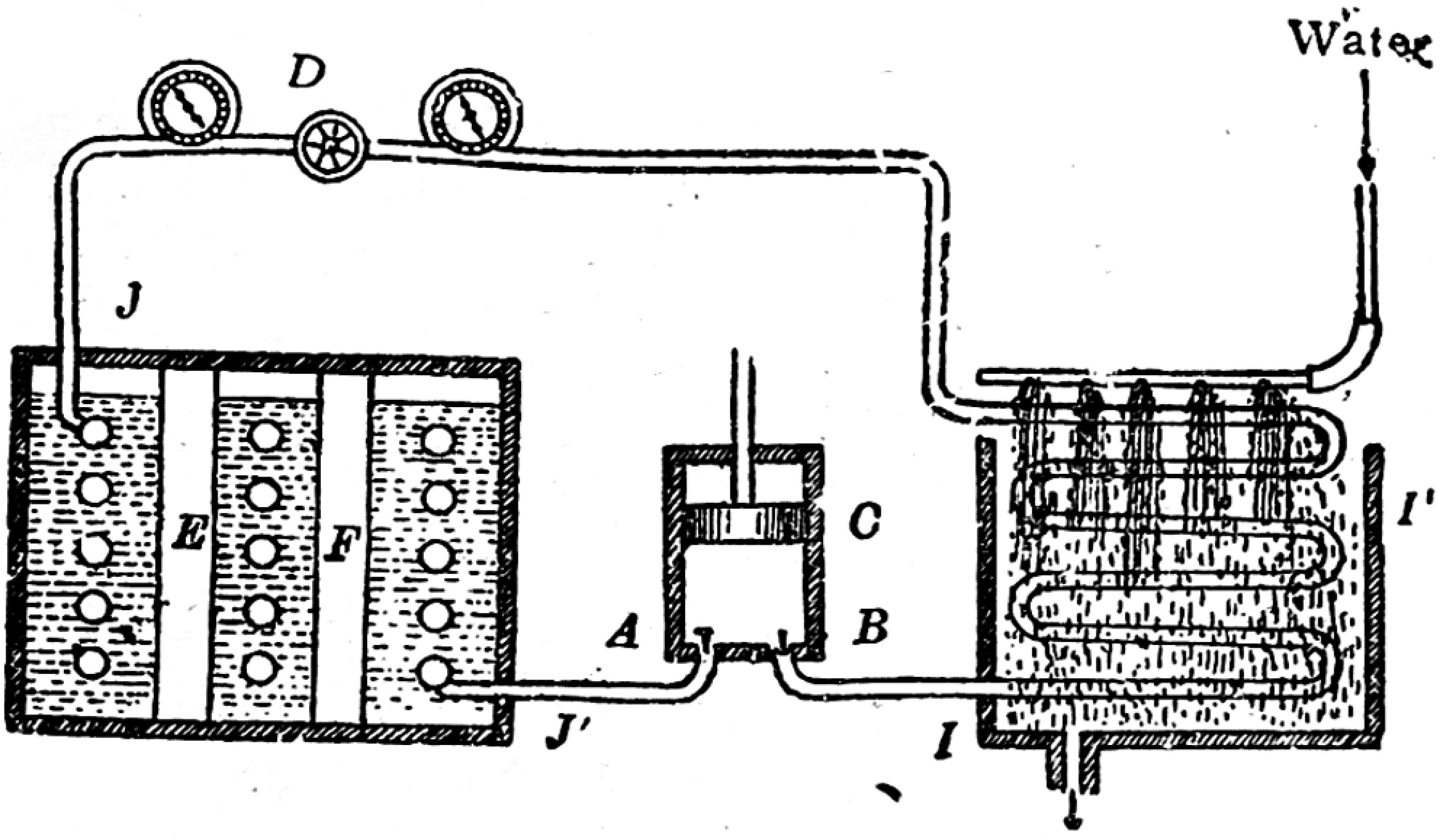

Fig. 40. Ammonia-Ice-plant.

The compressor *ABC* is worked by an engine. The pipe connections at *A* and *B* are provided with valves, that at *A* opening into the

cylinder, and that at B opening out from the cylinder into the condenser pipe I.

During the upstroke of the piston, the pump acts as an air pump, and during the downstroke it acts as a compressor.

The ammonia gas is drawn in at A, is compressed in the cylinder, and is forced under relatively high pressure (about 10 atmospheres) into the coils I, I', which are immersed in cold water. On account of compression the gas (ammonia) gets heated, but the cold water running over the condenser pipes cools it to a temperature, one or two degrees below the room temperature. Due to compression and cooling to the room temperature the ammonia is liquefied.* The liquid ammonia is allowed to escape slowly through a regulating valve D into the coils J, J' immersed in brine (*viz.*, salt solution). The coils are being continually exhausted by the pump ABC, which, as has been said, works during the upstroke as an exhaust pump. As soon as liquid ammonia passes through the valve D into the coils J, J', it evaporates rapidly, cooling thereby the salt solution to a temperature of $-8°$C.† Small tanks full of drinking water are placed in salt solution at places like E and F, shown in Fig. 40. Since the freezing point of water is 0°C., and the salt solution is at $-8°$C., the water in contact with the solution freezes to ice.

As soon as the water is frozen, the tanks are pulled up and the ice removed. They are again filled up with drinking water and placed back in position.

The same ammonia is used over and over again. For this purpose power must be constantly supplied to the compressor. This is generally done by an internal combustion or a steam engine.

The principle of the working of a domestic electric refrigerator is similar to the ammonia ice-plant. It consists of a small compressor run by an electric motor, cooling coils, expansion valve, and refrigerating coils. The substance used may be sulphur dioxide or methyl chloride or ammonia.

EXERCISES

1. An aluminium calorimeter (sp. heat 0·2) of mass 13·1 gm. contains 175 gm. of water at 8°C. If 5 gm. of steam at 100°C. are passed into the water, what will be the final temperature ? Take the latent heat of steam as 536.

Let the final temperature be θ.

Heat given out by the steam $=5(L+100-\theta)=5(636-\theta)$.

Heat taken up by water and calorimeter

$$=175(\theta-8)+13{\cdot}1\times0{\cdot}2\times(\theta-8)$$
$$=(\theta-8)(175+2{\cdot}62)$$
$$=(\theta-8)(177{\cdot}62).$$

*At a temperature of 20°C. ammonia is liquefied by a pressure of 8·5 atmospheres and since the gas in the above case is at a pressure of 10 atmospheres, and is at the room temperature it must be in the liquid condition.

† Brine does not freeze at this temperature.

Therefore $$3180-5\theta=177\cdot62\times\theta-1420\cdot96,$$

or $$182\cdot62\,\theta=4600\cdot96,$$

Therefore $$\theta=\frac{4600\cdot96}{182\cdot62}=25\cdot2°\text{C}.$$

2. Seven grams of ice float in water in a calorimeter of thermal capacity 5 calories. If, when steam 4·5 gm. in weight (at 100°C.) is passed into the calorimeter, the final temperature becomes 50°C., how much water was there in the calorimeter ? (Given that latent heat of steam =540.)

Let the mass of water be M gm.

The heat given out by steam $=4\cdot5\times540+4\cdot5\times50=2655.$

The heat taken up by water and calorimeter

$$=(M+5)50+7\times80+7\times50$$
$$=50M+1160.$$

But *heat gained* = *heat lost*

Therefore $$2655=50M+1160.$$

or $$M=\frac{1495}{50}=29\cdot9\text{ gm.}$$

3. Combustion of 1 lb. of coal generates heat sufficient to raise 8,000 lb. of water through 1°C. How much coal will be required to convert 56 lb. of ice at 0°C. into steam at 100°C. ?

If in place of using grams we use pounds throughout, this will make no difference in the result. Conversion is simply tedious and fruitless.

Heat consumed by 56 lb. of ice in passing into steam would be

$$56\times80+56\times100+56\times536=56\times716\text{ lb. degrees.}$$

Combustion of 1 lb. gives out heat enough to raise the temperature of 8,000 lb. of water through 1°, *i.e.*, 8000 lb. degrees ; therefore the quantity of coal required for our purpose

$$=\frac{56\times716}{8000}=\frac{5012}{1000}=5\cdot012\text{ lb.}$$

4. 50 gm. of steam at 100°C. are passed into a mixture of 100 gm. of ice and 200 gm. of water. Find the rise of temperature produced. The water equivalent of the calorimeter=15 gm.

Ans. 65·2°C.

5. A calorimeter of iron weighing 149·41 gm. contains 251·64 gm. of water at 9°C., 25·83 gm. of steam are passed into the water ; the final temperature rises to 64°C. Calculate the latent heat of steam. (Take the specific heat of iron as 0·113.) *Ans.* 535·8

6. At a certain place water boils at 60°C. It was found there that 10 gm. of steam (at 60°C.) when passed into 600 gm. of water at 4°C., raise the temperature of water to 14·18°C. Find the latent heat of steam at 60°C. *Ans.* 565.

7. One lb. of coal gives out heat enough to evaporate 15 lb. of water at 100°C. How much heat does it give out in burning ?

Ans. 8040 pound-degrees of heat.

8. One lb. of coal gives out heat enough to raise the temperature of 8240 lb. of water through 1°C. How many pounds of coal are required for converting 15·4 lb. of water at 100° to steam at 100°C. ?
Ans. 1 lb.

9. An iron calorimeter contains 220·56 gm. of water at 13°C. When 23 gm. of steam are passed into the calorimeter, final temperature becomes 69°C. Find the weight of the calorimeter. The latent heat of steam is 539·4, and specific heat of iron 0·114.
Ans. 120 27 gm.

10. 20 grams of steam at 100°C. are condensed in a calorimeter containing 200 gm. of water at 10°C. The temperature is raised to 64·43°C. The latent heat of steam is 536. Find the water equivalent of the calorimeter.
Ans. 10·02 gm.

11. How many kilograms of ice at 0°C. can be made from water at 20°C. by the evaporation of a kilogram of liquid ammonia at 0°C. assuming that all the heat lost by the water is taken up by ammonia. Take latent heat of water as 80 calories per gram and latent heat of vaporization of ammonia as 300 calories per gram. *Ans.* 3 kilograms.

12. Why does hot-tea cool more quickly in a saucer than it does in a cup ?

13. Why does the application of Eau-de-colognegi ves feeling of relief to a person suffering from headache ?

CHAPTER VIII

Hygrometry

213. Moisture in the Air.—Evaporation is constantly going on at all temperatures from the surface of oceans, rivers, lakes, and all other wet bodies exposed to the air. The records of observations made during the past several years show that evaporation from free water surface amounts in a year to as much as 88 inches in depth at the equator. Even in London, where the climate is so cold, the evaporation from free water surface amounts annually to about 20 inches. These figures will help the student to form some idea of the amount of evaporation that is going on everywhere. It is on account of this constant evaporation that the moisture is always present in the atmosphere.

To demonstrate the presence of moisture in air experimentally take ice-cold water in a glass tumbler and place it on a table. Its outside will soon become covered with drops of water. It might at first be thought that the drops oozed out through the pores in the sides of the tumbler ; but this is not so, for even a microscope does not show pores in glass. These drops come from the surrounding air. We can prove it beyond doubt by exposing to the air a hygroscopic substance, *e.g.*, calcium chloride, which will soon be found to be damp owing to the absorption of moisture present in air. *The science which deals with the dampness of air is called hygrometry.*

214. Relative Humidity.—We generally remark that the "air is dry" when water evaporates quickly and in consequence a wet cloth dries rapidly, and that the "air is moist" when wet clothes dry slowly. A layman would think that the air when dry contains less moisture than when it is damp. But to a student of Physics these remarks tell that in the first case the air is far from saturation, and in the second case, it is almost saturated. But he cannot say how much water vapour is actually present in the atmosphere. For it is just possible that the actual amount present in the atmosphere in the "dry" condition may be greater than the amount in the atmosphere in the "damp" condition. In other words the state of dryness or wetness of the air depends upon the fact how far removed it is from saturation. This shows that we must consider two things when we talk about the dampness of the air, (*i*) the amount of water vapour actually present in the air and (*ii*) the amount of water vapour required to saturate it at the same temperature. It is *on the ratio of these two* that our idea of dryness or dampness depends and not upon the first alone.

The ratio is called the **relative humidity,** or simply the **humidity** of the air. It is defined as follows :

The relative humidity of the air at any time is the ratio of the mass of water vapour actually present in a given volume of air to the mass of water vapour required to saturate the same volume at the same temperature.

If m be the mass of water vapour actually present in a given volume of the air, and M be the mass required to saturate the same volume, the relative humidity h can be expressed as

$$h=\frac{m}{M}$$

The mass of water vapour actually present in a given volume of air is proportional to the density of the water vapour. And, since the water vapour obeys Boyle's law (even up to saturation) its density is proportional to the pressure. Therefore humidity can be defined also as the ratio of the actual vapour pressure to the saturation pressure at that temperature, *i.e.*,

$$h=\frac{p}{P}$$

where p=the actual pressure and P=the saturation pressure at the same temperature.

215. The amount of water vapour actually present in the air can be determined by a chemical hygrometer.

It consists of an aspirator (Fig. 41) connected to a trap consisting of a bottle containing pumice stones soaked in sulphuric acid. The trap does not permit the moisture to go from the aspirator to the U-tubes. It is connected to two U-tubes containing calcium chloride as shown in Fig. 41. The open end of the second U-tube is stopped with a plug of cotton wool. Before connecting the tubes to the trap they are weighed. The stop-cock of the aspirator is opened and at the same time the cotton wool plug is removed. The water begins to flow out and the air is sucked in. When four or five litres of air have passed, the stop-cock is closed, the U-tubes are disconnected and weighed. The increase in weight gives the amount of water vapour present in the air that has passed. The volume of air is determined from the weight of water that has escaped from the aspirator. Knowing the volume of the air and the weight of the moisture present, we can determine the mass of water vapour present in a cubic metre of air. This gives what is called the **absolute humidity** of the air.

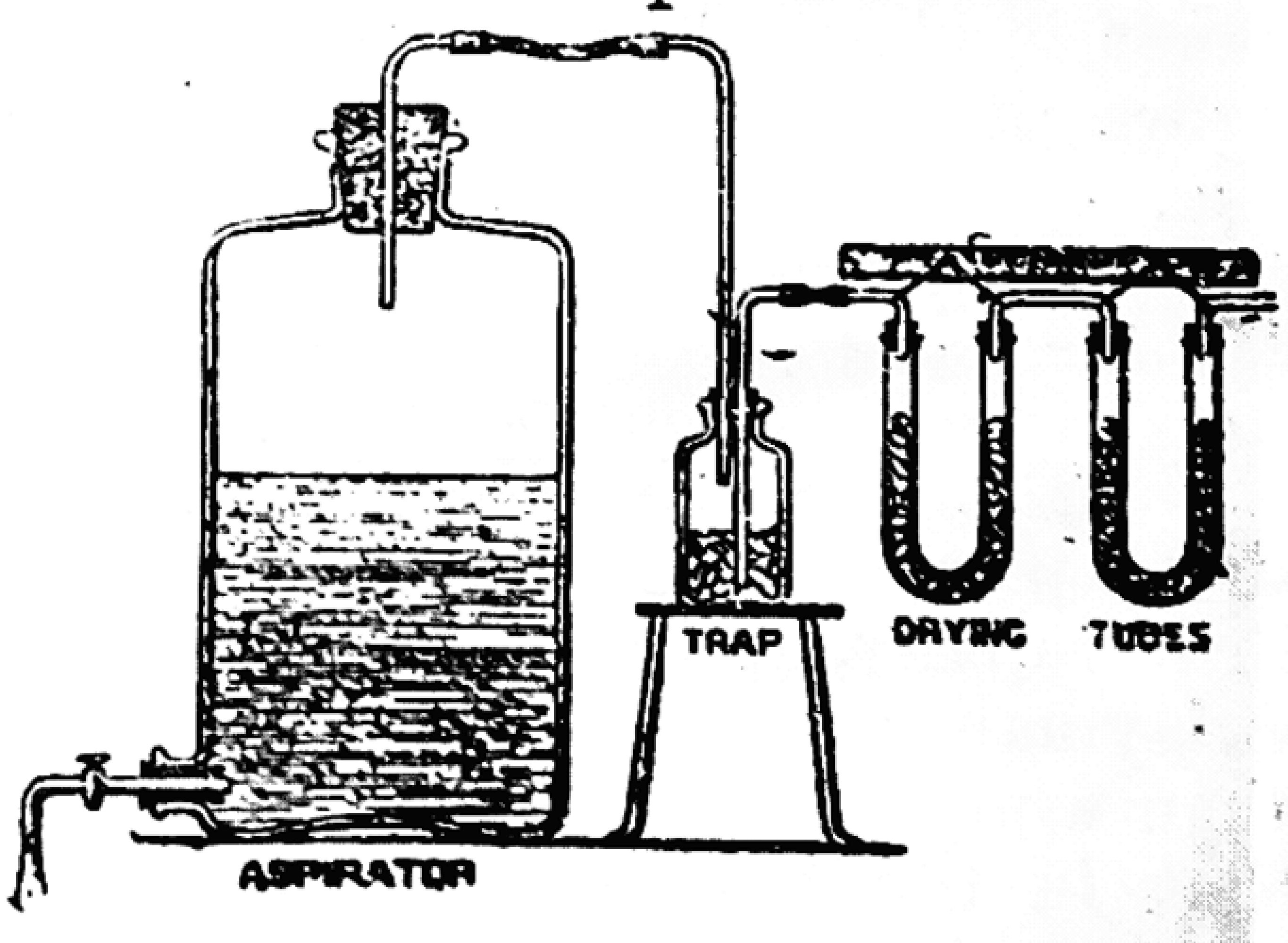

Fig. 41.

We can find from the tables the amount of water vapour required to saturate a cubic metre of air at the temperature of the experiment. *The ratio of the two gives the* **relative humidity.** The relative humidity is expressed sometimes as a fraction and sometimes as a percentage. For instance, suppose a cubic metre of air contains 15 gm. of water

vapour and the mass required to saturate it at the same temperature is 25 gm., then the relative humidity is either expressed as 0·6 or as 60 per cent.

216. Dew-point Hygrometers.—The above method gives the relative humidity accurately, but the method itself is rather tedious. The dew-point hygrometers enable us to determine the relative humidity in a far simpler manner. Before we consider the method of using them let us first understand the principle underlying their action.

The moisture, which is present in the air, is, as a general rule, far from being enough to saturate it. If we cool the air gradually, at a certain temperature the amount of water vapour present is sufficient to saturate it. As soon as this temperature is reached, condensation begins and we say that dew is formed. *The temperature at which vapour actually present in the air is sufficient to saturate it is called the dew-point.*

Now if we determine the dew-point of the air on a certain day, we can, by consulting the tables, find the maximum vapour pressure at the dew-point. This pressure is equal to the actual pressure of the vapour present in the air. We can further find from the tables the maximum pressure at the temperature of the air. The ratio of the two gives the relative humidity. So the method is to first determine the dew-point, and then find from the tables the maximum pressure at the dew-point and the maximum pressure at the actual temperature of the air and take the ratio.

The instruments which are used to determine the dew-point are called **dew-point hygrometers.** We shall consider two such hygrometers only.

217. Daniell's Hygrometer.—It is essentially a cryophorus containing ether and ether vapour only, all the air having been expelled. Inside the bulb A is mounted a thermometer dipping in ether and outside it a metallic strip is fused into the glass. The bulb B is covered with muslin. To find the dewpoint transfer all the ether inside the hygrometer into the bulb A. Pour ether from outside over the muslin. As this external ether evaporates the bulb B is cooled, which causes the vapour inside to condense and the vapour pressure inside the instrument to decrease. As a result of it the ether in the bulb A begins to evaporate, causing its temperature to fall. When the temperature of the bulb A and therefore of the metallic strip falls to the dew-point, the moisture in the air deposits on it and makes it lose brightness. Note the temperature at which the dimming of the strip occurs. Discontinue pouring ether on the muslin and note the temperature at which the dew disappears. The mean of the two gives the dew-point. This hygrometer is not very satisfactory to work with on account of the following disadvantages :—

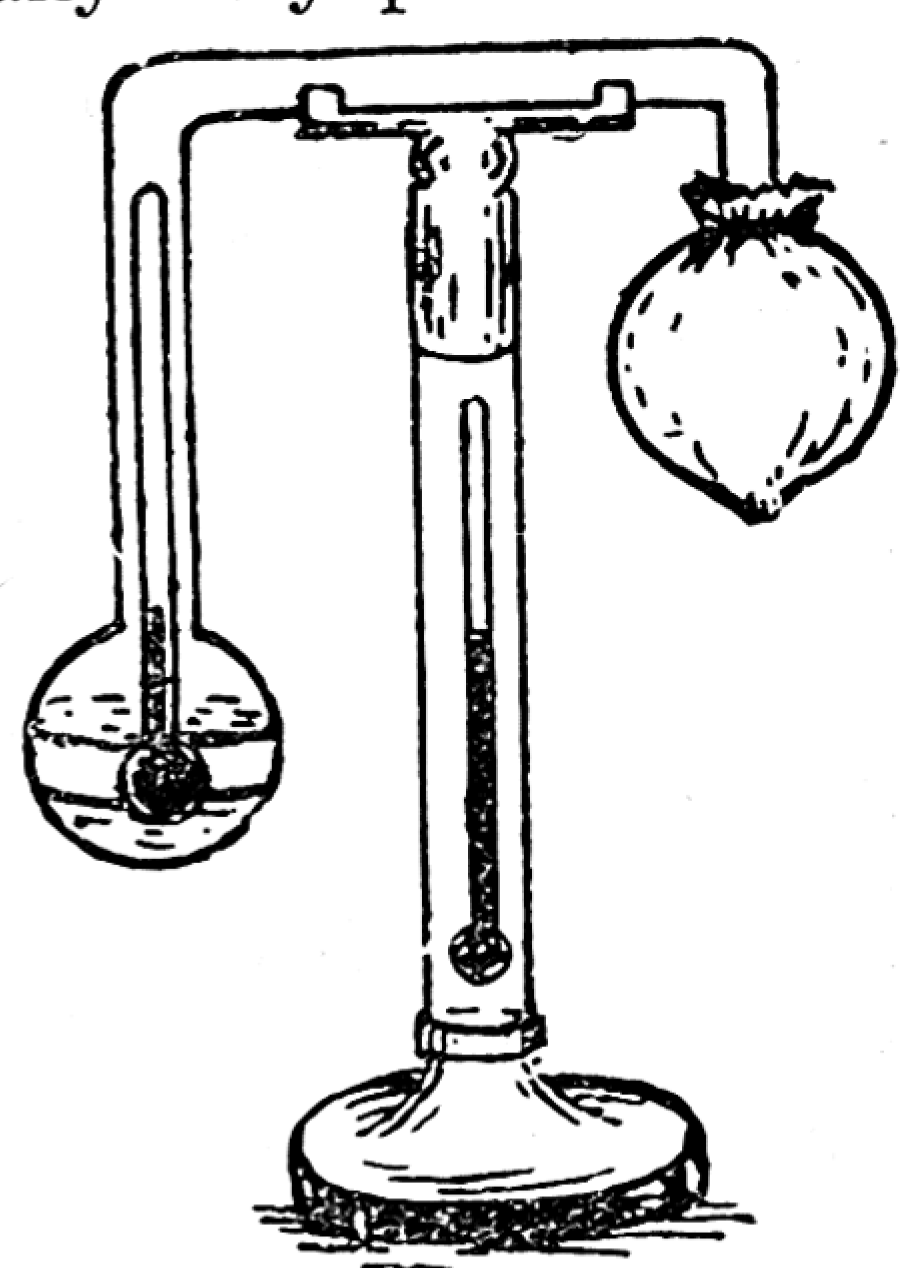

Fig. 42.

(*a*) The rate of cooling cannot be regulated.

(*b*) The temperature of the metallic strip is not the same as that of the liquid inside the bulb, owing to glass being a bad conductor of heat.

(*c*) The temperature at which dew appears or disappears cannot be noted easily.

(*d*) The proximity of the observer disturbs the hygrometric state of the air near the apparatus.

These disadvantages are removed in Regnault's hygrometer, which we shall now consider.

218. Regnault's Hygrometer.—It consists of a glass test-tube, *A*, having the lower portion *B*, made of silver. The outer surface of the silver part is polished. The mouth is closed with a cork having two holes. Through one passes a thermometer and through the other a bent tube. A side tube *CDE* leaves the test-tube near the top. It is connected to an aspirator. There is a similar test-tube mounted on the same stand but it is not connected to the aspirator. To work the hygrometer, ether is poured into the test-tube *AB* and a regulated current of air is passed through the ether with the help of the stop-cock of the aspirator. The air bubbling through the ether causes evaporation which cools the tube. As soon as the temperature of ether falls to the dew-point, the surface of *B* gets dim. This temperature is read with the help of a telescope from a distance so that the breath of the observer may not disturb the hygrometric state. The second tube enables us to know by contrast exactly when the portion *B* of the tube *A* begins to get covered with dew. Now the stop-cock of the aspirator is closed and the temperature at which dew disappears is noted. The mean of these two temperatures gives the dew-point from which the relative humidity is determined. Since the liquid inside the tube *A* is continuously agitated, it has always the same temperature throughout, and further, since the liquid is in direct contact with the silver part *B*, its temperature is the same as that of the liquid inside. This hygrometer is very elaborate; consequently it takes rather a long time to be set up, and moreover, it requires a good deal of attention. Hence it is not used by the meteorologists. They use the wet and dry bulb hygrometer, which is extremely simple, and depends on the principle of cooling caused by evaporation.

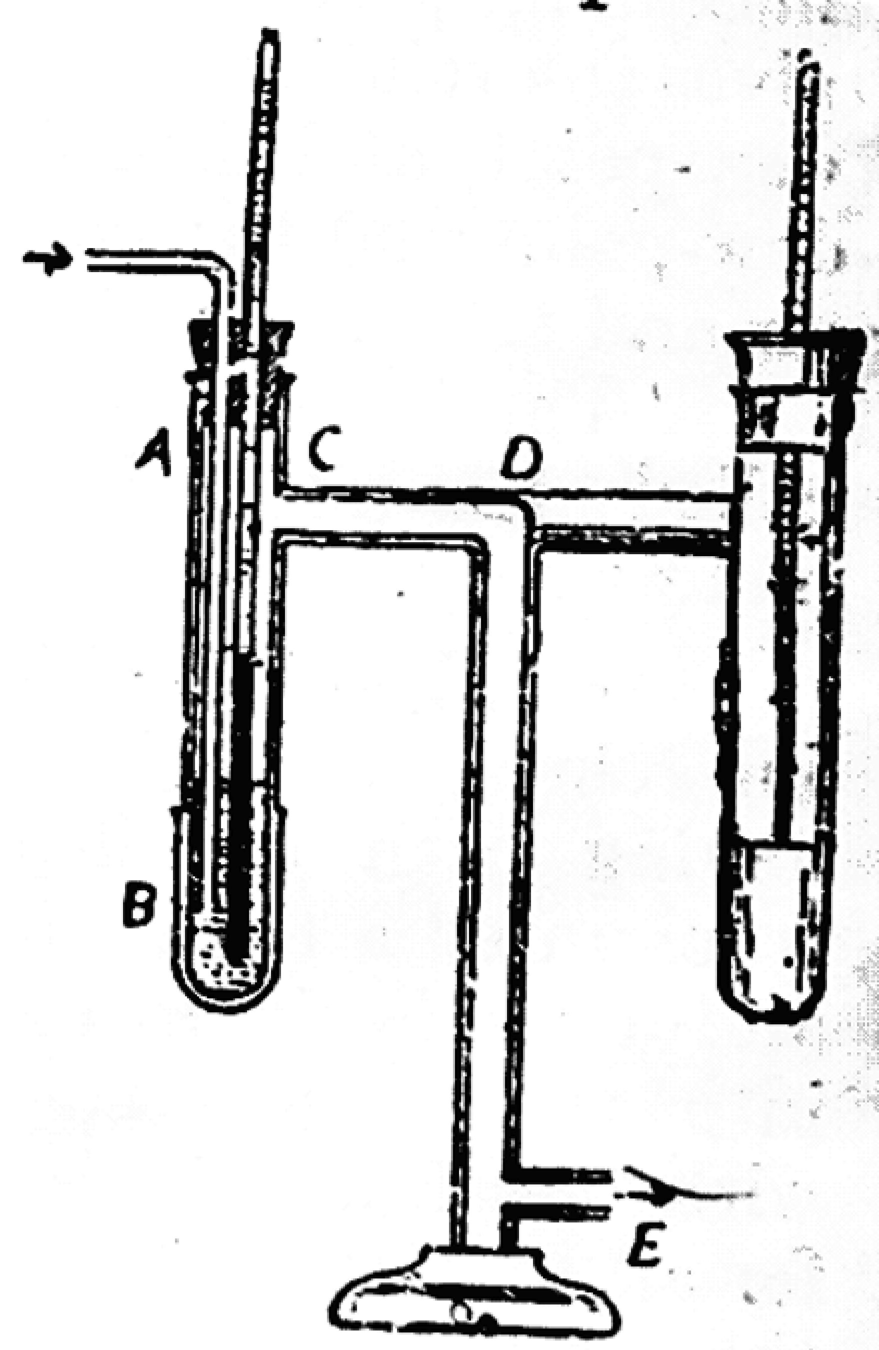

Fig. 43.

219. The Wet and Dry Bulb Hygrometer.—It consists of two thermometers mounted side by side. The bulb of one of them is wrapped with a muslin piece kept moist by dipping its lower end in water in a small beaker. The temperature of this thermometer falls, owing to the cold caused by evaporation. The difference between the reading of the two thermometers depends upon the state of the air. If it be dry the evaporation is brisk and the difference is great whereas if the air be damp the difference is comparatively small. In a general

sort of way it may be said that the greater the difference between the readings of the thermometers, the lower is the dew-point.

If the atmosphere is saturated with vapour there will be no evaporation, and hence the readings on both the thermometers will be identical. In this case atmospheric temperature itself would be the dew-point.

Tables have been prepared by means of which the relative humidity can be determined from the readings of the dry bulb thermometer and the difference of temperature between the dry and wet bulb thermometers. For detailed information about the tables the student should consult some book on Practical Physics.

220. Air Conditioning.—It is not unusual in our country to have as great a difference as 100°F. between the winter and summer temperatures. When the temperature is low we feel chilly and wish to heat our rooms and when it is high we like to cool them. It is only when the temperature of our rooms is about 75° to 80°F. in summer and 65° to 70°F. in winter that we feel comfortable. It was hardly 30 years back when it was realised that the feeling of comfort depended as much on humidity as on temperature. Let us see why ? Each one of us under normal conditions gives about 63 gm. of water vapour every hour. On a "muggy" day the rate of evaporation from our bodies is much less than this figure because the air is at or near the saturation point. Hence on such a day in summer even when the thermometer reads 90°F. we experience much greater discomfort than we do on a dry day at 100°F. We feel bad on a muggy day in spite of its comparatively low temperature because the higher humidity retards evaporation of moisture from the skin and so prevents the lowering of our body temperature. In a crowded room where the air becomes saturated with water vapour the evaporation from our skin stops and we feel the room to be *stuffy*. When the air is dry the evaporation from the pores is rapid so much so that sometimes the moist surfaces inside the nose and throat tend to dry and produce a feeling of irritation. In such environments the skin dries and cracks and the general health is impaired. People inhabiting such dry rooms sleep poorly, suffer headaches and are susceptible to colds. *Hence the desirability of controlling the humidity in our rooms.* It may be pointed out here that even when the humidity out of doors is very great the air indoors may be dry—perhaps too dry for good health. It is now agreed that the humidity in our rooms should be in the neighbourhood of about 50%. If it falls below 35% or rises above 80% conditions arise which are dangerous to health.

In case the air is saturated with water vapour, it is necessary to de-humidify it, otherwise it gives a feeling of stuffiness. The process of de-humidifying consists in mixing dry air and saturated air in proper proportions. To make air dry the saturated air is cooled by means of cooling plants, thereby making it deposit its moisture. On coming out the air is practically *dry*. It is then mixed with saturated air and the mixture is supplied to the rooms.

To have a comfortable feeling we should, therefore, not only cool our rooms in summer or heat them in winter but also control humidity. *The process of regulating the humidity, temperature, purity and circulation*

of air in a factory, a public building or a private house is called **air conditioning.**

There are many systems of air conditioning but in principle they are all similar. Air is sucked into a large duct by means of a blower and is cleaned, warmed or cooled to the required temperature and is brought to the correct relative humidity and is then circulated through the rooms.

EXERCISES

1. The dew-point on a certain day was found to be 12°C., the temperature of the air being 16·5°C. Find its humidity.

The maximum vapour pressure at 12°C.=10·46 mm. of (mercury), at 16°C.=13·64 mm. and at 17°C.=14·42mm.

The actual vapour pressure at 16·5 is equivalent to 10·46 mm. (of mercury). The max. pressure at 16·5° is 14·03 mm.

$$\therefore \qquad h=\frac{10{\cdot}46}{14{\cdot}03}=74{\cdot}5 \text{ per cent.}$$

2. On a certain day when the barometer was 760 mm. high, the temperature of the air was 20°C. and the relative humidity, 50 per cent. What was the pressure due to air alone ? The saturation pressure at 20°C., is 18 mm. (of mercury).

Since the relative humidity was 50 per cent., the actual vapour pressure was 9 mm. As the whole pressure was 760 mm. the pressure due to air alone was 751 mm.

3. Calculate the weight of 10 litres of air saturated with water vapour at 18°C. and 750 mm. pressure and compare it with the weight of 10 litres of *dry* air at the same temperature and pressure.

The maximum vapour pressure at 18°C. is 15·46 mm. of mercury.

Let us first find the weight of saturated air.

The pressure of air alone is 750−15·46=734·54 mm. We *know that 1 litre of dry air at N.T.P. weighs 1·293 gm. and that weight of 1 litre of water vapour under the same conditions is* $\frac{5}{8}$ *of the weight of 1 litre of dry air.*

Since $\frac{PV}{T}=\frac{P'V'}{T'}$, substituting the values we get

$$\frac{734{\cdot}54\times10}{291}=\frac{760\times V'}{273},$$

$\therefore$ V' or the volume of 10 litres originally at 734·54 and 18°C.

would at N.T.P. be $=\frac{10\times734{\cdot}54}{291}\times\frac{273}{760}$ litres.

$$\text{The weight of the air alone}=\frac{10\times734{\cdot}54}{291}\times\frac{273\times1{\cdot}293}{760}\text{ gm.}$$

$$=11{\cdot}73 \text{ gm.}$$

$$\text{The weight of the water vapour}=\frac{10\times15{\cdot}46}{291}\times\frac{273}{760}\times\frac{5}{8}\times1{\cdot}293$$

$$=0{\cdot}153 \text{ gm}$$

$\therefore$ Total weight $=11{\cdot}73+0{\cdot}153=11{\cdot}883$ gm.

Now let us calculate the weight of dry air.

10 litres of air at 80°C. and 750 mm. pressure would at N. T. P. be

$$=\frac{750\times10}{291}\times\frac{273}{760}\text{ litres.}$$

$$\text{The weight}=\frac{750\times10}{291}\times\frac{273}{760}\times1{\cdot}293\text{ gm.}$$

$$=11{\cdot}98\text{ gm.}$$

4. What is the relative humidity on a day when the temperature of air is 14°C. and the dew-point is 5°C. ? What is the actual vapour pressure on that day ?

Given that the max. vapour pressure at 14°C.$=11{\cdot}9$ mm. at 6°C. $=7{\cdot}0$ mm. at 4°C.$=6{\cdot}1$ mm. *Ans.* 55 per cent ; 6·55 mm.

5. Find the relative humidity if 20 litres of air at 18°C., when passed through the drying tubes increase their weight by 0·198 gm.

Given that 15·2 gm. of water vapour are required to saturate a cubic metre of air at 18°C. *Ans.* 65 per cent.

6. A quantity of hydrogen collected over water occupies 350 c.c., the temperature being 15°C. and the pressure 747·7 mm. Calculate the volume of dry hydrogen at N.T.P.

(The max. pressure of water vapour at 15°C.$=12{\cdot}7$ mm.)

Ans. 320·7 c.c

7. Find the mass of 5 litres of moist air at 18°C., the dew-point being 5°C. and the barometric height 759·5 mm.

Given the max. vapour pressure at 5°C. is 6·5 mm.

Ans. 6·04 gm.

CHAPTER IX

Transmission of Heat

221. Have you ever watched your mother while she boils milk for you in the kitchen ? Let us see what she does. She takes a cooking pot made of aluminium, copper, or some alloy, pours milk into it and places it over fire. The vessel gets hot very much earlier than the milk. She, too, feels warm although she may be sitting at a distance from the fire. The method by which the cooking pot gets hot is called **conduction**, the method by which milk gets heated is called **convection** whereas method by which your mother receives heat is called **radiation**. We shall show later that no material medium is necessary for this third method of transmission of heat. To make clear the distinction between conduction and convection imagine that at a mango party you are sitting with your friends in a row with mangoes lying in a bucket near one end. One method of distribution of mangoes is that the person nearest the bucket takes out mangoes and keeps one with him and passes on the rest one by one to the man next to him, who in turn keeps one with him and passes the rest on to the third man and so on. In this method the men do not change their positions. It is only the mangoes that move. Note the man nearest the bucket gets a mango first and then the second man and so on. The second method is that each man comes to the bucket and takes a mango from there, the nearest man takes a mango first of all and goes away, then the second man takes it and goes away and so on. In this case the carrier moves along with mangoes. Something similar happens in the transfer of heat by conduction and convection. We shall explain these ideas fully in what follows.

222. Convection.—Take a round-bottomed flask and fill it partly with water. Throw in it some pieces of solid colouring matter, say crystals of potassium permanganate. Heat it over a pointed flame. The water at the bottom, which becomes coloured on account of contact with the colouring matter, gets heated, and hence becomes lighter. It therefore, rises upward through the middle of the flask. The cold water at the top, being heavier, comes downward along the sides, gets heated and rises upward. Upward currents of heated water and downward currents of cold water are thus set up. These currents are called *convection currents*.

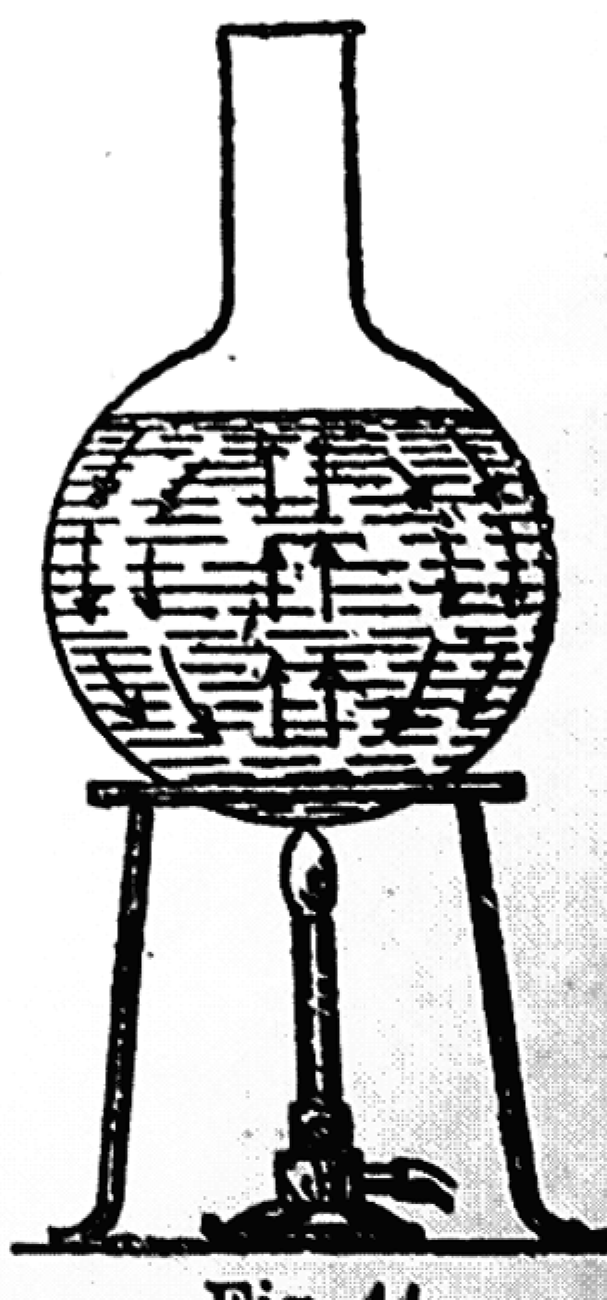

Fig. 44.

It is evident from the above experiment that this mode of transmission can occur only in liquids and gases, because convection currents cannot be set up in solids. As a matter of fact this is the only important mode of heating fluids. Remember, *Convection is the transmission of*

heat from one part of the body to another by the actual motion of the heated particles.

What has been said above about water is equally true of gases. They also are heated by convection currents. Ventilation is merely an application of the convection currents in gases. The air in a dwelling room is always warmer than the free air outside, and hence it rises upwards and passes out through the ventilators while cold fresh air comes into the room through the doors and windows to take its place. Trade-winds, land and sea-breezes, are all convection currents in the atmosphere.

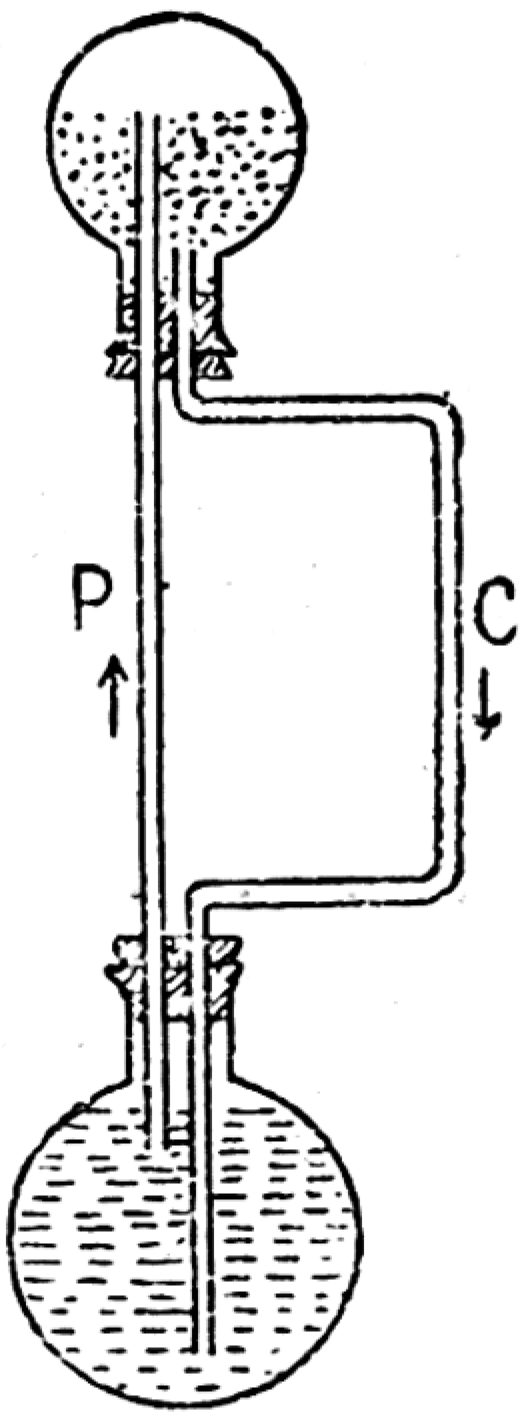

Fig. 45.

How the convection currents in liquids are used practically for heating rooms can be illustrated experimentally in the laboratory by the apparatus shown in Fig. 45. When the flask *B* is heated from below, the water begins to circulate in the direction of the arrows. To make the movement visible, colour the water in the upper flask. The flask *B* corresponds in the actual arrangement to the boiler in the basement, and the pipe *C* to the pipe of the radiators in the rooms. As water passes through the various rooms its heat is radiated from the surface of the pipe and by the time it reaches back the boiler, it is cool. The actual arrangement is rather complicated on account of the necessity of using safety valves and overflow pipes, etc.

223. Conduction.—Hold one end of an iron rod in a Bunsen flame. After a short time the other end becomes too hot to hold. If we touch the rod at different parts we find that the nearer is the part to the end in the flame, the hotter it is. In this case it is heat itself which moves from the hot to the cold end and not the heated particles. Such a method of transmission of heat is called *conduction*. It is defined as follows :

Conduction is the mode of transmission of heat from particle to particle in the direction of the fall of temperature, the particles themselves remaining at rest.

Conduction as will be seen later on, takes place mostly in metals. The facility with which heat can flow along a body is a measure of its conductivity. Various bodies differ enormously in their power of allowing heat to pass through them. For instance, an iron rod allows heat to flow alnog it much more easily than a glass rod, which means that iron is a much better conductor than glass. Before we explain how to measure the conductivity of a substance let us see how to define it. Take a cube of a certain substance (Fig. 46). Heat one of its faces. Due to conduction, the opposite face will also become heated. The

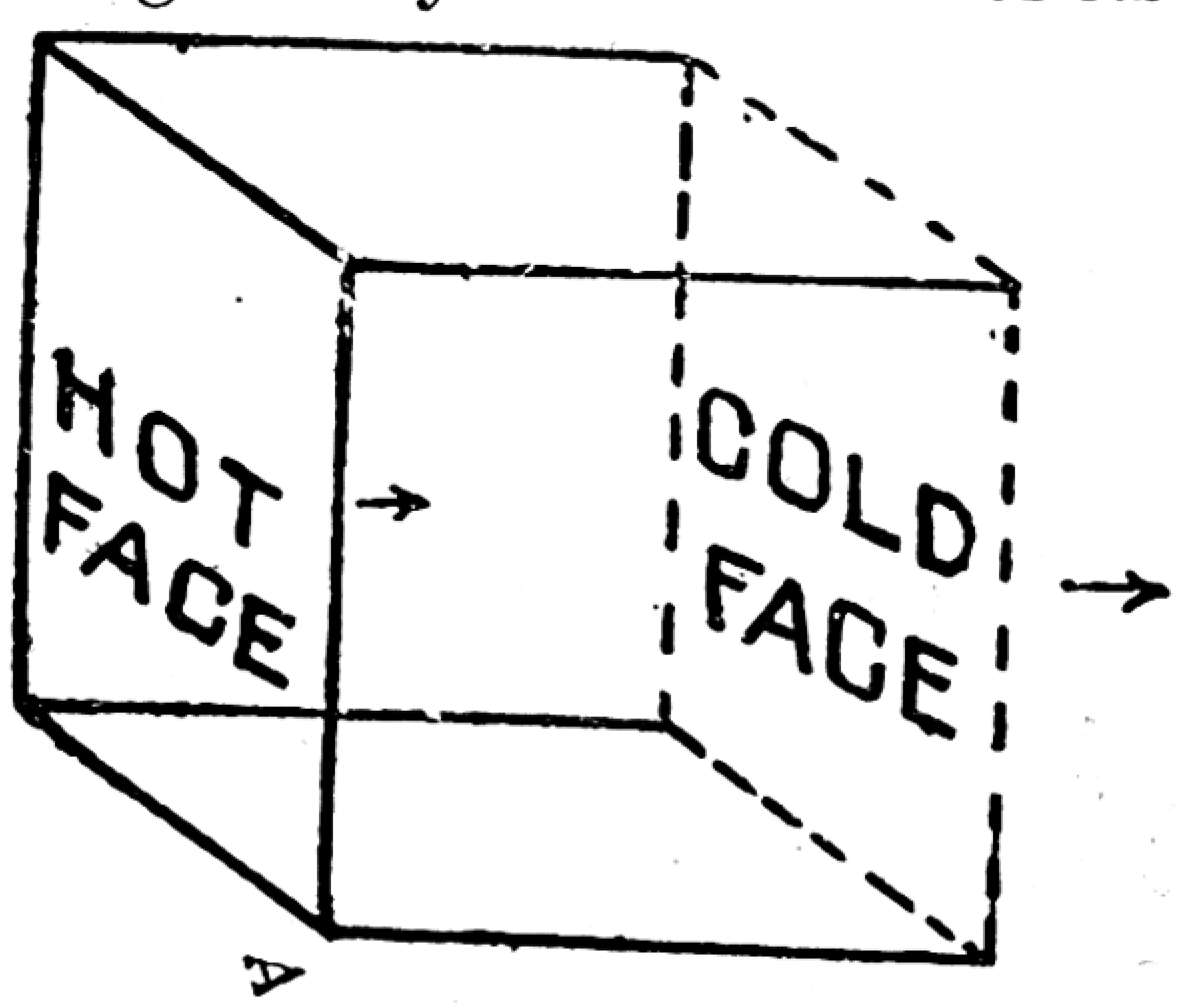

Fig. 46.

amount of heat that flows through the cube to the cold face depends upon the conductivity of the substance. The better the conducting power, the greater the quantity of heat that passes through. But at the same time we must remember that the amount of heat flowing through depends upon the area of the face : the greater the area, the greater the amount of heat passing through the cube. It also depends upon the difference of temperature between the two faces ; the greater the difference, the greater the flow of heat.

The thickness of the cube also affects the flow of heat ; the greater the thickness the less the flow. Moreover, the quantity of heat passing through the cube is proportional to the time : for instance, in two minutes twice as much heat will flow as in one minute. Mathematically we can express all the above-said conditions as

$$Q=\frac{KA(\theta_1-\theta_2)t}{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

where Q stands for the amount of heat that flows through the cube, A for the area of the face, θ_1 for temperature of the hot face, θ_2 for the temperature of the cold face, t for the time for which heat flows, x for thickness and K for the constant depending upon the nature of the substance.

The constant K is called the coefficient of thermal conductivity or, more simply, conductivity.

Formula (a) can be written as $K=\dfrac{Qx}{A(\theta_1-\theta_2)t}$ . . . . . . (b)

This relation helps us to find the conductivity of a material. If x be 1 cm., A be 1 sq. cm., $\theta_1-\theta_2$ be 1°C., and t be 1 second then $K=Q$. This shows that :

Conductivity of a substance is equal to the amount of heat that passes in one second through a centimetre cube whose faces are maintained at a difference of temperature of 1°C.

The quantity $\dfrac{\theta_1-\theta_2}{x}$ is called the *temperature gradient* ; it gives the rate at which temperature changes with distance in the direction of the flow of heat.

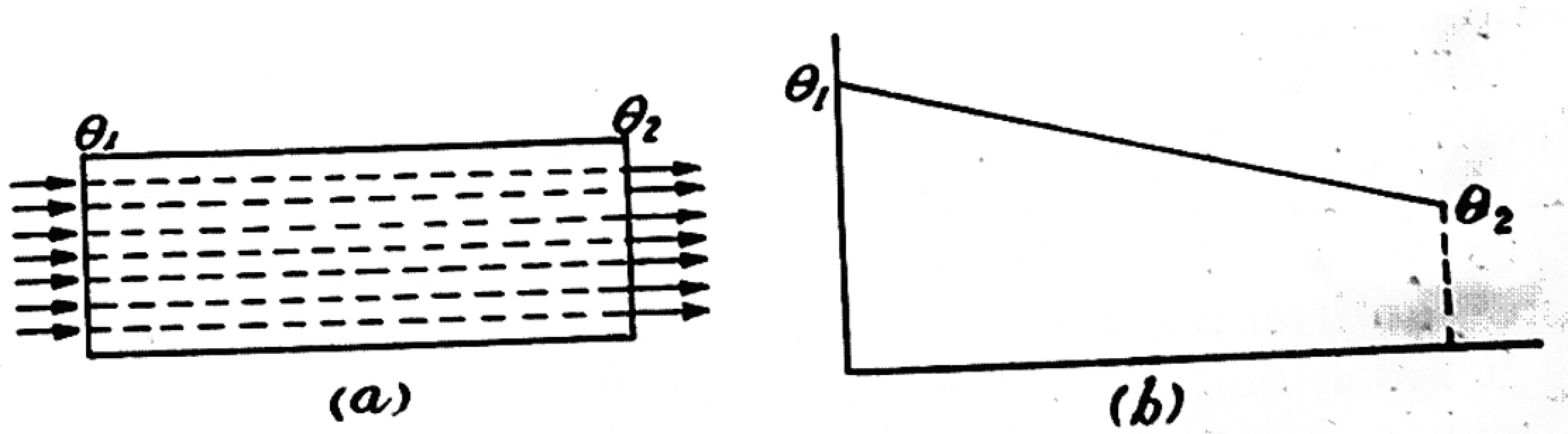

Fig. 47.

In the definition of the thermal conductivity given above it is taken for granted that the temperature gradient along the whole length x of the cube is uniform. In practice, this condition is difficult to realise, for it implies that there is no loss of heat at the sides.

Consider a bar whose ends are kept at temperatures, θ_1 and θ_2, θ_1 being greater than θ_2. If no heat is lost on the sides that is to say there is no loss due to radiation or conduction and convection—a condition impossible of fulfilment, the rate of flow of heat is uniform and is represented diagrammatically in Fig. 47 (*a*) and temperature gradient in Fig. 47 (*b*).

If the loss at the sides cannot be avoided the rate of flow of heat through a section at a distance x decreases with increase of x as shown in Fig. 48 (*a*). The temperature gradient at a point is now represented by $\frac{d\theta}{dx}$. Since K and A remain constant and rate of flow decreases, the temperature gradient must decrease with increase of x.

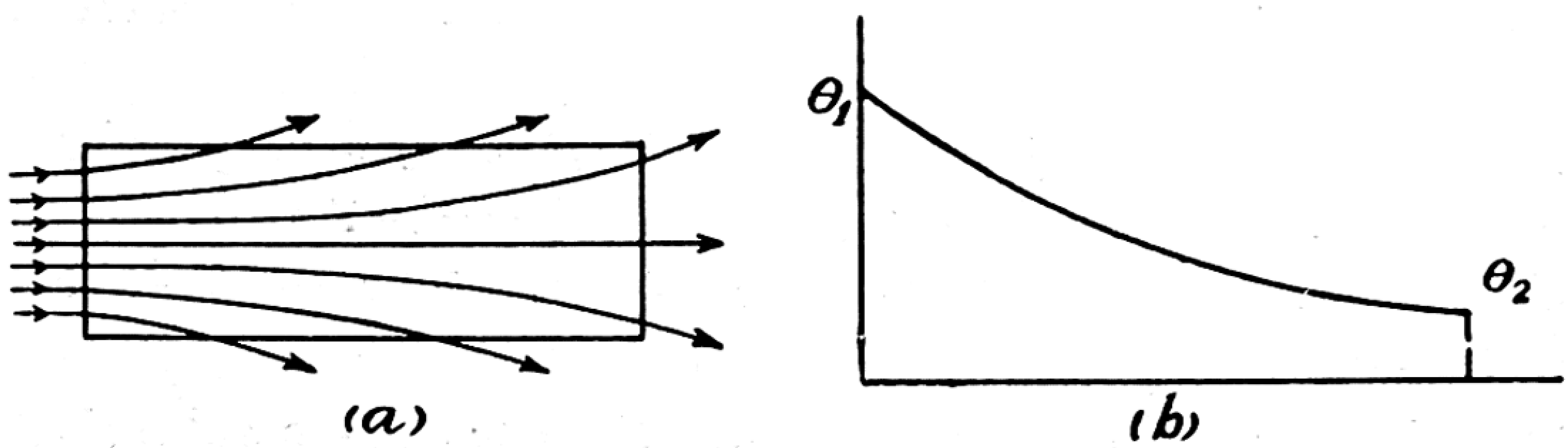

Fig. 48.

224. Flow of Heat along a Bar.—Let us discuss the case of flow of heat along a bar a little more in detail. The temperature distribution that we have talked of above holds good only when the bar has reached a steady state. How does the temperature change during the warming up and on what factors does it depend will now be explained briefly. Take a bar of iron, about a metre in length and drill holes in it at intervals of 10 cm. Fill them up with mercury. Heat its one

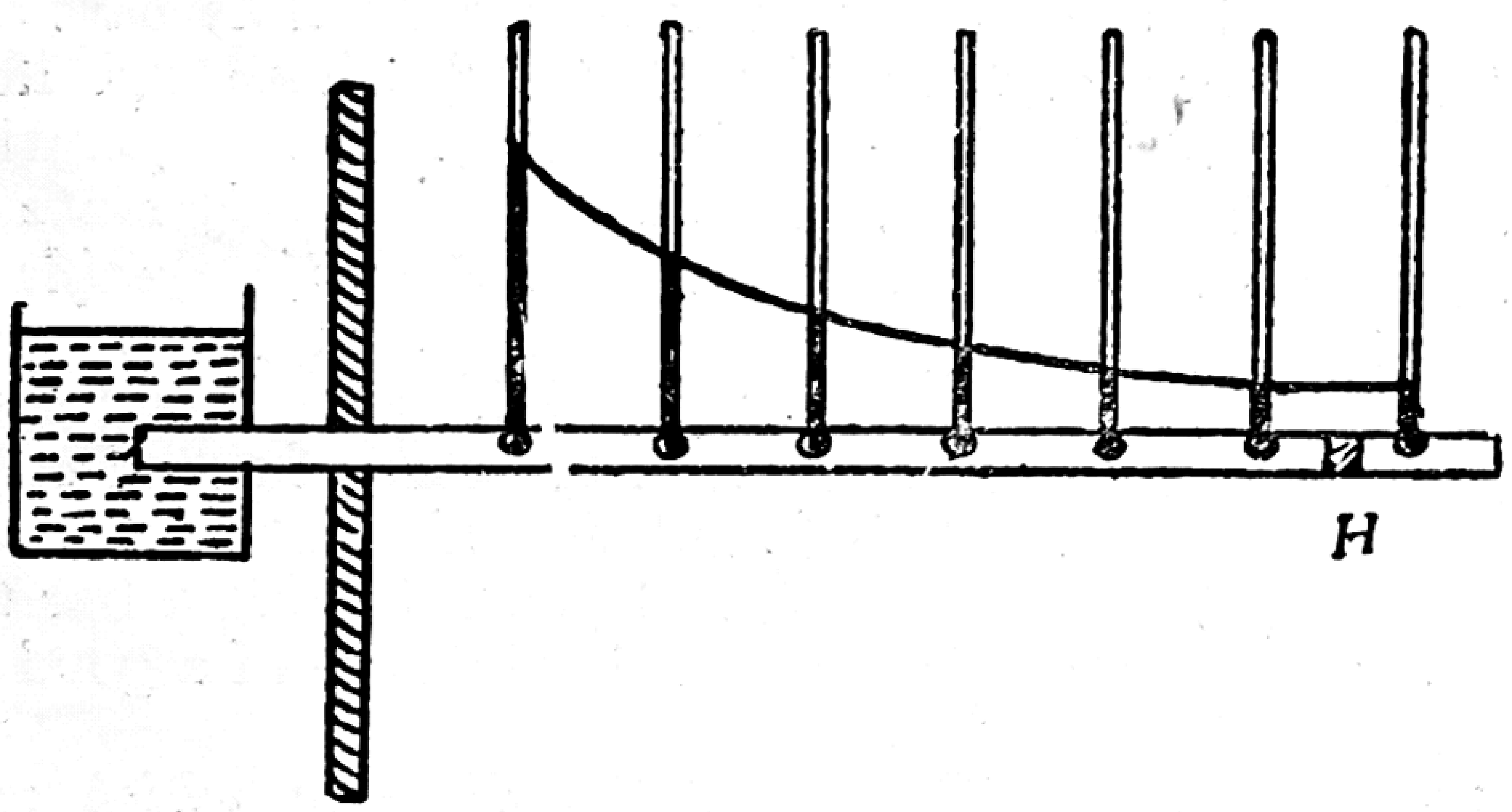

Fig. 49.

end, to a constant temperature, by immersing it, say, in a bath of molten lead. Place thermometers in the holes. As the bar warms up the thermometers begin to register a rise. Of course, the thermometer nearest the heated end is the first to indicate a rise in temperature, and is followed in succession by others. The end in the molten lead rapidly attains the temperature of the source. The part next to this end receives heat by conduction, absorbs part of this heat to increase its own temperature, loses a small portion from its exposed surface due to radiation

and convection, and passes on the rest to the adjacent part. The same thing happens with the next part, and so on. When this state of affairs has gone on for a sufficiently long time,* a stage is reached when a part like H ceases to absorb any more heat for raising its own temperature, but loses by radiation, etc., the whole of the heat that it receives. The readings of the various thermometers at this stage will cease to show any further rise and will be as shown in the curve (Fig. 49) which corresponds to curve in Fig. 48 (*b*). Since in the steady state, the thermometers show no rise of temperature, it must not be supposed that heat is not being absorbed at the hot end. As a matter of fact the bar as a whole is above the temperature of the atmosphere and is therefore losing as much heat on account of radiation and convection at the lateral surface and at the end as it receives at the hotter end.

This state, when there is no more absorption of heat to raise the temperature of any part of the rod, is called the **stationary state.**

The stage previous to the steady or stationary state is called the **variable state.**

225. The question now arises what effect does conductivity have on the flow of heat along a bar ? If the material of the bar is a good conductor it will allow heat to flow along it easily, so that the temperature of a part like H at a given distance from the hot end will be higher than what it will be when the material is a bad conductor, for in the latter case much of the heat will be lost before it reaches this part. But it should be noted that the rise of temperature of the part H during the time the bar is in the variable state depends upon the specific heat in addition to the conductivity of the material. To study the effect of the specific heat consider two exactly alike bars of different materials. The rise of temperature in that rod whose specific heat is low will be higher for the absorption of the same quantity of heat than in the case of the other bar. But when the steady state has been reached, *since there is no further rise of temperature, the specific heat of the material does not produce any effect.* So we see that the remark which we made above, that the temperature of the part H will be higher in the case of a good conductor than in the case of a bad conductor, is meant for the steady state. It is found that if other conditions are the same, the length (L) which we have to travel along a bar to attain a certain fall of temperature is greater in the case of a good conductor than in the case of a bad conductor.

It can be shown theoretically that *the distance which we have to travel along a rod to attain a certain fall of temperature is proportional to the square root of its conductivity.*

We can express it as

$$L \propto \sqrt{K}$$

or

$$K \propto L^2.$$

This relation forms the basis of an experiment for comparing the conductivities of different substances.

*It may be some hours.

226. Ingen-Hausz's Experiment.—Rods of different materials but of exactly the same dimensions are fixed into one of the sides of a rectangular metallic trough (Fig. 50) which is filled with water. The rods are coated with wax and the water is made to boil. The wax will be seen to melt on each rod. When no more wax melts that is to say, when the steady state is reached, measure off the length of the melted portion in each case. The conductivities will be proportional to the squares of these lengths.

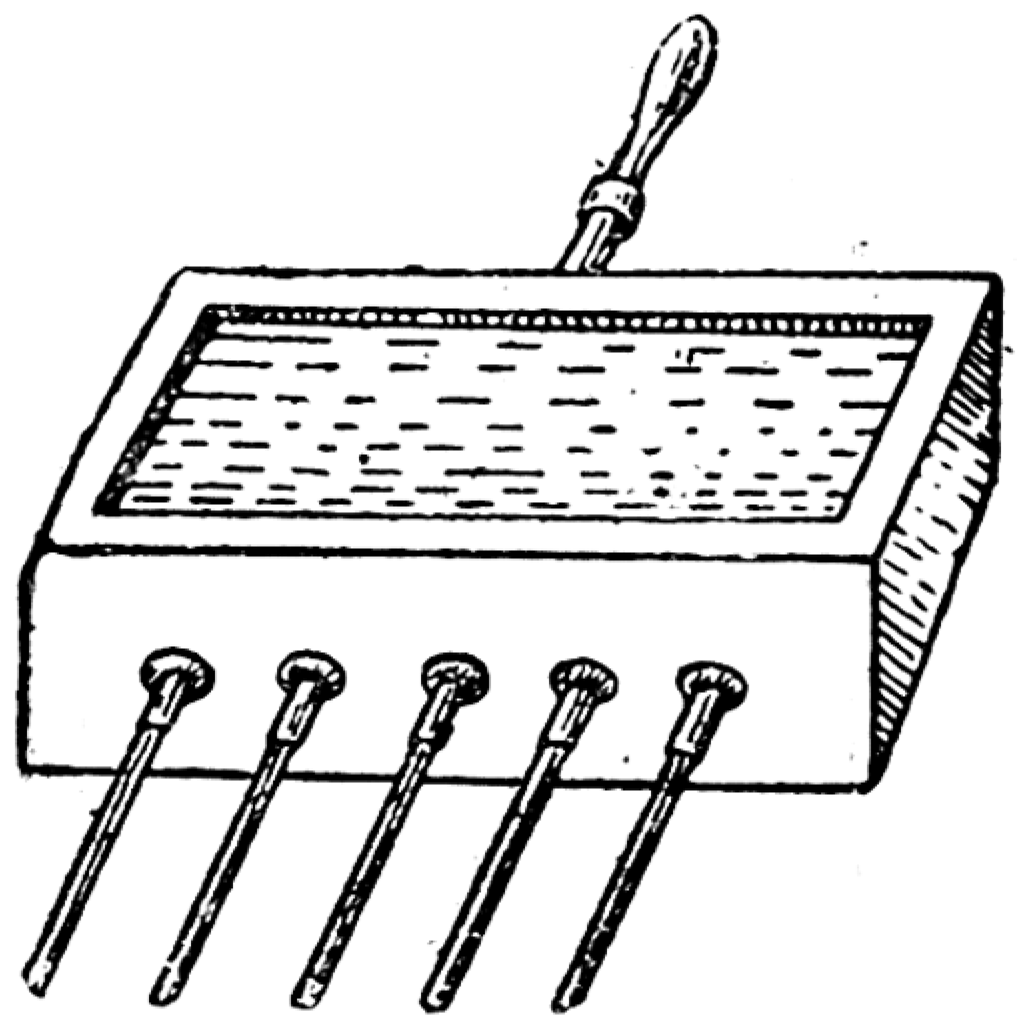

Fig. 50.

Note, the quickness with which the wax melts is no sure guide to conductivity, for the quickness depends also on the specific heat. Hence we must measure lengths only when the steady state has been reached.

As a result of experimental determination it is found that substances differ enormously in their conducting powers. We know this also from our every day experience. Who does not know, for instance, that wood or porcelain is a bad conductor of heat whereas metals are good conductors ?

226a. Determination of the Conductivity of a good Conductor by Searle's Method.—In this method an attempt is made to reduce the loss of heat at the sides by taking a thick bar whose cross section is sufficiently large for its length and by polishing its surface and surrounding it with felt. For a copper bar of this type, the ends of which are maintained steady at two temperatures, we can suppose the temperature gradient to be uniform. If we measure the temperature gradient at one part of the bar and the rate of flow of heat at another part the formula can be used for the determination of the conductivity of the material of the bar.

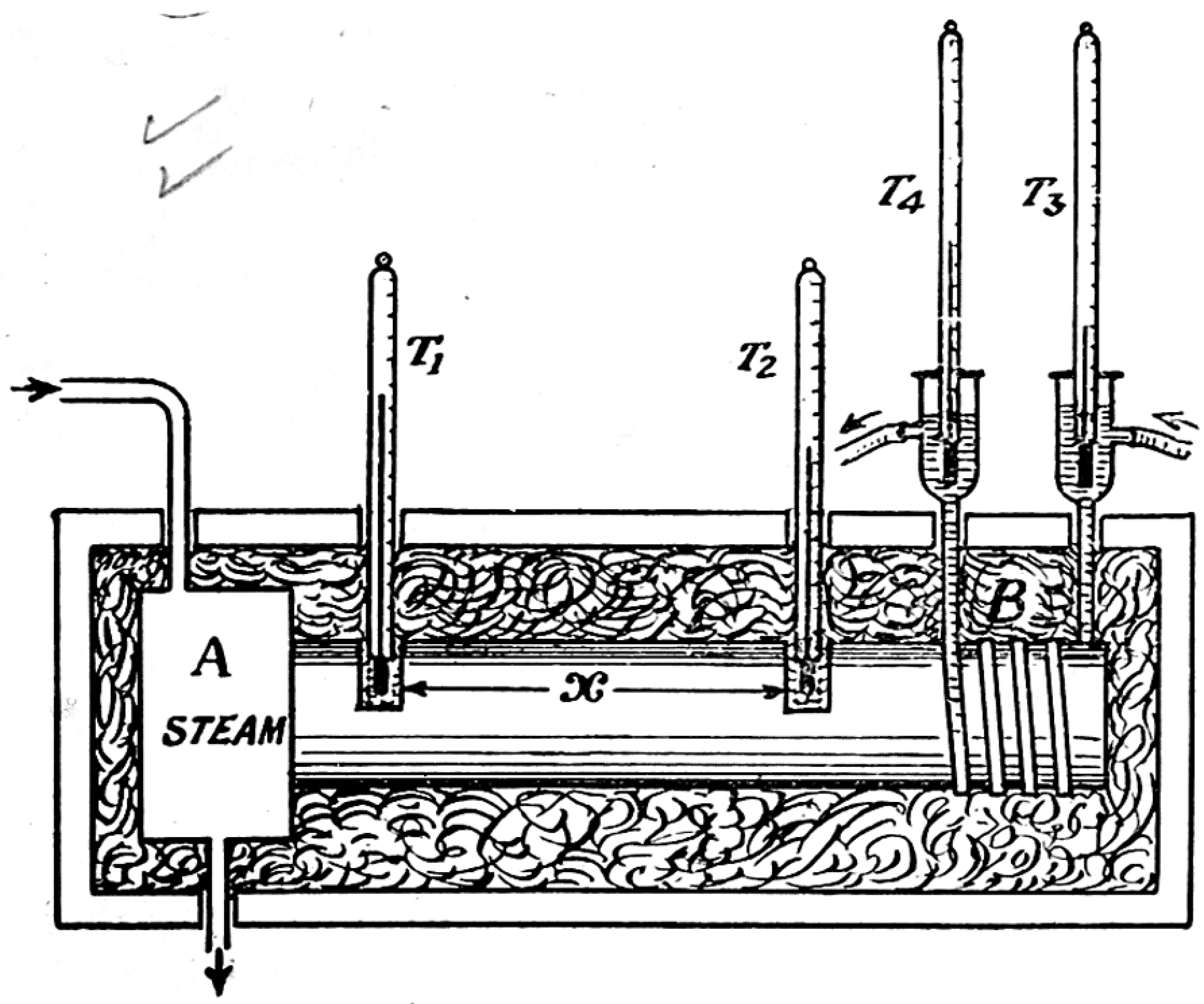

Fig. 50 (*a*).

A bar of copper highly polished about 4 to 5 cm. in diameter and 20 to 25 cm. in length is fitted with its one end into a cylindrical chamber *A* [Fig. 50 (*a*)] through which steam can be passed. Round the other end a spiral of copper tubing *B* is fixed, in which a steady stream of cold water circulates. The temperature of water as it enters and leaves the spiral is indicated by thermometers T_3 and T_4.

Two holes are drilled in the bar a known distance apart and thermometers T_1 and T_2 reading to $\frac{1}{10}$th of a degree are placed in them. The bar is lagged with felt to reduce the heat losses to a minimum.

Steam is passed through the chamber A and the flow of water through the spiral is adjusted so that when the steady state is reached the difference of temperature between the water as it enters and leaves the spiral is 7 to 8°C.

When the steady state is reached suppose m grams flow through the coil in t seconds and the initial temperature of water is θ_3, final temperature is θ_4, temperature registered by T_1 is θ_1 and by T_2 is θ_2 and the distance between the thermometers T_1 and T_2 is x cm. Applying equation (a) we get

$$m(\theta_4-\theta_3)=K\frac{(\theta_1-\theta_2)A}{x}t$$

or

$$K=\frac{m(\theta_4-\theta_3)x}{At(\theta_1-\theta_2)}.$$

Since in practice some heat is always lost on sides the value of K obtained by this method, is somewhat *low*.

In Forbe's method heat is allowed to escape freely from the surface of the bar but this introduces a disadvantage in that it is no more possible to use the simple equation of conductivity. The theory of the method is beyond the scope of this book. In spite of this advantage it must be said that Forbe's method is a tedious one and is hence of historical interest only.

227. The Davy Safety Lamp.—A mixture of air and an inflammable Forbe's does not burn unless it is heated to its *ignition temperature.* If somehow or other the temperature of the mixture is kept below the ignition point, it does not catch fire. Davy made use of this fact in the construction of his safety lamp. To understand the principle of Davy's safety lamp perform the following experiment : Place a piece of wire-gauze a little above the tube of a Bunsen burner, turn the gas on and set fire to the gas above the gauze. It will be noticed that the flame does not pass downwards through the gauze. Since the heat produced by the burning of the gas is conducted away by the gauze, the temperature of the mixture below the gauze does not reach the ignition point. The fact, *i.e., the property of the wire-gauze to prevent the flame from passing through it to the other side is the basis of Davy's safety lamp.* The flame is surrounded completely by wire-gauze of copper. Even when the outer air contains an explosive gas the lamp burns quietly because the explosive gas penetrates the gauze and burns inside with a blue flame.* But as the flame cannot pass through the gauze, the explosion which would otherwise have occurred is averted.

228. Conduction in Liquids.—Liquids, with the exception of mercury, are bad conductors of heat, and in order to experimentally study their conductivity we must avoid the complications arising due

* An indication of danger to the miner.

to (a) the convection currents, and (b) the disturbing effects of the walls of the vessels in which they are enclosed. In order to get over the first difficulty, liquids are heated from above and to remove the second difficulty, a vessel made of a bad conducting material such as glass or wood is used.

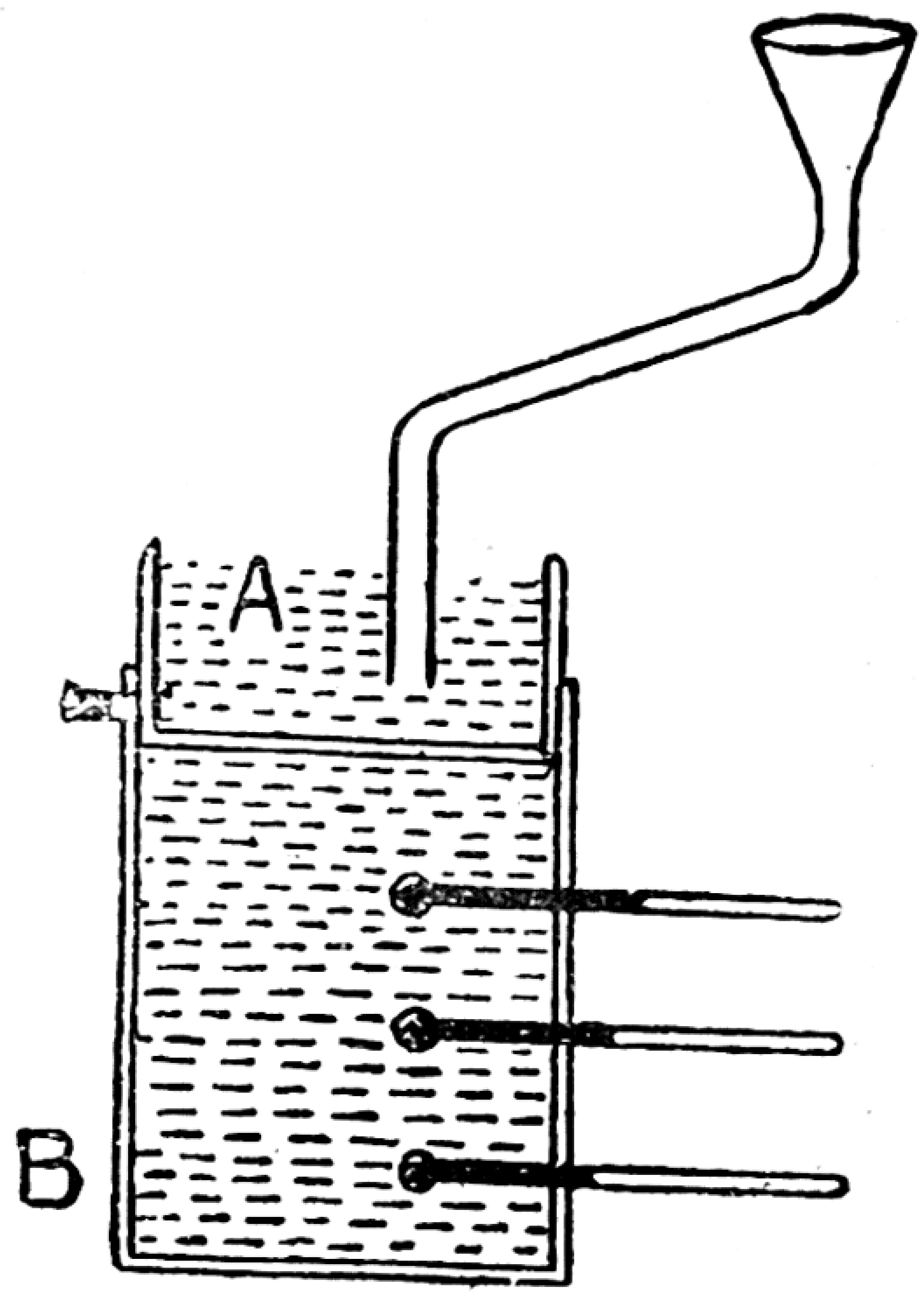

Fig. 51.

Despretz's Method.—In order to compare the conductivities of different liquids we make use of the principle of the Ingen-Hausz's experiment. We take a cylindrical wooden vessel *B* (Fig. 51); furnished with a row of holes through which the thermometers pass. At the top a metallic vessel *A* is placed. The wooden vessel is filled with the liquid whose conductivity is to be determined. Hot water at 100°C. is poured into the copper vessel *A*. It is renewed after every five minutes. It is found experimentally that heat travels slowly downwards. When the steady state is reached (which might take as long a time as 40 hours) the temperatures indicated by various thermometers are noted.

Fill the wooden vessel with various liquids, turn by turn, the hot water in *A* being in each case at the same temperature. The conductivities are proportional to the squares of the distances (measured downwards from the bottom of the vessel *A*) which give the same fall of temperature.

229. Conduction in Gases.—The experimental determination of the conductivity of gases is far more difficult than that of liquids. Here we have to avoid not only the convection currents and the effect of the vessel but also the effect of radiation. We shall explain outlines only of a simple method of determining the conductivity of gases.

The rate of cooling of a thermometer bulb of unknown thermal capacity heated to say 100°C. is found in a vessel filled at low* pressure with the gas whose conductivity is to be determined. The outer surface of the vessel is kept at 0°C. by immersing it in ice-cold water. Next, the vessel is exhausted to as perfect a vacuum as possible and the rate of cooling of the thermometer bulb is determined again. This gives us the radiation loss. Subtracting it from the loss of heat in the first case we get the rate of loss due to conduction alone, from which we can find the conductivity of the gas.

It is found that gases are very poor conductors of heat. To give an idea of their conductivity it may be mentioned that the conductivity of air is less than ten-thousandth part of copper. The warmth of woollen clothing is mostly due to the presence of air in the minute spaces in the cloth. People in Arctic regions make double-walled

*By taking the gas at low pressure the error due to convection is rendered negligible.

houses of ice to live in. The air between the walls being a bad conductor, does not allow the heat from inside to go out.

Table of Conductivities

(C. G. S. Units)

Silver	1·006	Wood	0·0006
Copper	0·92	Linen	0·00021
Zinc	0·265	Wool	0·0001
Platinum	0·166	Silk	0·000095
Iron	0·158	Felt	0·000087
Water	0·0014	Air	0·000054

230. Radiation.—It is a matter of daily experience that we feel a sensation of warmth when we expose ourselves to the sun's rays. That it is not due to the temperature of the air is clear from the fact that as soon as we place a screen between the sun and our body we do not feel the warmth. Hence the sensation is due to the heat energy that comes from the sun to us across a distance of 90,000,000 miles. Since our atmosphere practically ceases to exist at a height of about 300 miles we can regard the space beyond this distance as empty. Evidently heat cannot come to us through this empty space by convection and conduction. So there must be a third method of transmission of heat which requires no material medium for its propagation. Davy proved experimentally that heat can be transmitted across space without the help of a material medium. He enclosed a thermometer (whose bulb was painted with lamp-black) in a vessel at a distance from a platinum wire which could be electrically heated. The vessel was exhausted of air, and the wire was heated. The thermometer indicated an appreciable rise of temperature, although there was no air left inside the vessel. In this mode of propagation the medium is not affected, for, as said above, on screening ourselves we do not feel any warmth, showing thereby that the air is not sensibly affected by the heat of the sun which passes through it.

The third mode of propagation of heat is called *radiation* and is defined as follows :

Radiation is the process by which heat is transmitted from one point to another in a straight line without heating the medium.

All substances do not allow the heat radiation to pass through them with equal facility. Some allow it to pass through more readily than others. Those substances which allow the radiation to pass through without getting themselves heated are called **diathermanous ;** those which absorb the radiations are called **athermanous.** No material substance is *perfectly* diathermanous ; air is only approximately so.

Liquids and solids are mostly athermanous. It is not necessary that a body which does not allow light to pass through must cut off the heat rays as well. Solution of iodine in carbon bisulphide is perfectly opaque to light rays, but is diathermanous to heat rays. Before we consider the properties of heat radiation we shall explain the method of detecting it.

If sunlight or the radiation from a piece of heated metal be

allowed to fall on the bulb of an ordinary mercurial thermometer, it indicates a rise of temperature. If the bulb is coated with a black paint* the rise of temperature is much more marked. Hence such a thermometer can be used to detect the heat radiation; but in actual practice we use the Differential Air Thermoscope described below.

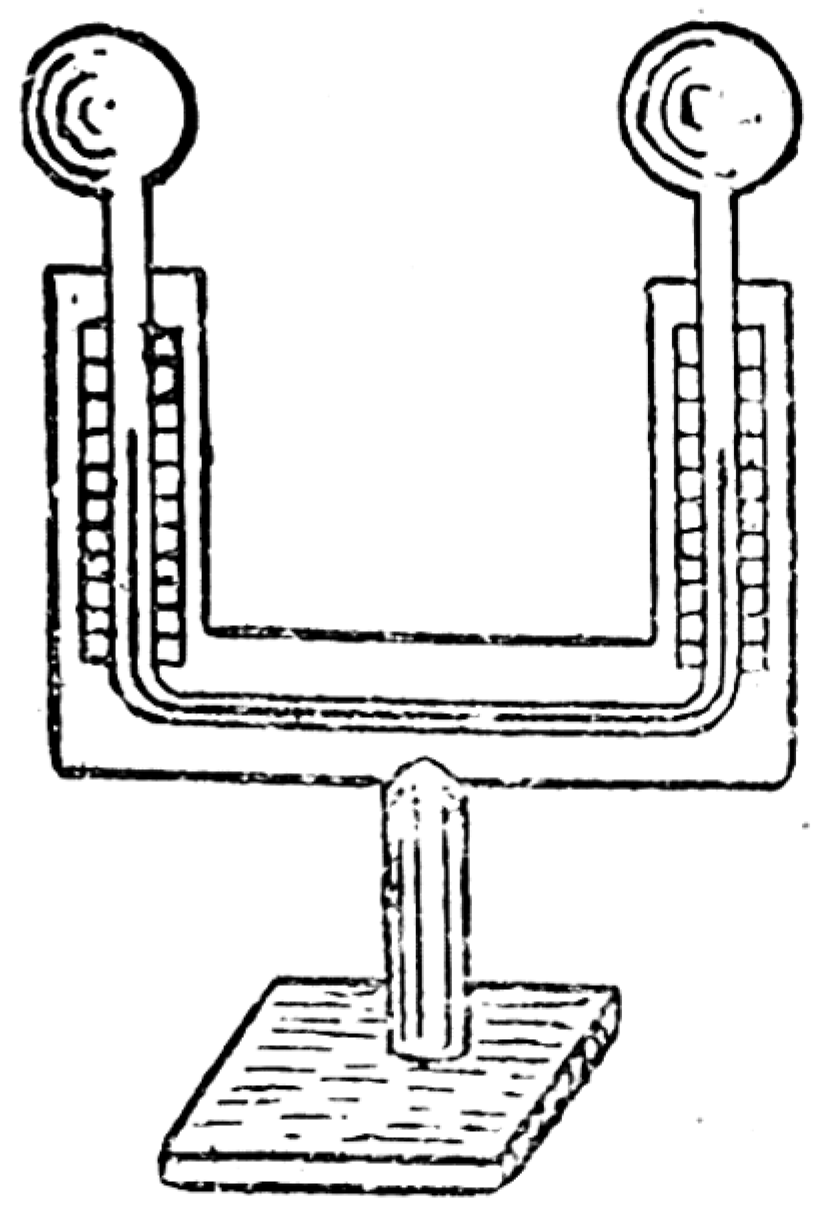

Fig. 52.

231. Differential Air Thermoscope.—It consists of a glass tube bent twice at right angles terminating in equal bulbs containing air (Fig. 52). The bend of the tube contains a coloured liquid.† The quantity of the air in the bulbs is so arranged that when the bulbs are at the same temperature, the liquid in the two arms stands at the same level.

If there be even a slight difference of temperature it is indicated by the difference in the levels of the liquid in the arms, for the air in the bulb on which heat radiation is incident expands and causes the liquid to go down and move over to the other arm.

If one bulb be coated with lamp-black the apparatus becomes very sensitive.

232. Properties of Heat Radiation.—These should be studied after reading Light.

(1) *Thermal or heat radiation requires no material medium for its transmission.*

We have already said in §230, how Davy proved it experimentally.

(2) *Thermal radiation travels in straight lines.*

To prove it experimentally take two wooden screens, each one of them having a small hole. Arrange them one behind the other in such a way that the holes are at the same level.

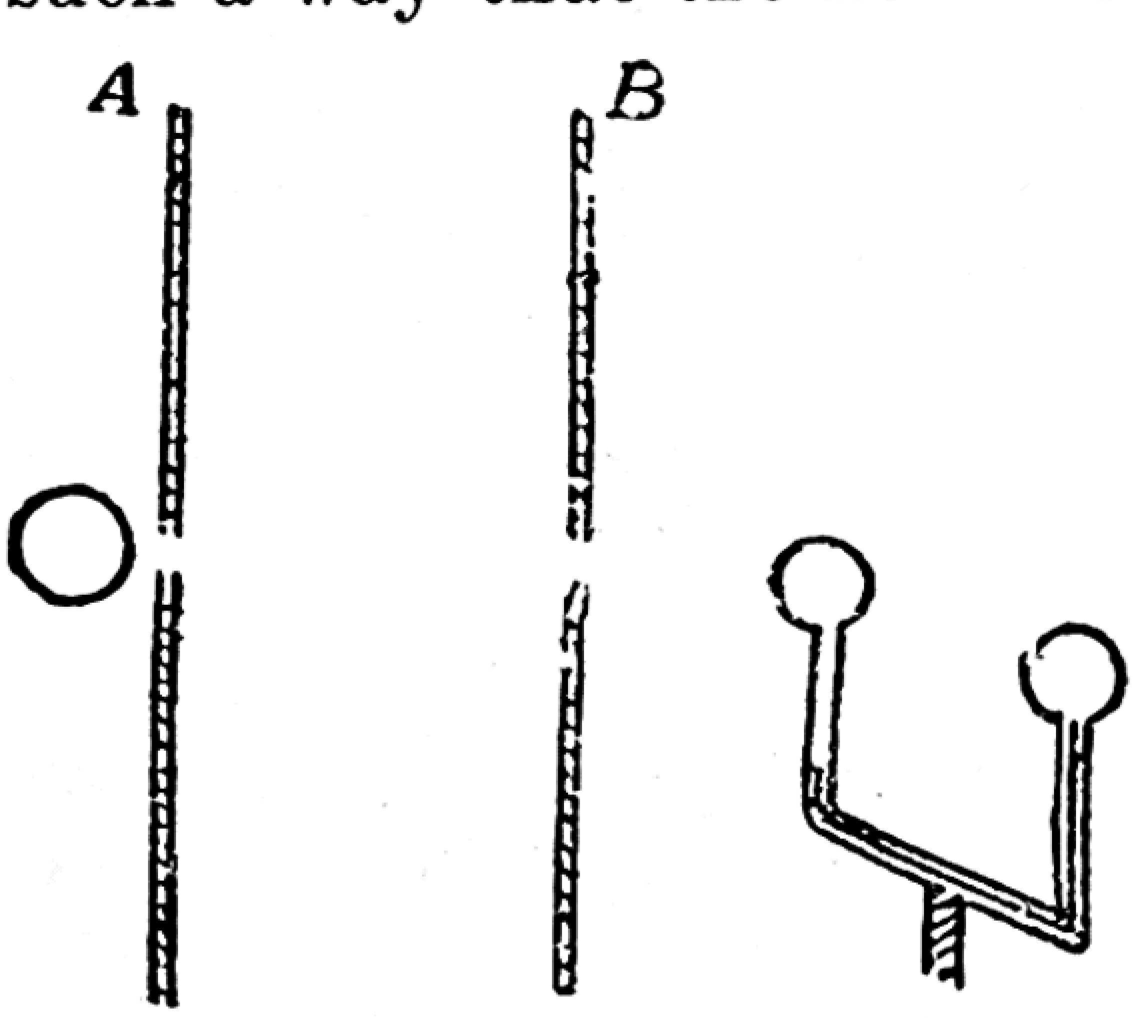

Fig. 53.

Place in front of one say, A (Fig. 53) a red-hot iron ball and behind the other, B, the black bulb of the differential thermoscope. The liquid moves at once. Now disturb the arrangement so that the holes are at different levels, and note that the liquid column remains undisturbed.

(3) *Thermal radiation obeys inverse square law.*

It should be noted that heat is radiated in all directions and in straight lines. Keeping this in view it is very

*The black paint generally used is made by mixing lampblack with alcohol in which a small amount of shellac has been dissolved.

†Generally coloured sulphuric acid is used, for it is neither volatile nor heavy and viscous.

easy to show that thermal radiation obeys the inverse square law like any other type of radiation say light or sound.

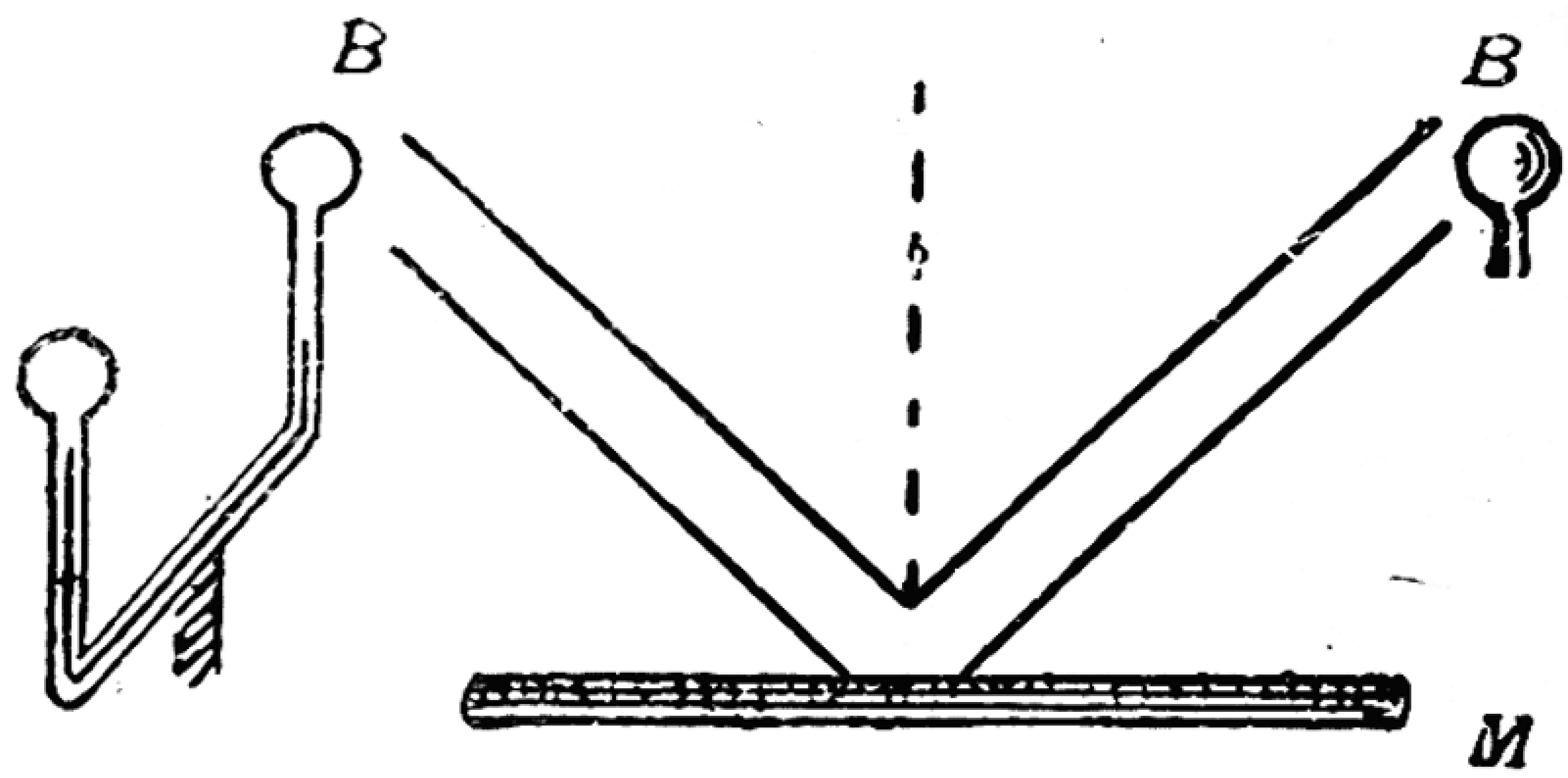

Fig. 54.

(4) *Thermal radiation is reflected from polished metallic surfaces in accordance with the laws of reflection.*

Two tubes hinged together are fixed, just above a polished metallic plate M as shown in Fig. 54. Place a red-hot ball at A, and the black bulb of a differential air thermoscope at B. It will be seen that the column of liquid remains unaffected so long as the tubes are not equally inclined, and moves down as soon as the angles are equal, showing thereby that the angle of reflection is equal to the angle of incidence.

To show in a more striking manner that heat rays suffer reflection at metallic surfaces, take two parabolic mirrors, M_1 and M_2, mounted on stands. Arrange them at a distance of about fifteen feet from each other. Place an iron ball A heated to redness at the principal focus of M_2 (Fig. 55). Place a piece of phosphorus on an adjustable stand B, and adjust the height of the stand so that the phosphorus is a little below the principal focus of the other mirror M_1. The phosphorus

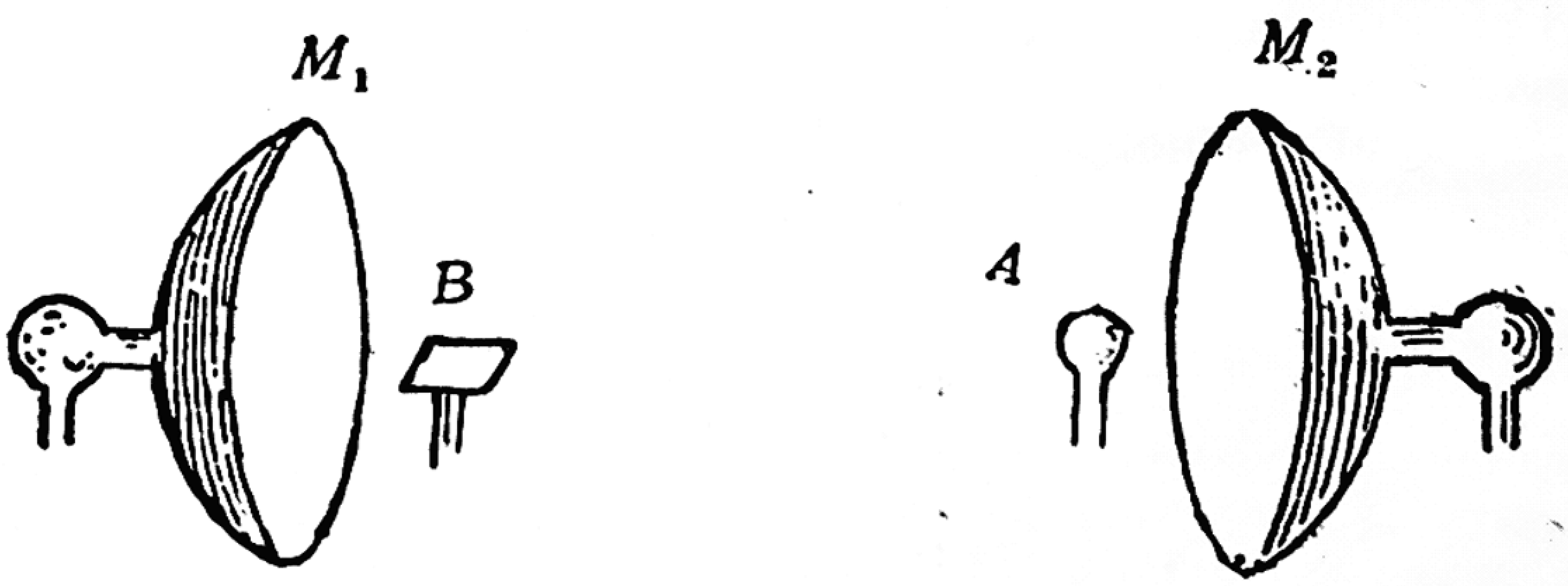

Fig. 55.

will not be affected. Raise the stand and it will be seen that as soon as the phosphorus is at the principal focus of the mirror M_1 it at once catches fire. Let us see what happens. The heat radiations fall on M_2, get reflected as a parallel beam, fall on M_1 and come to focus at its principal focus.

Very powerful effects may be produced by concentrating the sun's rays at a point with the help of a parabolic mirror ; with a mirror of 6 ft aperture copper and silver have been melted.

(5) ***Thermal radiation can be refracted.***

The easiest method of showing it is to hold a convex lens in the sun and to focus the sun's rays on a piece of black cloth. After a short time the cloth will catch fire due to concentration of the thermal radiations after refraction through the lens.

(6) *Thermal radiation does not affect the medium through which it passes.*

In the preceding experiment we have seen that at the principal focus of a lens a piece of cloth burns, but if the lens itself be touched, it will be found that the lens is practically unaffected. Only a very slight rise in temperature due to the absorption of the heat by glass will take place.

(7) *Thermal radiation travels with the speed of light.*

It has been proved experimentally that so long as the total solar eclipse lasts no heat radiation reaches the earth, and that as soon as light comes to the earth the heat radiation also comes along with it, showing thereby that heat radiation travels with the same speed as light.

233. Reflecting and Absorbing Power.—When heat falls upon a body, a portion of it is reflected and the rest is used up in raising the temperature of the body. The proportion between the quantities reflected and absorbed is different for different substances. There are some which reflect more than they absorb ; they are said to be *good reflectors* (and *bad absorbers*), while there are others which absorb more than they reflect ; they are said to be *good absorbers* (and *bad reflectors*). If a quantity of energy equal to Q units falls on a body in one second and a quantity Q_1 is reflected, the ratio $\frac{Q_1}{Q}$ is called the *reflecting power* of the body. To define the absorbing power of a substance, we have to consider the quantity absorbed in place of reflected. If Q units of energy fall on a body in one second and Q_2 units are absorbed, the ratio $\frac{Q_2}{Q}$ is called the Absorbing power. To study experimentally the reflecting power of bodies let the rays emitted from a red-hot iron ball fall upon a concave mirror M (Fig. 56.) They come to focus at P. Just in front of P, towards M, interpose a plate of the substance whose reflecting or absorbing power is to be determined. The rays after reflection at the plate will come to focus at Q. Place there the black bulb of a thermoscope and note the difference produced in the levels of the liquid. Keeping every other thing the same, use plates of different materials. The substance which causes the greatest difference in level is the best reflector, and therefore the worst absorber.

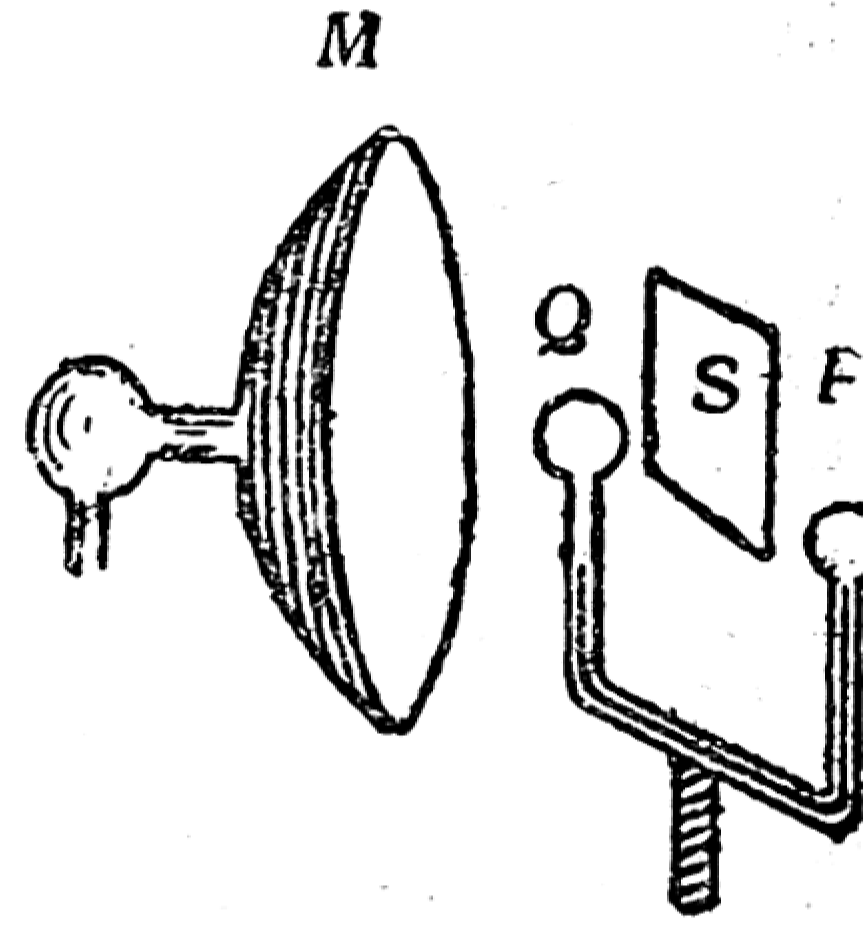

Fig 56.

It was found as a result of experiment that a polished brass plate produced the greatest difference, whereas plates coated with lamp-black and moisture practically did not affect the level, showing thereby that polished brass is the best reflector and therefore the worst absorber, and lamp-black and water the worst reflectors and hence the best absorbers.

234. Emissive or Radiating Power.—The quantity of heat radiated per second by a particular body depends in addition to the area of its surface and the temperature of the atmosphere upon the nature of its material and its own temperature. In order to compare the radiating powers of various substances we generally take the surface to be equal to 1 sq. cm. and difference of temperature between the body and its surroundings as 1°C. *The quantity of heat which a unit surface of a body emits in one second when the difference in temperature between the body and the surroundings is 1°C. is called the* **radiating power** *of the body.*

To compare the radiating powers of various bodies Leslie proceeded as follows :—

He took a cube whose one side was covered with polished tin, the second with a white paper, the third with a glass plate, and the fourth with lamp-black. It was placed at a distance from a concave mirror *M* (Fig. 57), at whose focus was placed the black bulb of a thermoscope. Boiling water was poured into the cube. The black side was first turned towards the reflector and the column of liquid was at once seen to go down considerably, indicating an appreciable rise of temperature. Next the paper side was turned ; the rise of temperature was found to be less than in the first case. The glass side came next to the paper side in producing the rise of temperature. The polished tin surface produced the least rise.

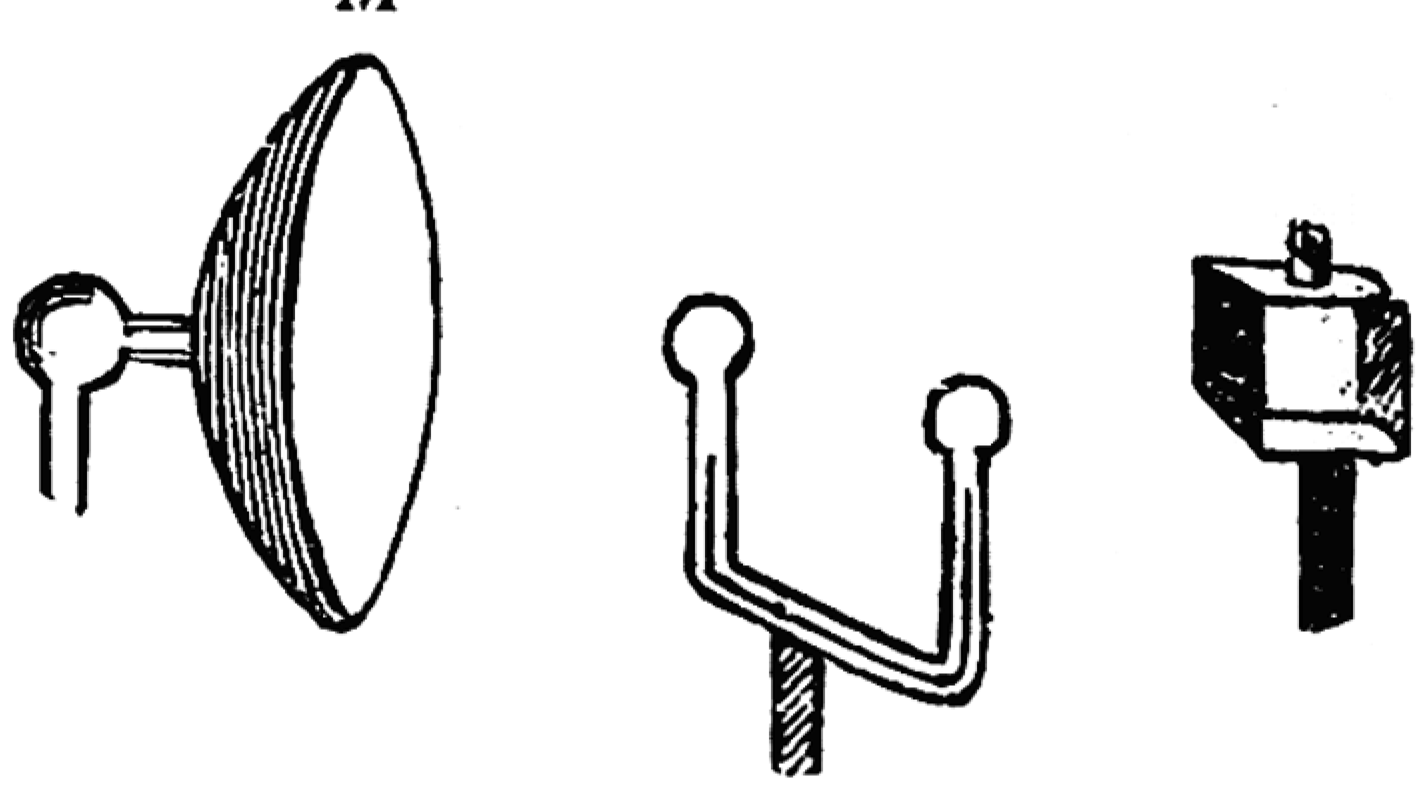

Fig. 57.

Leslie concluded from his experiments that lamp-black is the best radiator, and a polished metallic surface the worst radiator.

But we have already seen that lamp-black is the best absorber, and a polished metallic surface the worst absorber. Combining these two results we come to the conclusion that **a good absorber is a good radiator.**

A very striking experiment can be arranged to show the relation between the radiating and absorbing powers.

In a differential thermoscope the glass bulbs are replaced by cylindrical metallic vessels having flat sides. Between them is placed a cylindrical metal canister which can be filled with hot water. The faces of *A* are exactly equal to the faces of *B* and *C*, the dark faces are coated with lamp-black, and the others are polished. Let us suppose that the black face of *A* is opposite the polished face of *B*, and the polished face of *A* is opposite the black face of *C* (Fig. 58).

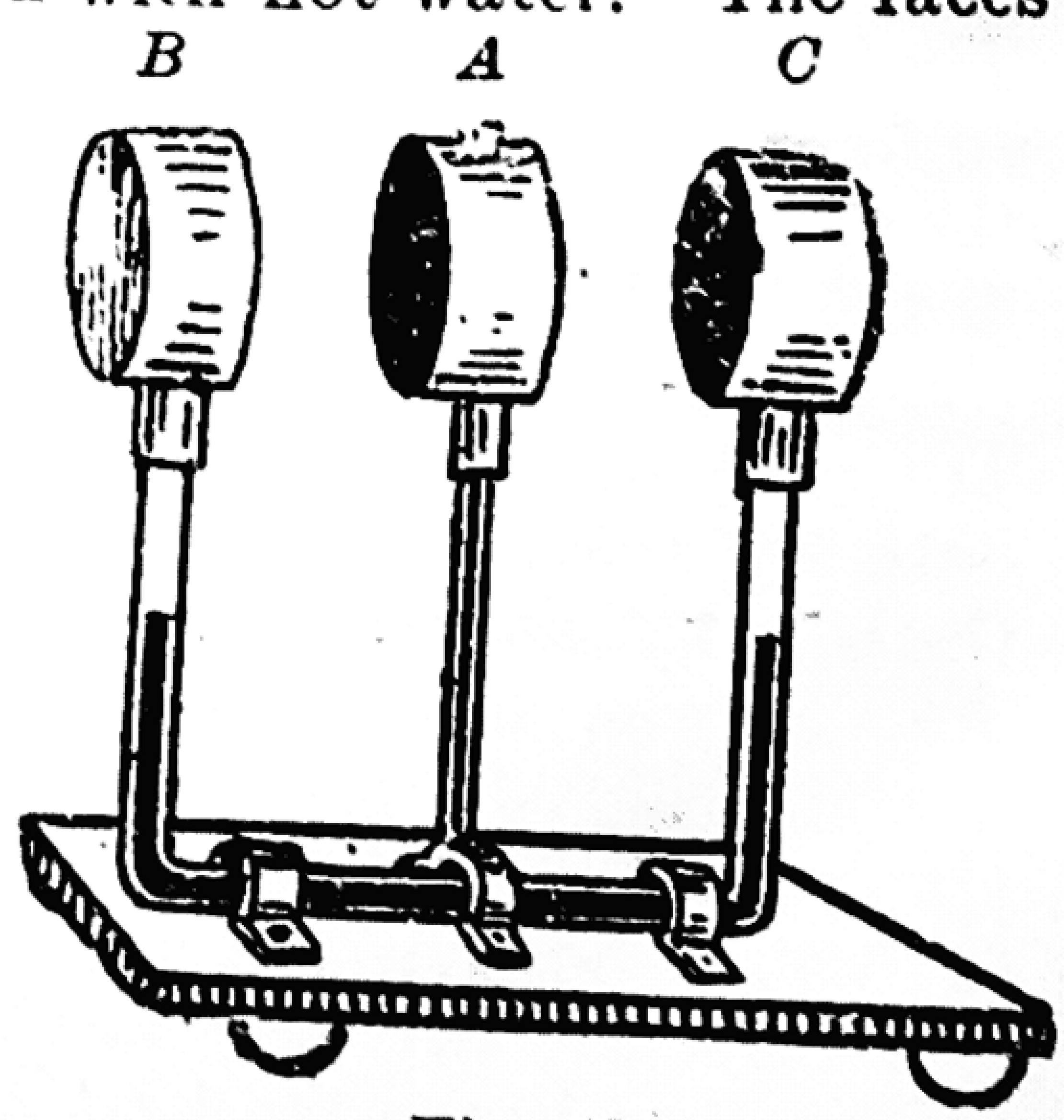

Fig. 58.

When *A* is filled with hot water, its polished face radiates heat towards the black face of *C* and its black face radiates heat towards the polished face of *B*.

With this arrangement it is found that the column of the liquid does not move, showing thereby that the great

radiating power of the black face is balanced by the weak absorbing power of the polished face, and that poor radiating power of the polished face is balanced by the good absorbing power of the black face. If, however, the canister be turned round, the liquid moves at once, for now a good absorber is opposite a good radiator and a bad absorber is opposite a bad radiator. The experiment shows clearly that *good radiators are good absorbers*.

235. Dewar Flask.—The fact that the radiation from a surface depends upon its nature, being greatest from the black surface and least from the polished surface, is made use of in the construction of a Dewar flask, in which a liquid can be kept either hot or cold without any change of temperature for a considerable period. The Dewar flask is ordinarily called **thermos flask.**

It is essentially a glass vessel with double walls, the space between the walls being vacuum. The inner surface of the outer wall and the outer surface of the inner wall are silvered. The vacuum does not allow conduction to take place, and the polished surfaces minimise the effects due to radiation. To make the effect of radiation still less the glass vessel is further surrounded by a polished cover.

236. Prevost's Theory of Exchanges.—Consider a ball at a high temperature placed in an enclosure. The ball constantly loses heat on account of radiation and the enclosure gains heat, till both are at the same temperature. When the temperature of the ball is constant, we think that the ball is not radiating any energy. But on surrounding this enclosure by another enclosure and filling the space between the two with ice, we find that the ball again begins to radiate heat energy, and its temperature falls. To explain how the presence of a cold body which has no action directly on the ball can make it emit radiation and further, how the hot enclosure can stop the body from emitting heat, which it begins to emit again as soon as the temperature of the enclosure falls, *we assume that a body radiates heat energy at all times whatever be its temperature.* A cold body of course radiates less heat than a hot body.

On this theory it is very easy to explain the heat radiation from a body. It is supposed that a hot body falls in temperature because it gives out more energy than it receives from the cold bodies around it. When the two bodies have the same temperature, each emits as much radiation as it receives. A cold body, on the other hand, receives more energy than it gives out to other, and hence its temperature rises.

This theory of exchanges of heat radiation is known as **Prevost's theory of exchanges.**

EXERCISES

1. The thermal conductivity of felt is represented by 0·000087. Calculate the heat lost in one hour through a felt sheet 1 cm. thick and 100 sq. cm. in area when its opposite faces are kept at a difference of 30°C.

We know that $$K=\frac{Q\cdot x}{A(\theta_1-\theta_2)t}.$$

Substituting the various values we get

$$0{\cdot}000087=\frac{Q\cdot 1}{100\times 30\times 60\times 60},$$

Therefore $Q=0{\cdot}000087\times3600\times3000$
$=8{\cdot}7\times108=939{\cdot}6$ calories.

2. In an experiment it was found that the underground temperature in a sand-stone district increased 1°C. for 27 metres descent. The amount of heat lost per hour per square kilometre of the earth's surface was $3{\cdot}6\times10^7$ units. What is the conductivity of the sandstone ?

$$K=\frac{Q\,.\,x}{A(\theta_1-\theta_2)t}$$

$$=\frac{3{\cdot}6\times10^7\times27\times100}{100000\times100000\times1\times60\times60}$$

$$=\frac{3{\cdot}6\times27}{36000}=0{\cdot}0027 \text{ C.G.S. units.}$$

3. It is found that 109,083 calories of heat are transmitted per second across a sheet of silver 100 sq. cm. in area and 1 mm. thick when the difference of temperature between the two faces is 100°C. Find the absolute conductivity of silver. *Ans.* 1·09.

4. The opposite faces of a metal plate 20 cm. thick are at a difference of temperature of 100°C., the area of the plate being 2 square metres. If the conductivity be 0·2, find how many calories of heat will flow across the plate per second. *Ans.* 20,000 calories.

5. Calculate the quantity of heat which flows per hour through a window pane of glass 7 mm. thick, 1 sq. metre in area, the temperature difference between two sides of the glass pane being 32°C. Take the thermal conductivity of glass to be 0·0005. *Ans.* 8,22,857, calories.

6. Find the conductivity of an iron plate 8 cm. thick and 1 sq. metre in area, the opposite faces of which are kept at a difference of 80°C. when 9,60,000 units of heat flow across the plate per minute.
Ans. 0·16

7. The two faces of a metal plate 100 sq. cm. in area are kept at a temperature difference of 100°C. It is found that 18,000 units of heat pass across the plate in one hour. If the conductivity be 0·02, what will be thickness [illegible]he plate ? *Ans.* 40 cm.

8. Find the the[illegible] conductivity of an iron plate 30 cm. long, 40 cm. broad and 28 cm. [illegible]ck whose one face is exposed to steam at 100°C. and the other face is in contact with ice, and 810 gm. of ice are melted in one minute. *Ans.* 0·18

9. Estimate the rate at which ice will melt in a wooden box 2 cm. thick of inside measurement $100\times60\times60$ cm. assuming that external temperature is 20°C. and the conductivity of wood 0·0006.
Ans. 2·34 gm./per sec.

10. Explain the use of surrounding the flame in Davy's safety lamp by a wire gauze.

11. Why do we heat our kettles and sauce pans from below while cooking, and not from above ?

12. Why does a chimney help a fire burn better ?
Does the chimney of a kerosene table-lamp serve the same purpose as the chimney in a kitchen or in a factory ?

13. The thermal conductivity of air is far less than that of cotton or wool. Why then we use cotton or woollen clothes to keep us warm ?

CHAPTER X

Mechanical Equivalent of Heat and Heat Engines

237. In describing the phenomena of expansion of bodies and change of state of bodies with heat and in stating the laws governing them it was not necessary for us to know any thing more about heat than that it is something which is associated with the sensation of warmth (*i.e.*, temperature), and can be measured by the methods dealt with in the chapter on Calorimetry. In other words we have ignored so far the question of the nature of heat, but now we shall try to discuss it briefly. There are two theories regarding the nature of heat. One of them is the caloric theory, which is as old as the ancient Greeks, and the other is the modern one. It will not be possible to go into details as regards these theories, we shall have to content ourselves with their outlines merely.

238. Caloric Theory.—Heat, according to this theory, is an elastic, imponderable fluid called caloric. The particles of this fluid are supposed to repel each other and attract the particles of ordinary matter It is supposed to pervade all bodies. Further, it is considered to be indestructible and uncreatable. With these assumptions we can explain quite satisfactorily the ordinary phenomena of heat, as, for instance, the flow of heat from a body at a high temperature to a body at a low temperature or radiation of heat from the flames or the expansion of bodies when heated. If we suppose further that the attraction of caloric for different kinds of matter is different, we can explain easily the fact that equal amounts of different substances require different amounts of heat to be heated through the same range of temperature. With other assumptions, the phenomena of latent heat, the expansion or contraction on solidification, can also be explained.

But the production of heat by friction cannot be explained on tihs hypothesis. It was Lord Bacon who first called attention to this fact and made it one of the strongest points against this hypothesis.

In 1798 Count Rumford, while engaged in boring cannon at Munich, was struck by the considerable rise of temperature of the cannon, especially of the chips cut out by the boring tool. By immersing the cannon and the blunt boring tool under water he actually boiled water.

The upholders of the caloric theory explained this rise of temperature by supposing that the thermal capacity of a body is less in the form of a powder than in the form of a solid block. Rumford modified his experiment in such a way that the amount of metal abraded was very little and even then he found a very considerable rise in the temperature of the block of metal. Rumford's idea was that anything

which could be produced without limit could not be material in nature. The fatal blow to this theory, however, was served by Davy, who showed, that two pieces of ice could be melted by rubbing together. Now surely, since water contains a far greater amount of heat than ice,* liquefaction cannot take place unless there is *generation of heat*. According to caloric theory, the generation of heat was impossible, and hence, Davy put forward the hypothesis, that it was the work done against friction which was converted into heat.

Before we go further we shall study first the connection between the amount of work done and the heat produced.

239. Heat and Mechanical Work.—Dr. Joule of Manchester between 1843 and 1848 carried out a series of experiments and showed that a definite quantity of heat was always produced by a definite amount of mechanical work. It was he who first measured the amount of work which must be done to produce a given quantity of heat. His apparatus is shown diagrammatically in Fig. 59. It consists essentially of a copper-calorimeter C to the sides of which the vanes V, V...are soldered and a set of paddles P, P,... carried by a spindle which is free to revolve about the axis ED.

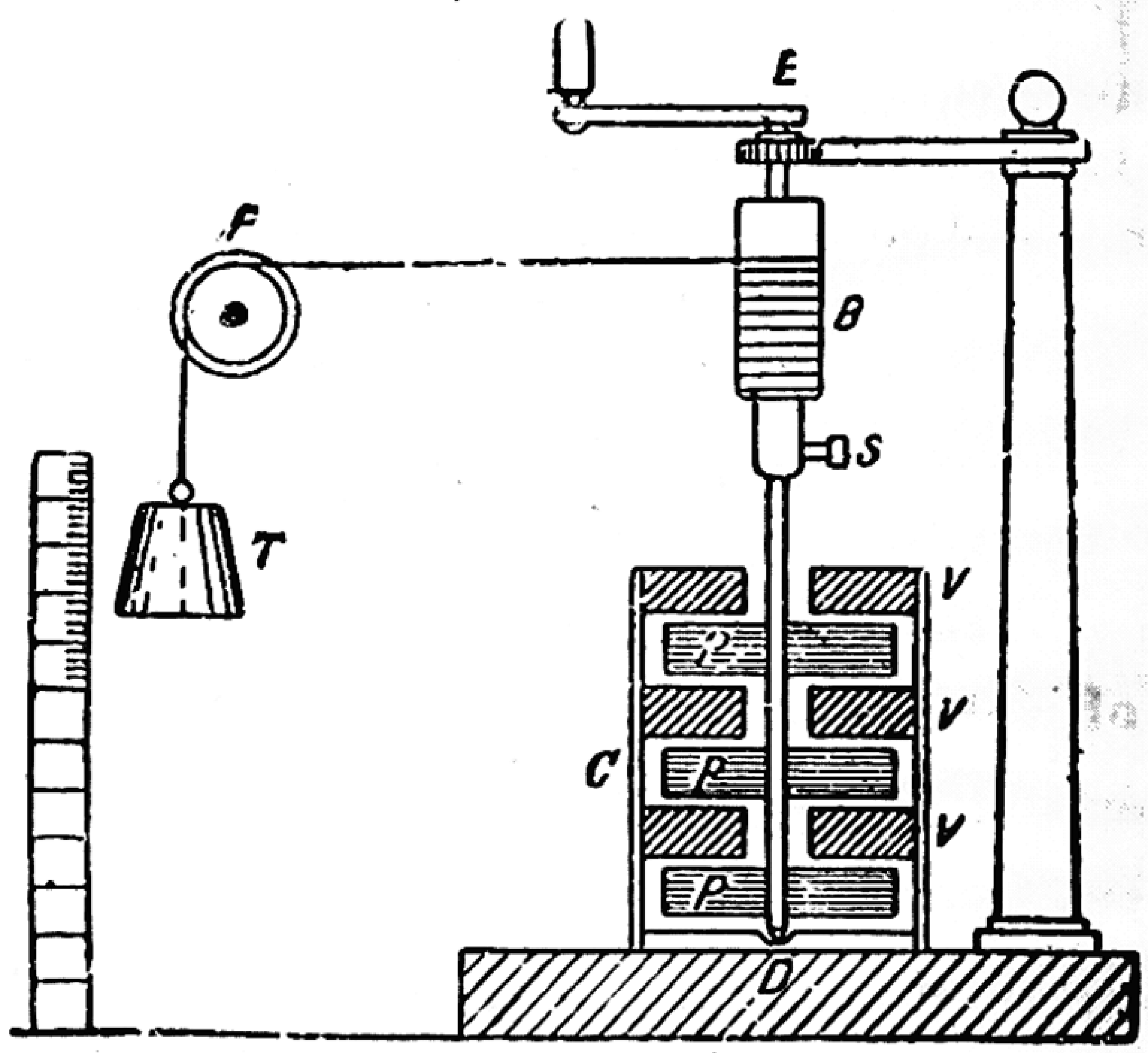

Fig. 59.

The calorimeter contains a known mass of water, whose temperature is read with a very sensitive thermometer. When the weight T attached to the end of cord wound round the drum B is allowed to fall, the spindle turns and the paddles move the water. The water is at once stopped by the vanes, and heat is produced. When the weight reaches the floor, the drum B is disconnected from the spindle by unscrewing the pin S, and the cord is again wound round it. The drum is fixed again upon the spindle without moving it, and the experiment is repeated again and again, till the rise of temperature is appreciable.

Without going into the corrections which Joule had to apply before he got the result, we shall explain in outline the principle of the method by which the calculation is made. Suppose the weight T lb. falls each time through a vertical height h ft. and that the experiment is repeated n times. Evidently the total work done is nTh foot-pounds; call it W.

If the thermal capacity of the calorimeter, vanes, water, spindle and the paddles be M and the rise in temperature be θ°C., the heat produced $=M\theta$: call it H.

* If ice be heated we get water, therefore it is obvious that water contains more heat than ice.

It is found that the ratio $\frac{W}{H}$ is always the same, *i.e.*, $\frac{W}{H}=$ a constant.

Joule repeated the experiment a large number of times, sometimes using water in the calorimeter, sometimes mercury, and so on, but he found that the ratio in every case was the same. This means, in other words, that to produce one unit of heat the same amount of mechanical work must be done. This amount of work is equal to the value of the constant. Joule found from his experiment that 772 foot-pounds of work must be done to produce enough heat to raise the temperature of 1 lb. of water through 1° Fahrenheit. More recent determinations show that 778 foot-pounds of work must be done. In the C.G.S. units it is found that $4{\cdot}2\times10^7$ ergs of work must be done to produce 1 calorie of heat. This constant is called the Joule's **mechanical equivalent of heat.** It is generally denoted by J.

The recent figures for J are as follows :—

$4{\cdot}185\times10^7$ ergs per calorie.
777·9 foot pounds per British Thermal unit.
1400 foot pounds per pound-degree-centigrade* unit.

We can write the above result as

$$W=JH.$$

The law that **the heat produced is proportional to the amount of work done,** is quite a general one ; it is immaterial whether the work is done in one way or the other.

On caloric theory conversion of heat into mechanical work is impossible, and hence the idea that heat is fluid had to be given up.

240. Modern Theory.—According to this theory the molecules of all bodies are in a state of continued agitation. In the case of solids the molecules are supposed to be vibrating to and fro about their mean positions. The velocity with which the molecules of a body move depends upon the temperature of the body. With rise of temperature the velocity as well as the amplitude becomes greater. Since kinetic energy depends upon the square of the velocity it is obvious that with rise of temperature the kinetic energy of the molecules becomes greater. On this theory there is no difficulty in explaining the generation of heat when two bodies are rubbed together. The work that is done against friction is transformed into molecular energy.

Let us see how this theory enables us to explain the laws of gases which we have seen experimentally to be true. We shall suppose that

(*i*) the molecules of a gas are all alike and that they behave like elastic spheres and are free to move in all directions ;

(*ii*) their velocity increases with increase of temperature ;

(*iii*) they continually collide with one another and with the walls of the containing vessel and that the collisions are instantaneous *i.e.*, take no time ;

---

*1400 foot-pounds of work must be done to produce enough heat to raise the temperature of 1 pound of water through 1°C.

(*iv*) they exert no force on each other, in other words between two successive collisions the molecules move in straight lines ;

(*v*) the actual volume of the molecules is small compared with the volume of the vessel, in other words, the molecules are comparatively far apart.

Let us consider a unit cube containing n molecules each of mass m and moving with velocity u. Consider a molecule which moves from face A to face B and strikes against it normally. Since the molecule is perfectly elastic it rebounds in the opposite direction with the same velocity and its momentum changes from $+mu$ to $-mu$, the change of momentum being equal to $2mu$. After collision at face B it goes to face A collides there and comes back to B. In other words it moves 2 cm. between two successive collisions on the same face and strikes a given face $\frac{u}{2}$ times a second.

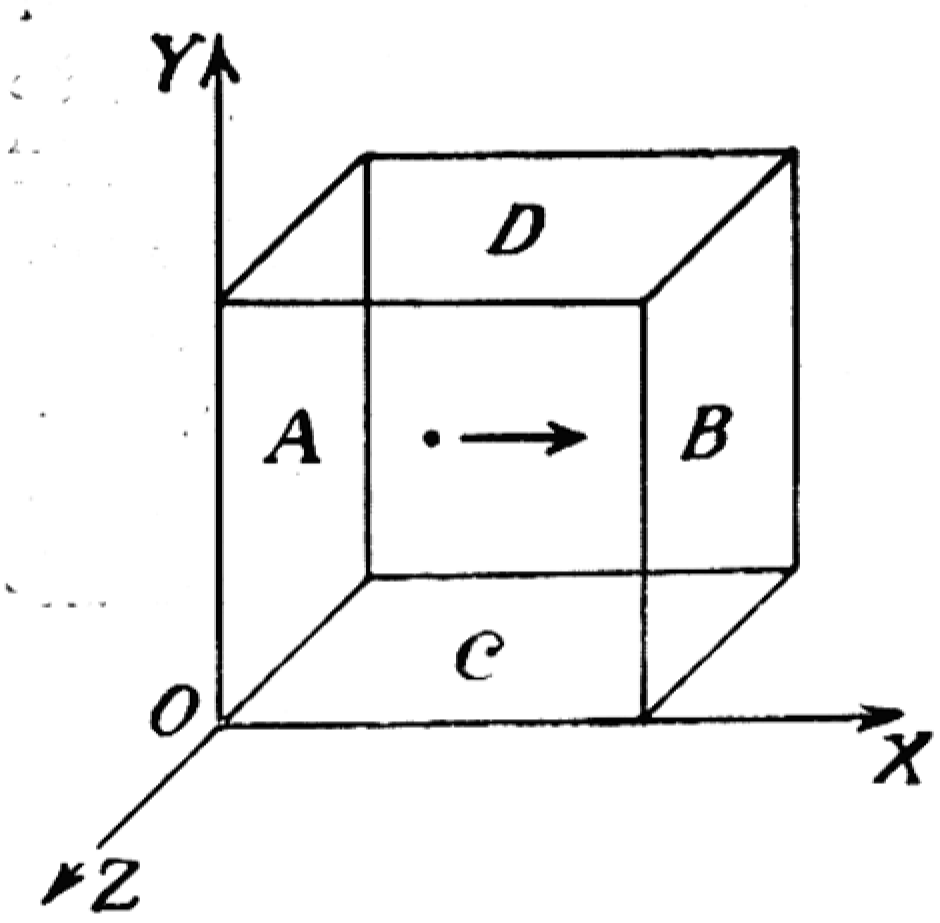

Fig. 60.

The impulse imparted to face B per second by the collisions of this single molecule $=2mu \times \frac{u}{2} = mu^2$.

Since the molecules are equally free to move in all directions we can suppose one-third of them *i.e.*, $\frac{n}{3}$ to move parallel to X-axis, $\frac{n}{3}$ parallel to Y-axis and $\frac{n}{3}$ parallel to Z-axis.

Confining our attention to the motion along X-axis we find that in one second the total impulse acting on face B

$$=\frac{n}{3}\,mu^2$$

$$=\tfrac{1}{3}mnu^2$$

But the total impulse acting per second on face B is equal to the pressure. Hence we find that

$$p=\tfrac{1}{3}mnu^2 \qquad \ldots\ldots\ldots\ldots \quad (i)$$

Writing ρ for nm, we get

$$p=\tfrac{1}{3}\rho u^2 \qquad \ldots\ldots\ldots\ldots \quad (ii)$$

This relation enables us to calculate the velocity of the molecules of a gas. For instance the velocity of oxygen molecules at $N.\ T.\ P.$ will be

$$u=\sqrt{\frac{3\times 76\times 13{\cdot}56\times 981}{1{\cdot}434\times 10^{-3}}}$$

$$=4{\cdot}6\times 10^4 \text{ cm. per second.}$$

If v=volume of 1 gm. of gas, the density $\rho=\frac{1}{v}$

Hence relation (2) can be written as

$$p=\tfrac{1}{3}\frac{u^2}{v}$$

or
$$pv = \frac{1}{3}u^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (iii)$$
$$= rT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (iiia)$$
or
$$u^2 = 3rT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (iv)$$

Equation (*iv*) shows that the velocity of molecules of a gas is proportional to its absolute temperature.

Boyle's Law.—Equation (*iv*) leads us to the conclusion that so long as temperature remains constant, u remains constant, hence from equation (3) we find that pv remains constant. This is Boyle's Law.

Charles's Law.—Let a given mass of gas be heated at constant pressure p and the volume increase from v_1 to v_2 and temperature change from $T_1°A$ to $T_2°A$. If we suppose that the mass of the gas is a grams, we can write equation (*iiia*) for the gas as

$$pv_1 = arT_1$$

and
$$pv_2 = arT_2$$

$\therefore$
$$\frac{v_1}{v_1} = \frac{T_1}{T_2},$$

which is Charles's Law.

Graham's Law of Diffusion.—Graham found that the rate at which a gas diffused through a porous pot into or another gas was inversely proportional to the square root of its density.

Let us apply the principles of kinetic theory and see if we can deduce Graham's Law.

From relation (*i*) it is obvious that the rate at which a gas will diffuse through a porous pot will depend upon the velocity of its molecules, the greater the velocity the greater the rate of diffusion.

But velocity given in equation (*ii*) $= \sqrt{\frac{3p}{\rho}}$. Hence the rate of diffusion of gas 1 will be proportional to $\sqrt{\frac{3p}{\rho_1}}$, and of the gas 2 to $\sqrt{\frac{3p}{\rho_2}}$. Dividing the first relation by the second, we get

$$\frac{\text{Rate of diffusion of gas 1}}{\text{Rate of diffusion of gas 2}} = \frac{\sqrt{3p/\rho_1}}{\sqrt{3p/\rho_2}} = \sqrt{\frac{\rho_2}{\rho_1}}$$

That is, the rates of diffusion of two gases into each other are inversely proportional to the square roots of their densities, which is Graham's Law.

240a. Van der Waals' Equation.—In the above discussion we have assumed that the molecules do not exert any force of attraction on one another and that their volume is small compared with the volume of the vessel. These assumptions are very nearly true at low pressures or high temperatures, but at high pressures and low temperatures they do not hold good and hence gases exhibit considerable deviations from Boyle's law. To represent the behaviour of real gases Van der Waals replaced the pressure p in the expression for Boyle's law by $\left(p+\frac{a}{V^2}\right)$

where a is a constant depending upon the mass and nature of the gas taken and replaced V by $(V-b)$, where b is another constant, also depending upon the nature and mass of the gas. The gas equation $pV=RT$ was hence modified by him to

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT.$$

If p be expressed in atmospheres, the mass of gas taken be 1 gm, the constants a, b and R for carbon dioxide will have the following values :—

$$a=0{\cdot}00874$$

$$b=0{\cdot}0023$$

and $$R=0{\cdot}00368^*$$

This equation is called *Van der Waals' equation* and it represents the behaviour of gases much more exactly than the equation $pV=RT$.

240b. First Law of Thermodynamics.—The transformation of mechanical work into heat or heat into mechanical work forms the basis of the science of *Thermodynamics*. This branch not only deals with the conversion of heat into mechanical work or *vice-versa* but also between heat and any other form of energy. Its study began with the experiments of Rumford and Joule which showed as said above, that when heat is produced at the expense of mechanical work, the ratio of heat produced to the mechanical work done is constant. This is known as *First Law of Thermodynamics*. It is in effect a statement of the principle of conservation of energy and is usually stated as follows :—

Whenever heat energy is transformed into any other form of energy or *vice-versa*, the quantity of energy which disappears in one form is exactly equivalent to the quantity which makes its appearance in the other form.

It is often written as a mathematical equation :

$$dQ=dU+dW,$$

where dQ is the heat supplied or taken away from a system, dU is the increase or decrease in internal energy and dW is the external work done by or on the system.

If external work done is zero the heat supplied is spent in increasing the internal energy of the system. Suppose, for instance, our system consists of 100 gm. of water and 500 calories are supplied to it. Neglecting the work done as a result of very slight increase in the volume of water we can say that the internal energy of the mass of water has increased by 500 calories and the evidence for the change in its internal energy is the rise in its temperature.

Now let us apply this law to a system consisting of 1 gm. of water at 100°C, which is supplied with 536 calories. We know that 1 gm. of water at 100°C changes into 1700 c.c. of steam. This means that the external work done *i.e.*

$$Pdv=76\times13{\cdot}56\times981\times1700 \text{ ergs.}$$

---

*or $\dfrac{1{\cdot}00646}{273}$

$$=\frac{76\times13{\cdot}56\times981\times1700}{4{\cdot}2\times10^7}\text{ calories.}$$
$$=41\text{ calories.}$$

The remaining calories (*i.e.* 495) are spent in increasing the internal energy of the system.

240c. The Second Law of Thermodynamics.—The first law is one of equivalence and does not indicate any preference for one direction or another. But it appears nature favours one kind of change more than the other. For example, when a bullet is stopped by a block of wood, its kinetic energy is changed into heat and as a result of it the wood and bullet both become warmer. The conversion of mechanical energy of the bullet into heat takes place in accordance with the first law. But the reverse case of the conversion of the heat energy of the wood and bullet into mechanical energy of the bullet which is equally in accordance with the first law never happens. Nobody has ever seen a bullet flying out of a block of wood at the expense of heat of the wood and bullet. In other words we find that in practice nature puts a limitation on the convertibility of heat into mechanical work.

The Second Law of Thermodynamics deals with the conditions which must be fulfilled for conversion of heat or other forms of energy into mechanical energy. For instance it says that it is not sufficient to have Q units of heat energy stored up in a body at temperature $T°$, we must also have an arrangement for the flow of some part of the heat to a body at a lower temperature for change of heat into work just as an arrangement in a steam engine is made for flow of heat from a high temperature boiler to a low temperature exhaust.

The second law further says that even under ideal conditions only *a part* of the heat that flows from a body at high temperature say $T_1°$ to a body at low temperature say $T_2°$ can be changed into work. The efficiency of the *ideal* heat engine working between these temperatures cannot exceed

$$\frac{T_1-T_2}{T_1}.$$

In a steam engine, for instance, in which steam is produced at a pressure of 225 lb./in², $T_1=200°C$ or $473°A$ and $T_2=100°C$ or $373°A$ the

$$\text{Maximum Efficiency}=\frac{473-373}{473}=21\%.$$

That is more than three quarters of the heat put into the engine, even when it is perfect, cannot be converted into work. It must go as waste The only remedy lies in raising the upper temperature or lowering the lower temperature or changing both to improve the efficiency.

The second law has been expressed in several forms, but all of them really mean that *heat tends to flow from higher to lower temperatures*.

It is sometimes expressed as

Heat does not flow from one body to another at a higher temperature unless work is done to accomplish this result.

241. Heat Engines.—We shall now briefly explain the

methods of conversion of heat into mechanical energy. The machines which enable us to do this are called *Heat Engines*. We shall explain only a few well-known forms of heat engines, *e.g.*, the Steam Engine, the Steam Turbine and the Internal Combustion Engine.

In spite of our scientific knowledge and mechanical skill, a small fraction of heat energy is converted into mechanical energy. One pound of good coal when burnt produces 15,000 B.Th.U. and if all this heat could be converted into work, it will be equivalent to $15{,}000 \times 778$ ft. lb.—an amount of work, which an engine of 8 H.P. would do in one hour. In practice, however, a locomotive uses about 9 lb of coal to do this much work in one hour which means an efficiency of about 11%. The rest of the energy so to speak is all wasted.

242. The Steam Engine.—It is used to convert heat energy obtained by the combustion of coal into mechanical work, steam being used for effecting the transformation.

It consists mainly of two parts, (1) the boiler in which steam is generated, and (2) the engine proper in which steam is allowed to expand and do work. In Fig. 61 is shown a cross-section of a locomotive. Its huge size is due to the boiler part, the engine proper forming only a small part. Why the boiler part is made so large in size will be clear to the student if he remembers that a locomotive converts from 5 to 10 tons of water per hour into steam. And since the capacity of a boiler to make steam depends largely on the area of its heating

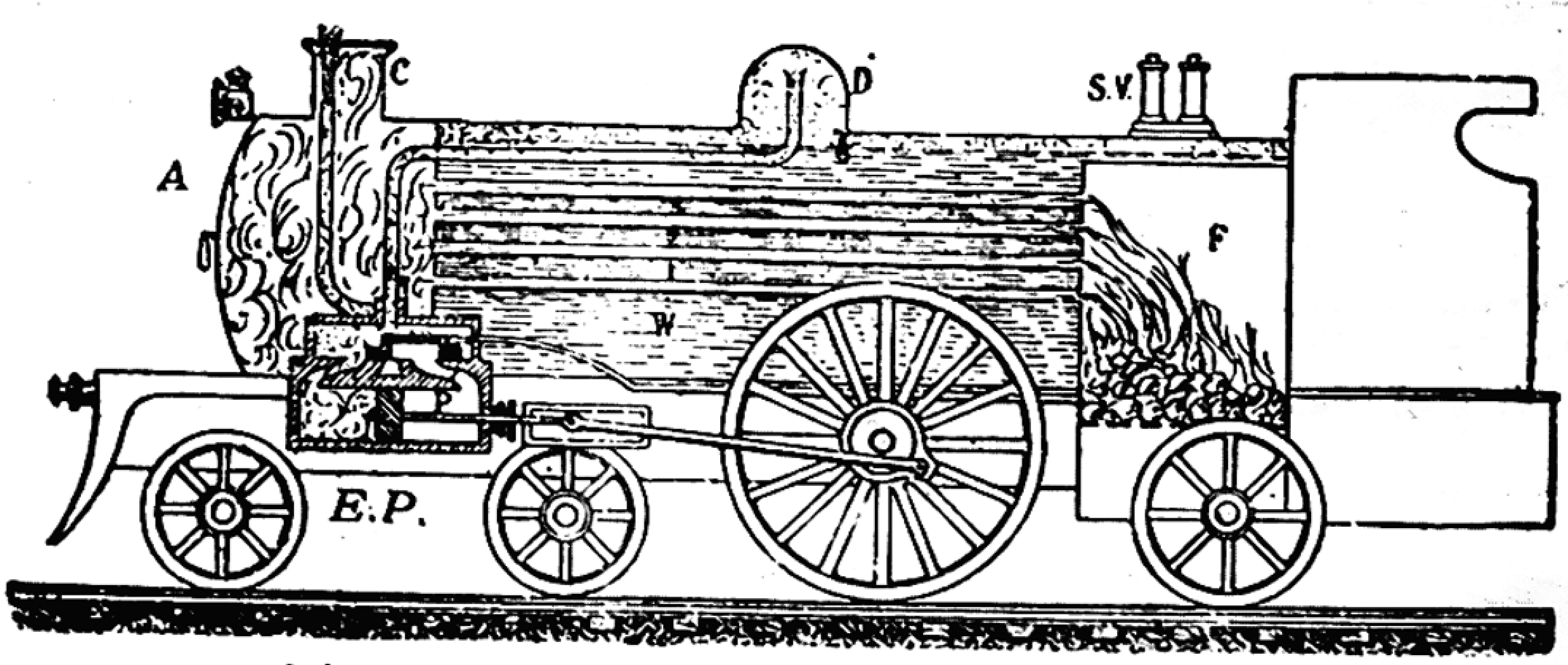

Fig. 61. Cross-section of a Locomotive.
[*F*, fire-box ; *W*, water ; *S.V.*, safety valve ; *A*, smoke-box ; *C*, Funnel ; *D*, Dome for steam regulator ; *E.P.*, Engine Proper *i.e.*, Slide valve and the cylinder ; *P*, Piston.

surface it is to increase this surface that the flames are made to pass through a large number of metal tubes varying in diameter from two to four inches immersed in water.

The steam formed is allowed to rise to a pressure of about 10 atmospheres and is then led through pipes placed in the fire tubes before it is admitted to the steam chest. By this device the steam is superheated and thereby the efficiency of a steam-engine is increased somewhat, but even then it is never more than 12% in the case of a locomotive.

Action of the Steam Engine.—To explain the action of a steam engine we shall take the help of a simplified sketch of a stationary

engine without the boiler part (Fig. 62). The engine proper consists essentially of the following parts :

(*a*) The cylinder *C* traversed by a steam tight piston *P* which divides it into two parts, upper and lower chamber.

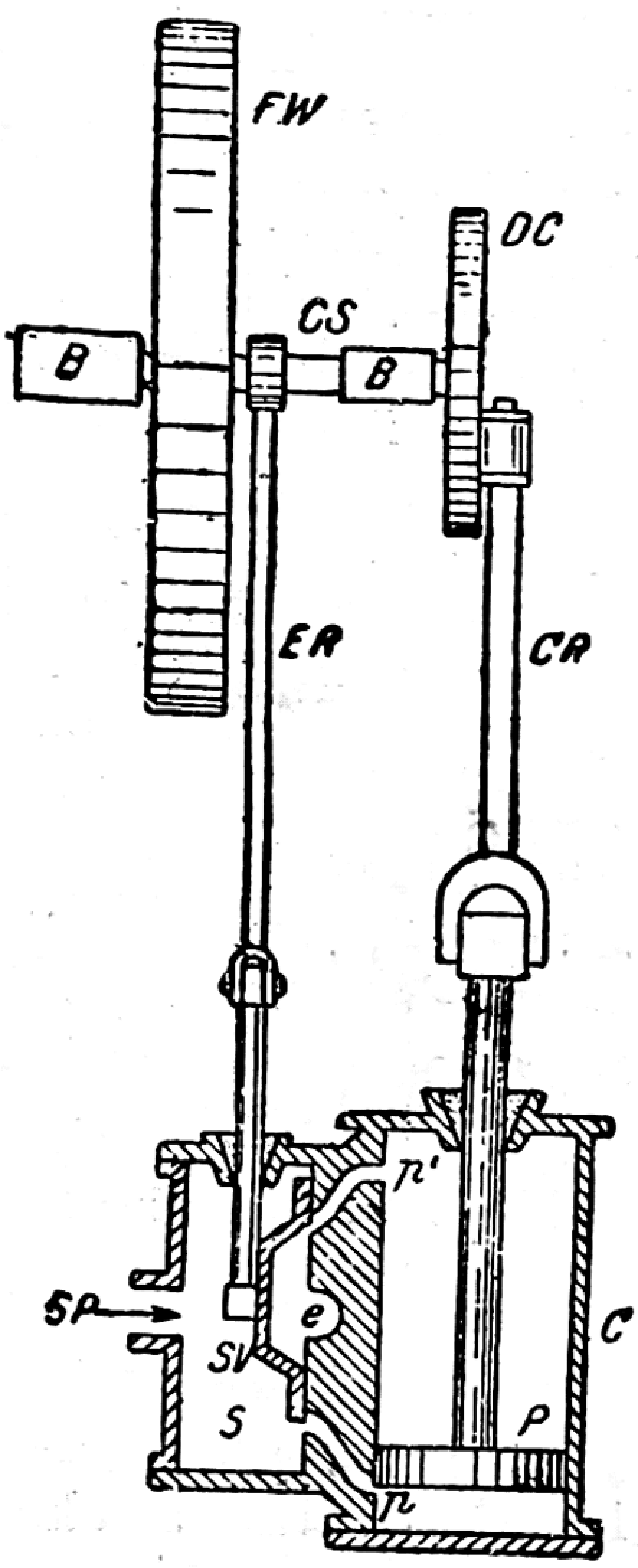

Fig. 62.
C=Cylinder
P=Steam-tight piston
S=Steam chest
SP=Steam pipe
SV=Slide valve
CR=Connecting rod
DC=Disc crank
B=Bearing
CS=Crank shaft
FW=Fly-wheel
p and *p'*=Steam ports
e=Exhaust port
ER=Eccentric rod

(*b*) The steam chest *S* into which live steam comes from the boiler at a pressure of about 10 atmospheres. From *S* the steam passes into the cylinder through the steam port *p* or *p'*, whichever of them is open.

(*c*) The exhaust port *e* through which the steam at a lower temperature is forced out in the exhaust pipe.

(*d*) The slide valve *SV* which is moved up and down by the eccentric rod *ER*.

To understand the action of a steam engine let us suppose that the piston is just beginning the upward stroke, and the sliding valve has just begun to open the lower steam port *p* admitting steam into the cylinder. As the piston *P* moves up, the valve opens the port to its full extent, and the steam present above the piston is forced out through *p'* into the exhaust pipe. The upward movement of the piston causes the flywheel *FW* to rotate. As the wheel rotates, it causes the sliding valve *SV* to move down by means of the eccentric rod *ER* and close the port *p* at an early stage of the upward stroke of the piston (it may be as much as $\frac{1}{4}$ or $\frac{1}{3}$), and thereby cut off the supply of the steam to the cylinder. At this stage both the ports are closed by the sliding valve. But since the pressure of the steam below the piston is greater than the pressure above it, the steam *expands* and becomes cooler thereby. In expanding it pushes the piston through the rest of the stroke. When the piston is at the top, the upper port *p'* is opened by the sliding valve, admitting the steam above the piston which pushes the piston down. When the piston begins to move down the dead steam from below is driven out through *e*. Thus we see that the steam acts on both sides of the piston, alternately working it upwards and downwards in the cylinder. An engine in which the steam acts on both sides of the piston is called reciprocating or double acting. If the steam acts only on one side, the engine is called single-acting.

Twice in each revolution the top of the connecting rod (*CR*) is at the same level as the crank-shaft (*CS*). At these positions which are called the "Dead centres," the piston exerts no turning effect on the shaft. In order to enable the crank-shaft to rotate smoothly without a jerking motion past these positions, a **flywheel** is fixed to the

shaft, the large inertia of which makes the motion quite uniform.

It must be remembered that it is not the steam which drives an engine, but the *heat which is* in the steam, for when the steam expands, it cools, and the heat which is made free is converted into the mechanical work. It is for this reason that the sliding valve closes the port after the piston has moved through $\frac{1}{4}$ or $\frac{1}{3}$ of its stroke so that the steam may expand enormously and the fall in temperature may be considerable. The greater the difference in temperature of the incoming steam and the exhausted steam, the greater the efficiency of the engine. Steam engines (stationary) possess an efficiency of 17 to 18 per cent whereas the best locomotives are only 12 per cent efficient, that is to say, they convert only 12 per cent of the heat produced by the combustion of coal into mechanical work.

243. Work done by the Steam in an Engine.—To calculate the work done by steam in an engine let us suppose that the piston just begins its upward stroke. The pressure which the steam exerts on the lower side of the piston instead of remaining constant falls off continuously* as the piston moves up. Further, there is some pressure on the upper side of the piston as well, due to the presence of the dead steam. These facts make it difficult for us to estimate the work done by the steam. To simplify the calculations it is usual to suppose that there exists an average effective pressure on the piston acting in the upward direction during the upstroke, and in the downward direction during the downstroke. With this assumption, work done by the steam in a double-acting engine per minute is $2PAln$ and the horse power is

$$\frac{2PAln}{33{,}000},$$

where P is the average effective pressure of steam *in lb. per square inch*, A the area of the piston in *sq. inches*, l the length of the stroke measured one way *in feet*, and n the number of revolutions made per minute.

Example.—A reciprocating steam engine has a mean effective pressure of 50 lb. per sq. in., the length of the stroke is 6 inches, the diameter of the piston is 5 inches, and the engine makes 500 revolutions per minute. What is the H.P. of the engine?

$$\text{H.P.} = \frac{2\times 50\times \frac{3{\cdot}142\times 25}{4}\times\frac{1}{2}\times 500}{33{,}000} = 14{\cdot}87.$$

243a. Steam Turbine.—The reciprocating steam engine is suitable for small factories or ships or locomotives. It is exceedingly reliable and robust and works well both at low and moderately high speeds. But for very high speeds or big factories or big ships the steam turbine is used. It converts 34 to 36% of the heat energy of the steam into work. Since in recent years the use of turbine has become popular it will be worthwhile for the student to understand its principle and construction.

*For as said above, the steam is cut off at an early stage of the stroke, and as the steam expands its pressure goes on falling.

Principle.—One of the earliest forms of the steam turbines is shown in Fig. 63. It consists of a wheel having a large number of curved blades fixed to its rim. The steam is blown against these blades through a nozzle in very much the same manner as water is directed against the buckets of a Pelton wheel. The impact of the steam pushes the blades forward at a very great speed. It is quite common in a turbine of this type for the wheel to run at 15,000—25,000 revolutions per minute. Such speeds are inconvenient and cannot be used directly to run machines. To reduce the speed the steam is allowed to expand in stages, each stage involving a small drop of pressure. With such arrangement the speed is less though the power developed is the same. This method of distributing the fall of pressure over a number of rings of blades is met with in the form of turbine developed by Sir Charles Parsons and called after him Parsons Turbine.

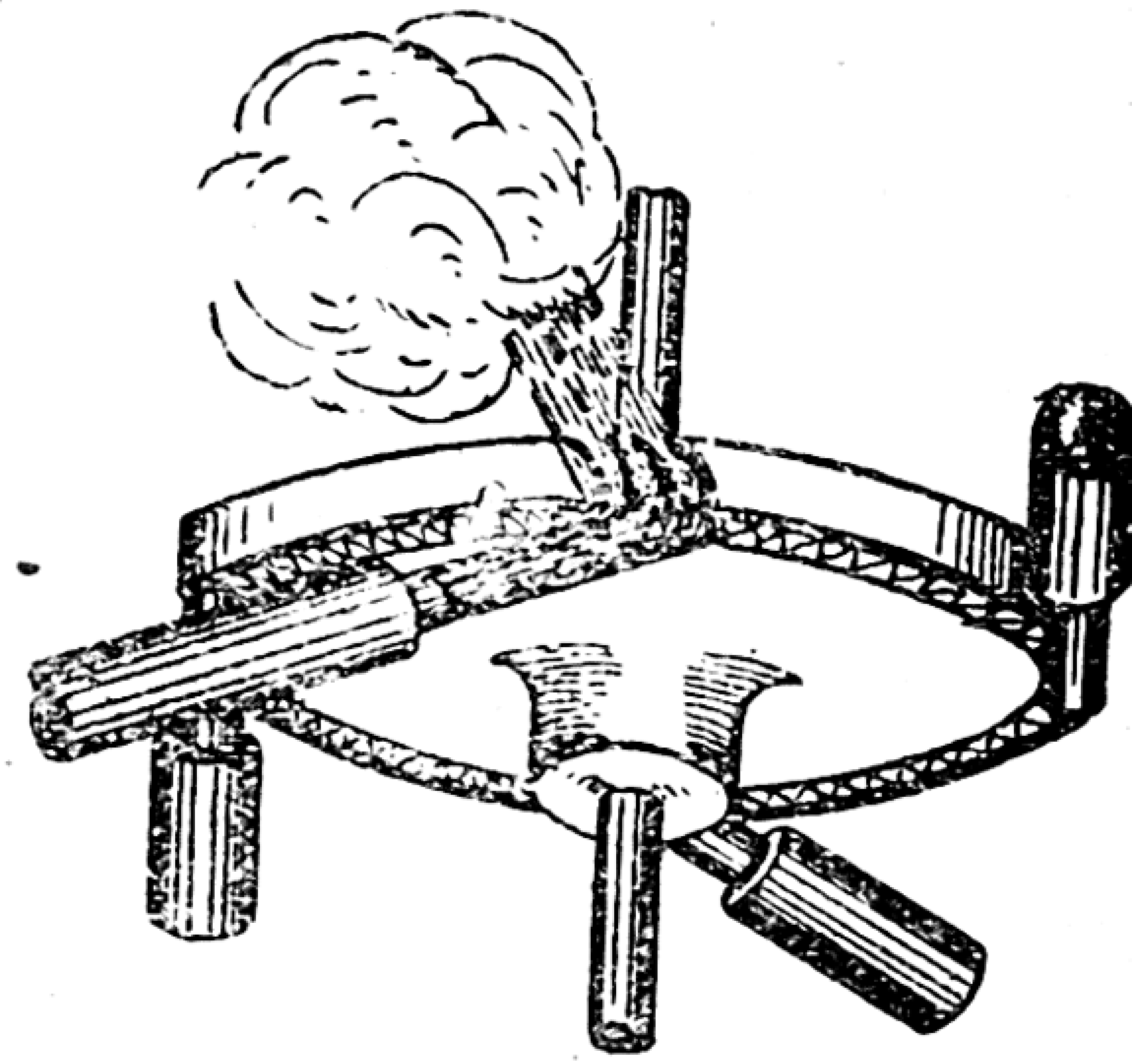

Fig. 63.

It consists of a long rotating shaft (called Rotor) having mounted on its entire length rings of curved blades. The shaft or rotor turns in a casing also set with blades, called guide blades, which are fixed approximately at right angles to those on the shaft. Fig. 64 shows the blades as they would look if the casing were transparent. The dark blades are guide or fixed blades whereas the shaded blades are rotor blades.

The steam enters at the narrow end of the turbine and meets the first ring of guide blades which deflect the steam against the first ring of rotor blades. The steam in expanding exerts thrust on these blades and causes them to spin round along with the shaft. From the first ring of rotor blades the steam passes to the second ring of guide blades and from them to the second ring of rotor blades. Here too the steam gives a kick to the blades and causes the rotor to spin round. This continues all the way as the steam passes along the space between the casing and the rotor. The casing and rotor are enlarged by steps so that the steam has room to expand and thus have its energy abstracted from it. The blades too, become larger and larger as steam passes on. In a modern turbine there may be as many as 1,500 rotor blades and the largest blades may be as big as 38 inches.

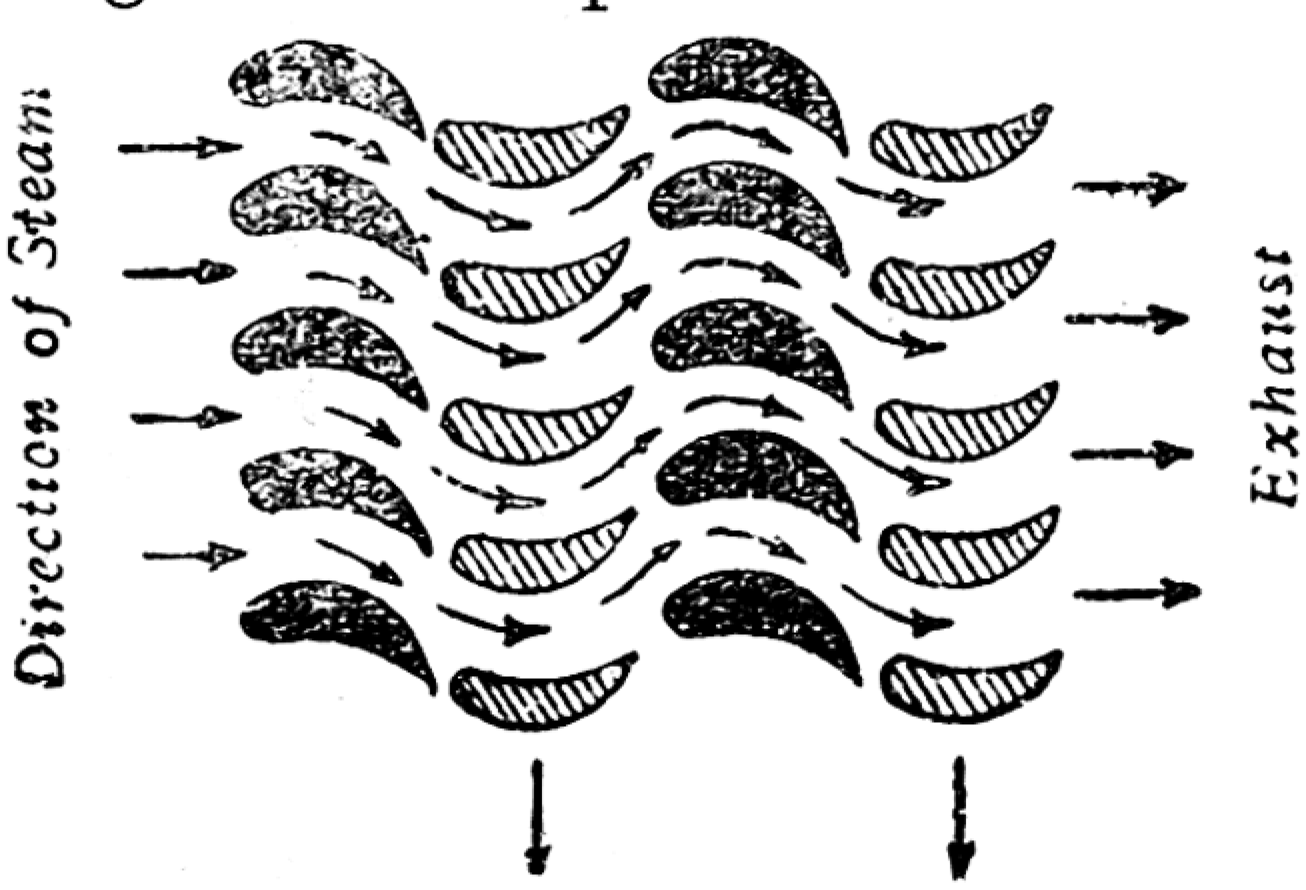

Fig. 64.
Direction of motion of blades.

The steam turbine is more efficient, takes less space, requires little attention and gives less vibration than a reciprocating steam engine. As a result of these advantages its use in ships as well as on land is becoming frequent.

244. Internal Combustion Engine.—In the case of a steam engine or a steam turbine the fuel is burnt under a boiler to produce steam which is the working substance and which is conducted to the cylinder of the engine or to the turbine through pipes. But in the case of a petrol or Diesel engine the fuel is burnt in the cylinder of the engine itself and the hot products of combustion are themselves the working substance. In other words, the steam engine or turbine is an external combustion engine whereas the petrol or a Diesel engine is an *internal combustion* engine.

Compared with the steam engine the internal combustion engine possesses many advantages. It is lighter and occupies less space, for no boiler is required. There is no loss of heat such as that occuring in a steam engine from the boiler or the steam pipes. It is free from smoke. It can be started at a moment's notice and does not involve the employment of men for handling fuel. Moreover, it utilizes about 25 to 40 per cent. of the available heat, whilst the steam engine converts about 12 to 18 per cent. of the heat into useful work. It is on account of these reasons that the internal combustion engines have become so important during the recent years. As a matter of fact the invention of the internal combustion engine has revolutionized the methods of locomotion. The automobiles and the aeroplanes owe their successful working to these engines.

We shall consider only two types of internal combustion engines, the petrol engine and the Diesel engine.

Petrol Engine.—In this engine, a mixture of vapour from petrol and air is exploded in the cylinder by an electric spark. In the form commonly used the complete cycle of operations can be divided into four stages and the engine is called four-stroke engine. We shall explain briefly how such an engine works.

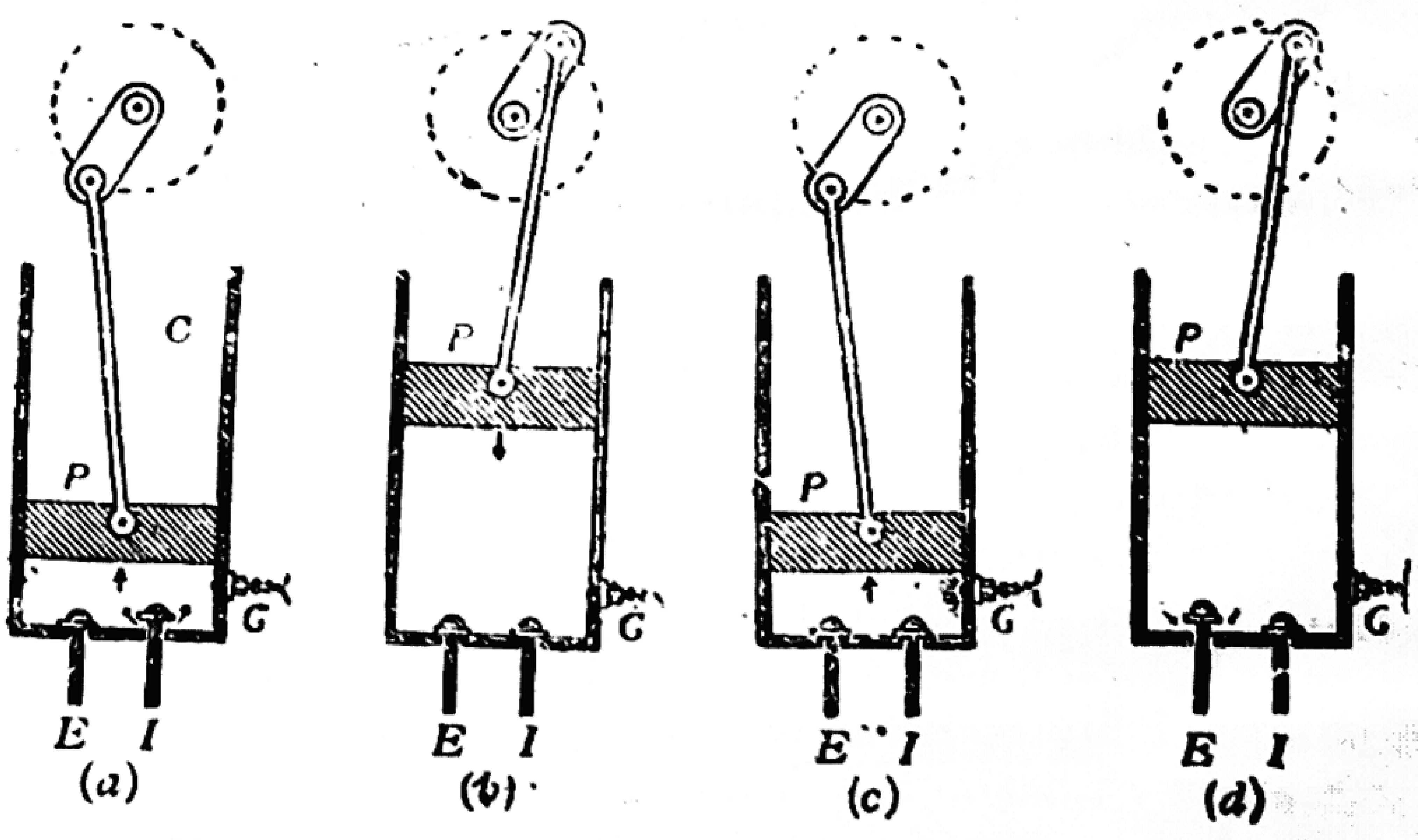

Fig. 65.

There are two valves in one end of cylinder (*C*), an inlet valve (*I*) which admits into the cylinder a mixture of the petrol vapour and air, and an exhaust valve (*E*) which allows the burnt gases to escape into the atmosphere. A spark plug (*G*) is inserted into the cylinder to ignite the mixture. To start the engine the crank shaft is turned by

rotating a handle* making the piston P move outwards and suck in a mixture of petrol vapour and air through the valve (I) into the cylinder. This stroke is called the *charging* or *suction stroke* [Fig. 65 (a)].

When the cylinder has been filled, the valve I closes and the piston moves inwards and compresses the mixture of air and petrol vapour to about $\frac{1}{5}$th of its original volume. This stroke is called the *compression stroke*. During this stroke both the valves remain closed [Fig. 65 (b)].

When the compression stroke is near its end, an electric spark is produced by the sparking plug, thereby igniting the mixture. As a result of this explosion the piston is driven outwards, performing useful work by imparting energy to the fly-wheel. This is called the *explosion* or *working stroke* [Fig. 65 (c)].

The rotating fly-wheel pushes the piston inwards, and the burnt gases are forced out of the cylinder through the exhaust valve E. This is the fourth stroke. It is called the *exhaust stroke*. Then the whole cycle repeats itself.

Since it is only during the explosion or the working stroke that the power is supplied to the fly-wheel for doing useful work, the fly-wheel is made heavy, so that energy stored in it may keep the engine running during the remaining three strokes. In motor car and aeroplane engines where it is not possible to carry heavy fly-wheels, four, six, eight, or twelve cylinders are used to drive the crank-shaft, and it is so arranged that the cylinders deliver power to the crank-shaft one after the other.

We have seen that the explosion takes place in the cylinder. It is evident, therefore, that unless some arrangement is made to cool the cylinder, the valves, the piston and other working parts, they will expand on account of the heat of explosion and cease to work satisfactorily. In addition to this the cylinder will become so hot that the gaseous mixture would explode immediately on being admitted into it. Hence the necessity of cooling the cylinder. In the case of motor cars the cooling is done by surrounding the cylinder with a jacket in which water is kept circulating whereas in the case of æroplanes to reduce weight the engines are air cooled.

Diesel Engine.—We shall now describe another form of an internal combustion engine called *Diesel engine* which is not only more efficient than petrol engine but is also cheaper because the fuel used in it is crude oil or petroleum residue (*i.e.*, the heavier oils remaining after all the petrol has been distilled off from natural oil), which is very much cheaper in cost as compared with petrol and does not so easily catch fire. It has still another advantage over the petrol engine in that it is more reliable, because there is no electrical arrangement for ignition, which, as every owner of a motor car knows, is perhaps the least reliable part of this engine. This engine was invented by Rudolph Diesel in 1895. Usually it is a fourstroke engine like the petrol engine.

(1) In the *first stroke* air is drawn into the cylinder.

(2) In the *second stroke* the air is compressed in the cylinder by a piston to a pressure of about 500 lb. per square inch. As a result of

*Or by an electric motor worked from accumulators. The arrangement is called self-starter.

this compression, temperature of the air rises above 600°C. which is higher than the ignition temperature of the fuel.

(3) In the *third stroke* the fuel valve opens and the oil under pressure* is blown in the form of a spray into the compressed air. On account of the high temperature of the air the oil at once begins to burn. There is no sudden explosion as in the petrol engine; on the other hand, the combustion here is a slow and continuous process. The gases produced as a result of burning exert a large pressure on the piston which is consequently driven outwards. This is the *working stroke.*

(4) In the *fourth stroke* the exhaust valve opens and the waste gases are expelled

The student might ask as to why a Diesel engine is more efficient than a petrol engine. The answer is that the efficiency of an engine depends upon the compression ratio of initial to final volume in the compression stroke. In a petrol engine this ratio is between 5 and 6, whereas in a Diesel engine it is between 12 and 15. On account of this high compression ratio the efficiency of a Diesel engine is as high as 40% as compared with 24% of a petrol engine and 10 to 18% of a steam engine.

On account of the high pressure developed in the cylinder it is necessary that the cylinder should have thick walls. This makes a Diesel engine very heavy as compared with a petrol engine. Hence Diesel engines were formerly made only in stationary units. But on account of less fire risk and higher efficiency and cheap cost of fuel their use in locomotives, ships, and buses is becoming increasingly popular these days.

EXERCISES

1. A waterfall is 50 metres high. The water falls below on a perfectly non-conducting surface. How much warmer will it be after the fall?

Let us see how much work is done by gravity on one gm. of water. The potential energy at the top will be mgh, or $1 \times 981 \times 50 \times 100$ ergs.

But we know that $4 \cdot 2 \times 10^7$ ergs of work must be expended in producing 1 calorie of heat, hence

$$981 \times 5 \times 10^3 \text{ ergs of work will produce } \frac{981 \times 5 \times 10^3}{4 \cdot 2 \times 10^7},$$

or
$$\frac{981 \times 5}{4 \cdot 2 \times 10^4} = 0 \cdot 117 \text{ calorie (approx.)}$$

Since the ground underneath is perfectly non-conducting, it does not take any heat from the water. Therefore every gram of water rises in temperature through 0·117°C., the sp. heat being one.

2. What is meant by the statement that heat is a form of energy?

A ball of iron (sp. heat=0·1) has its temperature raised through 0·6°C. on account of a fall through 25 metres. Calculate the value of J.

(*U. P. Intermediate Board.*)

Ans. $4 \cdot 1 \times 10^7$ ergs. per calorie.

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*Oil is compressed to a pressure of 800 pounds per square inch before it is blown into the cylinder.

3. A cylindrical tube 96 cm. long made of a non-conducting material and closed at both ends contains 500 grams of lead shot (sp. heat=0 03) which, when the tube is held vertically, occupy 6 cm. of the tube length. The tube is suddenly inverted and the shot fall to the other end of the tube. The tube is then again quickly inverted and this process is repeated 20 times. At the end the temperature of the shot is found to be raised through 1·4°C. Find the value of J. *Ans.* $4\ 2 \times 10^7$ ergs. per calorie.

4. Find the velocity of a lead bullet which warms itself upto 250°C. on striking an unyielding target. The whole of the heat generated by the impact is supposed to heat the bullet. Take the specific heat of lead as 0·03 and J as $4{\cdot}2 \times 10^7$ ergs per calorie. *Ans.* $2\ 5 \times 10^4$ cm/sec.

5. A raindrop falls on the ground from a height of 210 metres. What will be the increase in its temperature when it is stopped by the ground ? Suppose that $4{\cdot}2 \times 10^7$ ergs produce 1 calorie of heat. *Ans.* 0·491°C (approx.)

6. A block of ice at 0°C. is dropped in a well of water (the water also being at 0°C.) from a height. Find out what the height should be so that $\frac{1}{100}$ of it may be melted. Suppose that $4{\cdot}1 \times 10^7$ ergs produce 1 calorie of heat. *Ans.* 342·5 metres.

7. Describe the principle and action of (*i*) a steam engine, (*ii*) a petrol engine, and (*iii*) a Diesel engine.

One pound of anthracite coal when burnt can heat 14,000 lb. of water through 1°F. What is the efficiency of a locomotive if it consumes 2 lb. of coal per hour per horse power ? *Ans.* About 9 per cent.

8. In a double acting steam engine the average pressure was 40 lb. per square inch, the length of the stroke was 12 inches, the number of revolutions per minute 300, and the area of piston 125 sq. inches. Find the horse power of the engine. *Ans.* 90 9 H.P.

9. Describe an internal combustion engine.

Why does a one-cylinder stationary Diesel engine of the four-stroke need a heavy fly-wheel ?

10. Can you tell how many revolutions per minute the fly-wheel of a single cylinder four-stroke engine is making by counting the puffs of the exhaust ? If so how ?

11. A steam engine consumes 330 lb. of coal per hour. The calorific value of the coal is 12,000 B. Th. U. per lb. The horse-power developed is 125. What is the efficiency of the engine and boiler together ? *Ans.* 8%.

12. In Britain and the United States the horse power rating of a motor car engine is found from the relation, $\text{H.P} = \frac{2Nd^2}{5}$ where N is the number of cylinders and d is the bore in inches of each cylinder. Find the bore of (*i*) a 10 H.P. and (*ii*) a 40 H.P. four cylinder motor car. *Ans.* (*i*) 5/2 inches, (*ii*) 5 inches.

PART III

SOUND

CHAPTER I

Transverse and Longitudinal Wave-Motion

245. When a piece of stone is dropped into a pond of water, ripples travel outwards from the point where the stone strikes the surface. To a cursory observer it seems that water is travelling out from the point, but actually this is not the case. If a piece of cork be thrown at the surface it is found that when the waves reach it, it simply moves *up and down*, and the movement ceases when the waves have passed off. The position of the cork with respect to the centre of the disturbed area remains the same. This shows that it is not the water but the *disturbance* which travels outwards. This is an example of what is called a **wave motion.** It may be defined as a *form of disturbance which travels through a medium and is due to the repeated periodic motion of the parts of the medium about their mean positions, the motion being handed on from one part to the next.* Before we discuss how wave-motion is produced by the repeated periodic motion of the particles, let us first study the periodic motion of a single particle.

246. Periodic Motion.—It is a kind of motion in which a body continually returns to the same condition of motion after equal intervals of time. The motion of the earth round the sun or of the moon round the earth is, for example, periodic. There is, however, a special type of periodic motion which is of extremely great importance in science. It is called *simple harmonic motion.* In it a body moves backwards and forwards about a mean position. The motion of a simple pendulum is an example of such a motion. The motion of bodies producing sound is also in most cases simple harmonic. Hence it will be useful to know what is meant by simple harmonic motion.

Consider a particle P (Fig. 1) moving with a uniform speed round a circular path $ABA'B'$. As P moves in the direction of the arrowhead from A and completes a revolution, the foot N of the perpendicular drawn from P on the diameter BB' moves from O to B, from B to B' and back from B' to O. This to and fro motion of the point N along BB' is known as Simple Harmonic Motion. We usually define—

Simple harmonic motion as the projection of a uniform circular motion on a diameter of a circle.

The period of the motion of N is equal to the time taken by N to make one complete vibration along the diameter BB' to travel from O to B, from B to B', and back from B' to O. It is also equal to the time taken by P to travel completely round the circular path.

The *Phase* of the motion is the fraction of the time period that has elapsed since N last passed through its mean position in, say, the upward direction or since P passed last through the point A. Sometimes it is expressed in terms of angles. For instance, the phase of the motion when the particle is at B is 90° or $\frac{T}{4}$. The number of complete vibrations performed in a second is called the *frequency*. If T be the time period, the frequency $n=\frac{1}{T}$.

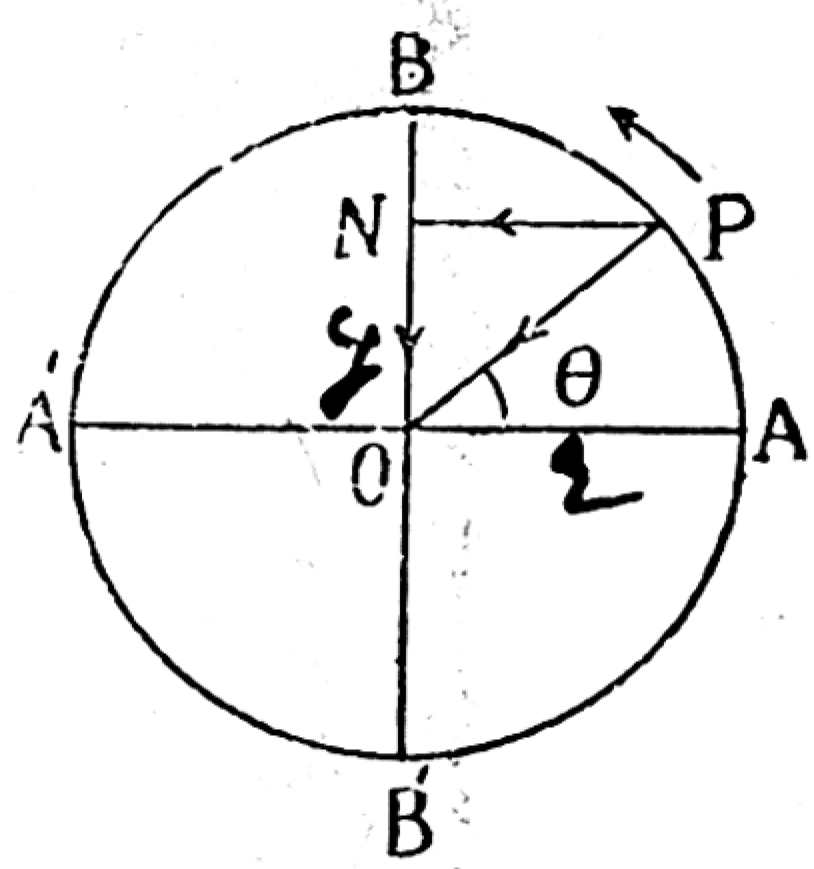

Fig. 1.

Let us now consider the characteristics of a simple harmonic motion.

The particle P has an acceleration $\frac{v^2}{r}$ acting inwards along PO, where v is the speed with which P moves round the circle and r is the radius. We can resolve it into horizontal and vertical components. The acceleration of N is equal to the vertical component only, for the horizontal component cannot affect motion in the vertical direction. The vertical component of the acceleration of $P=\frac{v^2}{r}\sin\theta=\frac{v^2ON}{rOP}=\frac{v^2y}{r\,r}=\frac{v^2}{r^2}y$, where y is the distance of N from the mean position, O.

Hence the acceleration of $N=\frac{v^2}{r^2}y$ and is directed towards the mean position.

Since v and r are both constant quantities, $\frac{v^2}{r^2}$ is also a constant quantity ; let us call it k.

$\therefore$ the acceleration of $N=ky$.

This shows that the acceleration of N at any instant is proportional to its distance at that time from the mean position. *We have said above that the acceleration is always directed towards the mean position.*

This enables us to define the simple harmonic motion in the following alternative manner. *When a body moves so that its acceleration is always directed towards a certain point and varies directlas its distance from that point the body is said to move with* **simple harmonic motion.**

The velocity of N is equal to the vertical component of the velocity of P because the horizontal component cannot produce any effect in the vertical direction. It is clear from the figure that when P is at A, the vertical component is equal to v itself and hence the velocity of N when it passes through O must be v. When P is at B, the horizontal component is equal to v and the vertical component is zero. Hence the velocity of N at B is zero. At intermediate points the value lies between v and zero. This shows that the speed

of N during the to and fro motion varies from point to point; it is maximum at the mean position and zero at the extreme points.

We have considered above the motion of the particle which is moving in a simple harmonic manner. If a large number of particles execute simple harmonic motion differing in phase regularly as we go from one particle to the next we get wave-motion. We shall discuss this point later on.

247. Types of Wave-motion.—Let us go back to the case of waves which are formed in water. As soon as a stone strikes the surface of water it causes a depression. The water tries to recover its original level (due to elasticity), and consequently flows into the depression. But owing to the kinetic energy of its particles it overshoots the mark, with the result that the depression is followed by an elevation which is again followed by a depression, and so on. As a result of this water-particles begin to move up and down. Due to cohesion the neighbouring particles are also affected, with the result that they in turn reproduce exactly the same motions. This explains how the wave motion is set up in a pond. From what has been said above it is clear that *for the production of wave-motion we require a periodic motion of the particles one after the other.*

When a body suffers disturbance at one part, the whole of it does not become immediately affected, but on the other hand the disturbance is handed on from one layer of particles to the next. Suppose we give a blow with a hammer to one end of a wooden rod held horizontally. The blow causes compression at the place where the hammer strikes, leaving the remaining part unaffected. The compression, however, quickly moves forwards. As soon as the force which produces the original disturbance is removed, the particles first affected try to resume their normal state, but in doing so, owing to inertia, overshoot the mark and go to the other extreme, with the result that a wave of compression is followed by a wave of rarefaction, or depression in case of water is followed by elevation. These opposite waves succeed each other rapidly; the amplitude, however, of each successive wave is smaller than that of its predecessor. This goes on till the whole of the body resumes its normal state.

We have seen that the particles move about their mean position while the disturbance moves forward. Now either the particles may move up and down at right angles to the direction in which the waves travel as in the case of water, or they may move to and fro about their mean position in the same direction in which the waves move as in the case of a rod. In the first case wave-motion is due to transverse vibrations, and the disturbanceis sa id to be **transverse wave-motion**; and in the second case the wave-motion is due to longitudinal vibration and the disturbance is said to be **longitudinal wave-motion.** We have given one example of each kind; let the student think out for himself other examples of transverse and longitudinal wave-motion. We

248. Transverse Wave-motion.—Let us consider in a somewhat greater detail the resultant motion when a large number of particles of a medium are executing periodic motion, the phases of the neighbouring particles being related to each other by a certain relation which, in

simpler language means that the disturbance reaches the particle turn by turn and that the time taken by the disturbance in going from one particle to the next is the same for any two neighbouring particles.

Suppose we have a number of particles (say 9) arranged originally at equal distances along a line [Fig. 2. (*i*)]. Imagine that the transverse vibratory motion begins at A. This motion, on account of the cohesion between the particles, will be handed on from one particle to the next with a certain phase retardation, say $\frac{1}{8}$th of the time-period. It is clear from what is said above that the disturbance will go from one particle to the next after $\frac{T}{8}$ sec. This phase retardation will be the same for any pair of neighbouring particles.

Initially the particles are all on the same line AB, as shown in Fig. 2 (*i*). After $\frac{T}{8}$ sec., the particle 1 is displaced upwards, whereas No.2 is just affected [see. Fig. 2(*ii*)].

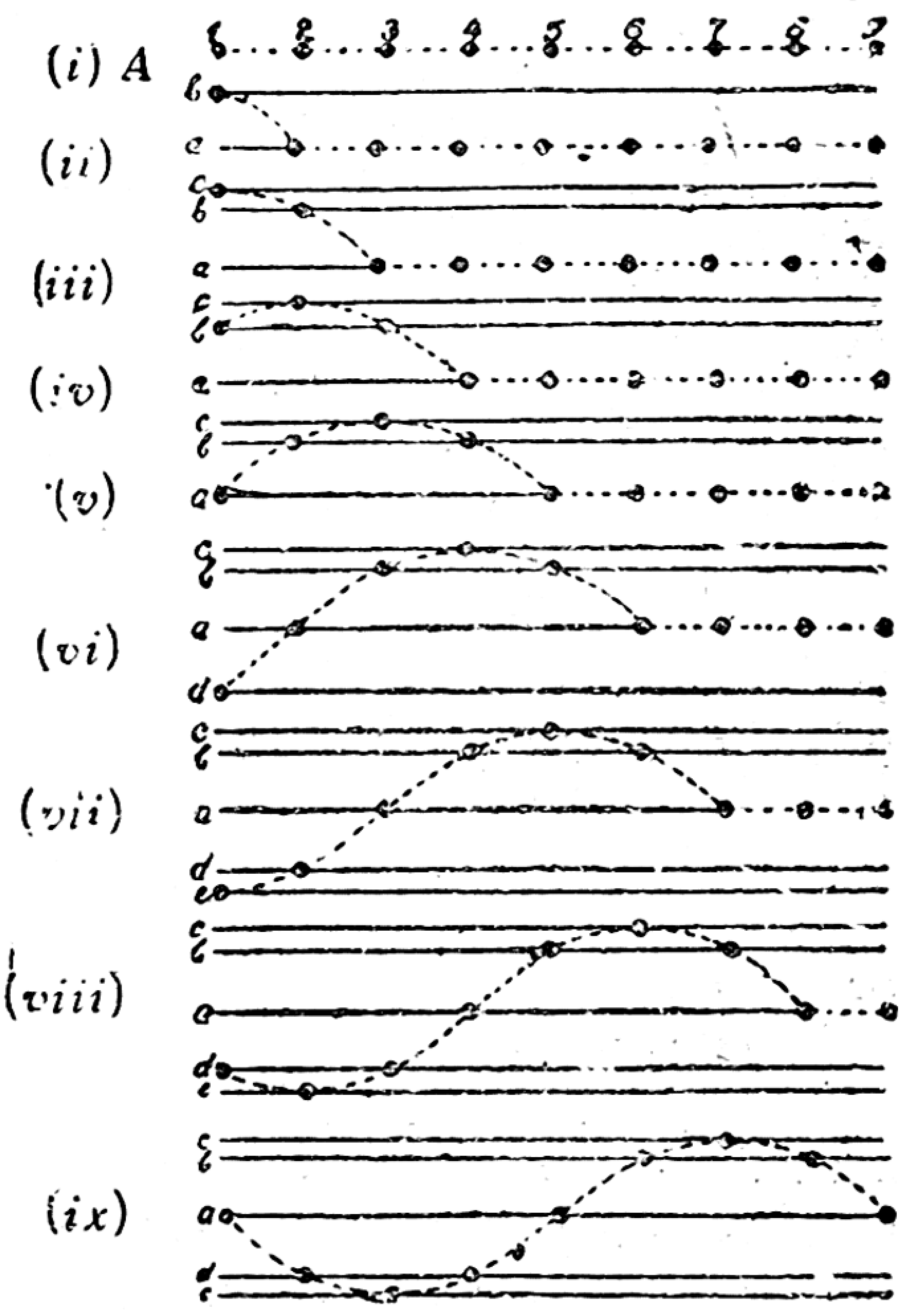

Fig. 2.

Since the velocity of the particle is not uniform, being maximum at the mean position and zero at the point of maximum displacement, the distance passed over by the particle during each fraction of $\frac{T}{8}$ sec. will not be the same. It will, obviously be greater when the particle is passing through the mean position than when it is near the point of maximum displacement. Suppose for the sake of simplicity that during $\frac{T}{8}$ sec. when the particle is near its mean position, it travels three times the distance which it travels when near the farthest point. Graphically, we can represent this by drawing two lines b and c at such distances from the line a that the distance between b and a is three times as big as that between b and c.

After a period $\frac{2T}{8}$ $\left(i.\ e.,\ \frac{T}{4}\right)$ sec., the first particle will go over to maximum distance from the mean position, *i.e.*, to the line c,* Fig. 2 (*iii*), the second particle will just reach the line b, whereas the third will be just affected. After $\frac{3T}{8}$ sec., the first particle will be back on the line b, particle No 2 will be at the maximum distance, *i.e.*, will be on

*After $\frac{T}{4}$ second the particle must reach its extreme position, corresponding to the position B or B' of the particle N (Fig. 1).

the line c, and the particle No. 3 will be on the line b on its outward journey whereas particle No. 4 will be just affected. This is shown in Fig. 2 (*iv*). When half the period is over, the particle No. 1 will be at its mean position but with a tendency to go downwards, the particle No. 2 will be on line b on its backward journey, the particle No. 3 will be at the maximum distance (line c), the particle No. 4 will be on the line b, but on the outward journey, particle No. 5 will be just affected. Similarly, the changes in the position of the other particles can be traced easily from Fig. 2. It will be seen that after a complete time-period the particle No. 1 is just back in its mean position with a tendency to move upwards in which direction it started at first. Particle No. 9 is also just about to move upwards, these two particles are said to be in the *same phase* ; and particles Nos. 1 and 5 are said to be in *opposite phases* for although they have numerically the same speed, their direction of motion is opposite. The point at the maximum distance above the line of mean position is called a **crest** ; and that at the maximum distance below the line of mean position is called a **trough.** The distance *between the two nearest particles in the same phase* (*as particles 1 and 9*) *is called* **wave length.** It is also equal to the distance between two nearest crests or troughs. It is evident from what we have said above that the wave-length is the distance through which the disturbance travels during the time a particle makes one complete vibration. It is another way of defining it. Let us call the wave-length λ. Bearing in mind that $\frac{1}{T}=n$, when n is the frequency and T the time period, we can easily find a relation between the velocity with which a wave travels, its wave-length and time-period (or frequency) of the body producing wave-motion. For

$$\text{velocity}=\frac{\text{distance travelled}}{\text{time taken}}=\frac{\lambda}{T}$$

or

$$v=n\lambda.$$

This is a very important relation for the wave-motion in general.

We have assumed that there are nine particles in the row, but actually there is an infinite number of particles ; the increase in the number of particles, however, does not affect our reasoning in any way.*

249. Longitudinal Wave-motion.—It has already been said that when the particles vibrate to and fro about their mean position in the same direction in which the waves travel, the wave-motion is said to be longitudinal. It can be very easily demonstrated by giving a slight tap to a long helix at one end, when a wave of condensation will be seen to run along its length.

In gases, since there is no cohesion, the transverse waves are not formed at all, only the longitudinal type of wave-motion is possible. To understand that kind of wave-motion let us suppose that a number of particles, say 11, (Fig. 3) are arranged at equal distances along a line,

*A curve drawn as explained above will not be smooth, but in reality the curves for wave motion are perfectly smooth. It is due to the fact that we have considered the average velocity during $T/8$ sec. to be constant, whereas actually the velocity is different at each instant and goes on increasing or decreasing regularly.

the particles being capable of a to and fro motion about their mean positions along the line itself. Let us suppose, as before that each particle begins its movement *a little later* than the particle preceding it, or, in other words, that there is a certain phase relation between two neighbouring particles. Suppose that phase relation is $\frac{1}{8}$th of the time-period as in the case of the transverse motion. Since a particle does not move with a uniform velocity, we shall suppose that during $T/8$ sec., when a particle is near its mean position, it covers three times as much distance as it does when near the extreme position. Bearing this in mind, imagine particle No. 1 to be first displaced towards the right. At the end of $\frac{1}{8}$th of the time-period it will be at $\frac{3}{4}$th of the amplitude towards the right. Just at this time the second particle will be affected [see Fig. 3 (*ii*)]. After $\frac{2}{8}$th of the time-period is over, the particle No. 1 will be at the maximum distance to the right whereas No. 2 will be at

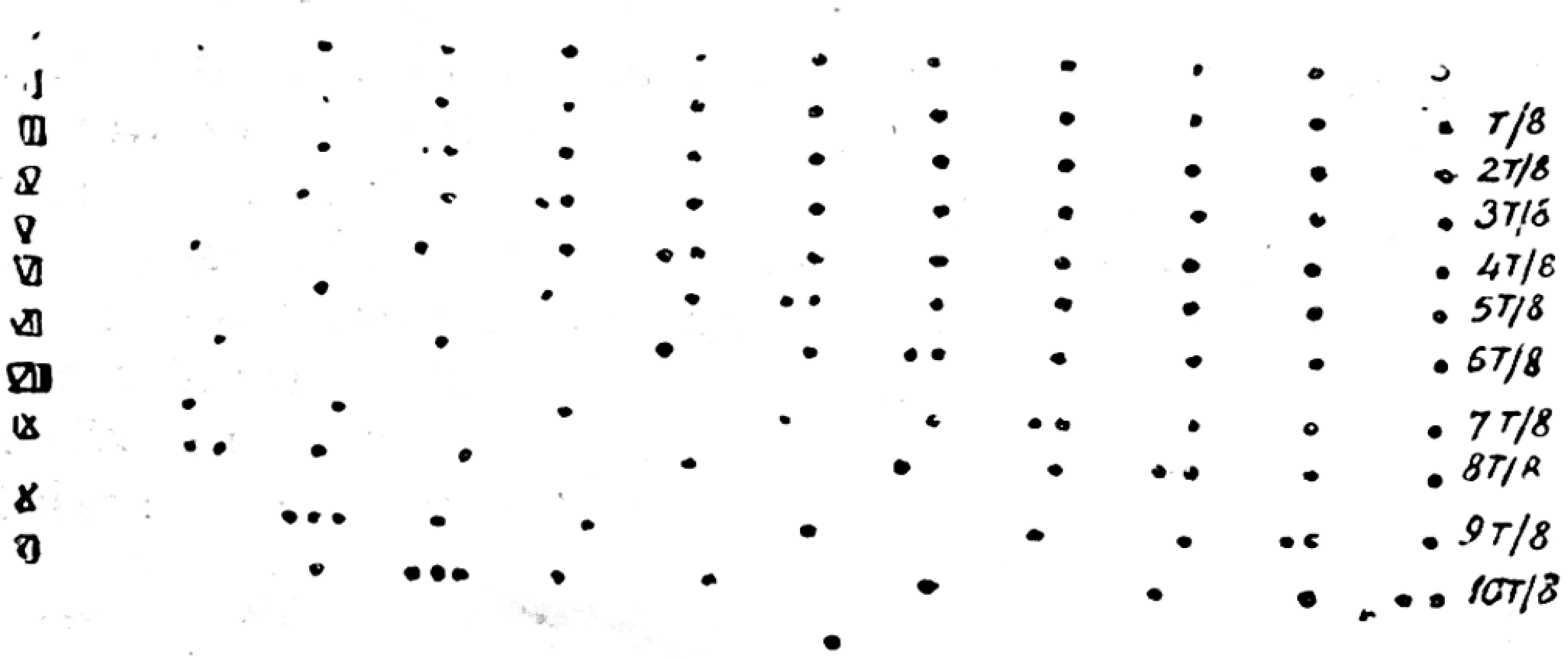

Fig. 3.

$\frac{3}{4}$th of the amplitude towards the right, and No. 3 will be just affected. It is shown in Fig. 3 (*iii*). After $3T/8$ sec. the particle No. 1 will be on its return journey and will be at $\frac{3}{4}$th of the amplitude to the right from the mean position, particle No. 2 will be at the maximum distance, particle No. 3 at $\frac{3}{4}$th of the amplitude but on outward journey, whereas particle No. 4. will be just affected. In this way we can go on tracing the transmission of disturbance from particle to particle. A glance at Fig. 3 shows clearly that after a complete time-period, the disturbance has reached the particles 7, 8 and 9 ; 1 and 9 being in the same phase. It should be noted that in this case, instead of getting crests and troughs, we get **condensations** and **rarefactions** of the particles. As before, *wave-length is the distance between any two nearest particles in the same phase.* It is also equal to the distance between the centres of any two nearest condensations and rarefactions.

Note that the overcrowding is maximum near the centre of condensation, and the distance between two neighbouring particles is greatest in the centre of rarefaction.

During the time the particle No. 1 completes the vibration, the disturbance just reaches the particle No. 9. Hence the distance between particles 1 and 9 gives one wave-length. As in the case of transverse wave-motion velocity of longitudinal wave-motion $=\dfrac{\lambda}{T}=n\lambda$.

Since it is longitudinal wave-motion with which we are chiefly concerned in Sound, the student is advised to understand clearly this type of wave-motion. We shall consider one more example to make clear as to how a wave of compression or condensation travels forward. Suppose three trucks and one engine are coupled together, and that, to begin with, they are stationary. If we take the length of a truck from buffer to buffer it is evident that when the buffers* are compressed by a push, they will shorten up and hence the length of a truck will become smaller. If the engine moves through, say, half a metre towards truck No. 1, the buffers between it and engine will be shortened up. As the springs shorten up they exert force on truck No. 1, and move it forward. As a result of it the springs on the side of the engine are relaxed to some extent, whereas the springs on the other side, *i.e.*, towards truck No. 2, are compressed. Truck No. 1 will not stop when the pressure on the two sides is equal but will continue to move forward on account of the kinetic energy that it possesses, till the springs between trucks (1) and (2) become compressed so much that their pressure brings truck (1) to rest, and moves truck No. 2 forward, compressing springs between trucks No. (2) and (3). This shows how compression travels forward from one truck to the next.

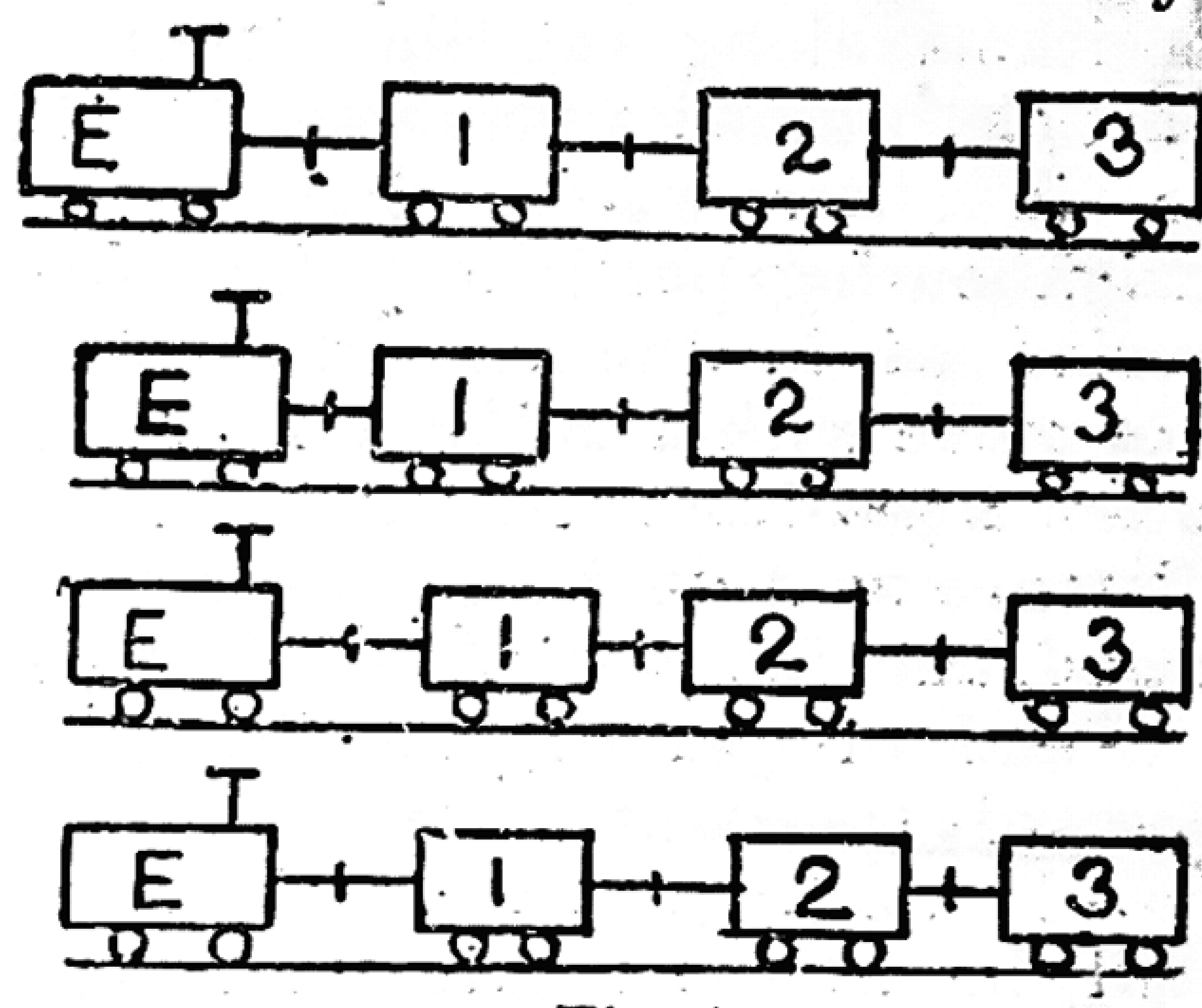

Fig. 4.

250. Characteristics of Wave-motion.—Let us carefully note the characteristics of a wave-motion. The first point to note is that it is the disturbance that travels forward and not any material particles or bodies. The second point is that the movement of a particle (or a body) begins just a little later than that of its predecessor. In other words, there is a definite phase relation between two neighbouring particles (or bodies). The third point is that the velocity of the wave-motion is entirely different from the velocity of bodies transmitting it. For instance, in our example of trucks, truck No. 1 moved 50 cm., whereas the disturbance or wave during that time moved through the complete length of the truck.

EXERCISES

1. Explain the meaning of the term amplitude, wave length, frequency and phase.

A sound produced in water has a wavelength of 5·8 metres. If the velocity of sound in water is 1485 metres/sec. find the frequency and wave-length of the note heard by an observer in air.

Ans. 256 ; 1·33 metres.

2. What do you understand by longitudinal wave-motion ?

When a train of transverse waves is passing along a row of parti-

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*Buffers contain springs.

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